Baryon number creation and phase transitions in the early universe

Hut, P.; Klinkhamer, F.R.

Astron. Astrophys. 106, 245–253 (1982)

ASTRONOMY AND ASTROPHYSICS

Baryon Number Creation and Phase Transitions in the Early Universe

P. Hut[1]* and F. R. Klinkhamer[2]

[1] Astronomical Institute, Roetersstraat 15, 1018 WB Amsterdam, The Netherlands
[2] Sterrewacht, Postbus 9513, 2300 RA Leiden, The Netherlands

Received June 10, accepted October 5, 1981

Summary. We aim at a consistent scenario for the generation of the baryon-antibaryon asymmetry in the early Universe. First we discuss recent calculations using unified interactions, especially how the required CP violation can be provided for. Secondly we consider finite temperature effects, namely the existence of phase transitions, i.e. changes in gauge symmetries in a cooling Universe. Although the expansion of the Universe can be affected significantly, the final baryon number is surprisingly insensitive to the nature of the phase transition, as numerically calculated in a simple model. Also we briefly discuss problems with monopole production and the generation of the density perturbations required for galaxy formation.

Key words: baryon number creation – early Universe – galaxy formation – cosmology

1. Baryon Number Creation

Grand Unified Theories (GUTs) have a single coupling constant g ($\alpha \equiv g^2/4\pi \sim 1/40$) at high energies ($> M_X \sim 10^{14}$–$10^{15}$ GeV) and have quarks and leptons in common representations, which may lead to baryon number violating reactions (for example $uu \to e^+ \bar{d}$). This symmetric (i.e. one simple gauge group G) theory is spantaneously broken (see Sect. 2) at high energies into the presently observed weak, electromagnetic and strong interactions. Low energy remnants of unified interactions predict proton ($p = uud$) decay into e^+ and $\pi^0 = d\bar{d}$, but with a large lifetime $\tau \sim \text{const}\ M_X^4/M_p^5 \sim 10^{31\pm 2}$ yr (Ellis et al., 1980a), because the relevant boson, generally denoted by X, received a large mass M_X by the symmetry breaking (for a review see Ellis, 1980). Here and in the following we take $\hbar = c = k_B = 1$, but keep $M_{Pl} \equiv G^{-1/2} = 1.22\ 10^{19}$ GeV.

It is well known that GUTs, which violate baryon number (B) conservation and the symmetries of charge (C) conjugation and charge + parity (CP) can produce a net baryon number density in the Universe during a period out of thermal equilibrium (Weinberg, 1979, and references therein). This last ingredient is provided by the mass of the X bosons ($Ta \neq \text{const}$) and the rapid expansion of the Universe [rate $H \equiv \dot{a}/a$, with $a(t)$ the scale factor; see Weinberg (1972)] compared to the interaction rates. In the simplest scenario the final ratio of densities of baryon number over entropy is produced by the decay of the lightest superheavy bosons (Nanopoulos and Weinberg, 1979)

$$n_B/s = \frac{45}{4\pi^4}\,\zeta(3)\,(N_X/N)\,\Delta B, \tag{1}$$

where ΔB is the average *net* baryon number produced in the decay of an $X-\bar{X}$ pair and N_X and N are the effective number of spin states of X and all particles (mass $< M_X$), respectively. This n_B will be generated at temperatures T_d (when $\Gamma_{\text{decay}}(T_d) \sim H(T_d)$) below the bosons mass ($T_d < M_X$) after a period of free ($\Gamma_{\text{decay}} < H$ for $T > T_d$) expansion giving the required non-thermal distribution ($n_X \sim n_\gamma \sim T^3 \neq n_{\text{therm}} \sim (M_X T)^{3/2} \exp(-T/M_X)$). The observed $n_B/s \sim 10^{-10}$ $(\Omega_{b0}/0.01)$ *thus requires* $\Delta B \gtrsim 10^{-8}$, since $N_X/N \sim 10^{-2}$ in typical GUTs, where the inequality is required if there are entropy increasing processes at lower temperatures (e.g. some super cooling in the Weinberg-Salam transition at $T \sim 300$ GeV, but see below).

ΔB is determined by the different branching ratios of X and $\bar{X}$ (see Appendix A), which come from higher order Feynman diagrams.

But in the minimal $G = \text{SU}(5)$ theory broken by *5* and *45* Higgs scalars (these numbers give the dimensionality of the representations) the produced $\Delta B \sim \Gamma_{\text{interference}}/\Gamma_{\text{total}}$ is too small [in Γ_{inter} is the imaginary part of the trace of 8 Yukawa coupling matrices (in generation space) f_i and in $\Gamma_{\text{total}} \sim \Gamma_{\text{tree}}$ the trace of $2f_i$'s (Barr et al. (1979)]. Already Nanopoulos and Weinberg (1979) had observed the need of different scalars in order to have a net ΔB within the numerator Im Tr of $4f_i$'s. In the Yukawa terms (i.e. scalar fermion-antifermion $f_i \bar{\psi} \phi_i \psi$) of the GUT Lagrangian density L there are two non-trivial unitary matrices in generation space ($n \times n$). These have a) $(n-1)^2$ parameters to be identified with the usual Kobayashi-Maskawa matrix (giving CP violation for $n \geqq 3$, i.e. $\geqq 6$ quark flavours) after breaking with a *5* of Higgs H ($\langle H \rangle = (0, 0, 0, 0, v_0)$ with $v_0 \sim 300$ GeV)), and b) $(n-1)$ parameters only observable at unification energies and essential for ΔB (Ellis et al., 1979). The simplest way to have a ΔB of the right magnitude is with two *5*'s of Higgs (Yildiz and Cox, 1979). Recently a direct connection was noticed (Ellis et al., 1981a) between the ΔB interference graphs and those leading to a finite renormalization ($\delta\theta_{\text{GUT}}$) of the θ parameter of the QCD (SU(3)) vacuum. (This θ defines the ground state when topologically distinct Yang-Mills vacua exist, analogous to the Bloch functions for periodic potentials.)

Assuming $\theta = 0$ at a high energy, say M_{Planck}, the present observational limits on the electric dipole moment of the neutron d_n, which gets a dominant contribution of the CP violating term with parameter θ (1 GeV) $\gtrsim \delta\theta_{\text{GUT}}$, allow for practically no entropy generation after the n_B generation at unification energies (Ellis et

---

Send offprint requests to: F. R. Klinkhamer

* Present address: Institute for Advanced Study, School of Natural Science, Princeton NJ 08540, USA

al., 1981a, b). The reasoning goes as follows: $d_n \gtrsim 4\ 10^{-16}\ \delta\theta_{\rm GUT}$ e-cm would violate the experimental upper limit ($2\ 10^{-24}$ e-cm) if the generated $\{n_B/s\}_{\rm GUT}$ had to be significantly larger than 10^{-10} to allow for later entropy generation. We remark that in comparing $\delta\theta_{\rm GUT}$ and ΔB graphs moduli of typical unitary matrix elements U_{dm} connecting the different contributing Higgses [see point b) above] were naturally assumed to be $O(1)$. If these are substantially smaller they could alleviate the nearly conflicting theoretical ($\delta\theta <$ const. $|U_{dm}|^2 n_B/s$) and observational d_n limits [see Eqs. (24) of Ellis et al. (1981a)]. Note that these arguments do not hold if the anomaly of a global $U(1)$ axial symmetry makes all θ equivalent to zero (Peccei and Quinn, 1977). Another possible alternative (see also the Note) for CP violating decays, namely mixing of X and $\bar{X}$ into X_a and X_b Hamiltonian eigenstates which are not CP eigenstates, is not allowed for superheavy bosons X coupled to the observed fermions, these bosons all being charged electrically (Kolb and Wolfram, 1980b; Nanopoulos and Weinberg, 1979). This needs not hold for a GUT with superheavy fermions (Barbieri et al., 1980).

Up till now we have considered the simple delayed decay scenario. More general calculations have been done, which show how truly primordial (quantum gravity epoch?) baryon-antibaryon asymmetry would be washed out at unification energies by a complicated interplay of decay ($X \to abc$), inverse decay ($a'b'c' \to X$) and X exchanging collisions ($ab \to cd$, $B(ab) \neq B(cd)$), after which a fresh asymmetry is created (Kolb and Wolfram, 1980b, Harvey et al., 1981; Fry et al., 1980). As expected (cf. Weinberg, 1979) these numerical solutions confirm the simple delayed decay estimates for small enough couplings of the relevant X to the fermions (e.g. $\alpha_{\rm eff} \lesssim 10^{-4}$, $M_X \sim 10^{14}$ GeV). If the lightest X decays with a larger $\alpha_{\rm eff}$ a considerable dilution of the generated baryon number takes place (Kolb and Wolfram, 1980b, Fig. 4). The reason that these Yukawa couplings could very well be smaller than the gauge coupling g is the following: these terms in $L_{\rm GUT}$ give at lower energies *simultaneously* with the Weinberg-Salam breaking to the weak and electromagnetic forces the observed fermion masses. This breaking (Higgs doublet with $\langle 0|\phi|0\rangle = v_0\binom{0}{1}$, $v_0 \sim 300$ GeV) gives the weak boson W the mass $M_W \sim g v_0 \sim 80$ GeV (cf. example in Sect. 2) and the small effective Fermi coupling $G_F = g^2/2M_W^2$. The Yukawa couplings $f\bar{\psi}_1\phi\psi_1$ give after spontaneous symmetry breaking a mass term for lepton l of $f v_0 \bar{\psi}_l \psi_l$, hence $f \sim m_l v_0^{-1}$ is very small (e.g. Taylor, 1976).

2. Phase Transitions

In Sect. 1 all calculations described were done in a flat space zero temperature formalism (as laboratory physics) with as only cosmological effect the use of a statistical ensemble of massive particles driven out of equilibrium by the expansion of the Universe. However an important phenomenon is not taken into account yet: symmetry restoration of spontaneously broken gauge theories at high temperatures $T_c \sim v$. Let us briefly explain the mechanism of spontaneous symmetry breaking (e.g. Taylor, 1976). If the effective potential $V(\phi_{c1})$ of the Higgs scalars ϕ (V is a function of the classical, i.e. not operator valued, fields ϕ_{c1}) has a minimum at $\phi_{c1} = v \neq 0$, than the vacuum expectation values is non-zero: $\langle 0|\phi|0\rangle = v$. The field theory is most simple with shifted fields $v \equiv \phi - v$, which behave properly, i.e. annihilate the vacuum $\langle 0|v|0\rangle = 0$. The Lagrangian as function of v, which remains renormalizable after the symmetry breaking, shows that the gauge fields corresponding to the broken symmetries [remember the gauge fields are $A_\mu = A_\mu^a t_a$, with t_a the (n^2-1) generators of the Lie group $G(=SU(n))$ describing the symmetry] have become massive, cf. M_W of Weinberg-Salam. No massless scalars appear, the Goldstone theorem being invalidated by the existence of long-range gauge interactions. Let us give the simplest example possible $G = U(1)$ (e.g. O'Raifeartaigh, 1979): we have a complex scalar field $\phi = \phi_1 + i\phi_2$, one gauge field A_μ ($G = e^i\phi$ has one generator) with field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, the gauge covariant derivative $D_\mu\phi = \partial_\mu\phi + igA_\mu\phi$ with coupling constant g and the Lagrange density (from which the equations of motion for A_μ and ϕ follow by functional derivation)

$$L = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}|D_\mu\phi|^2 - V(\phi), \tag{2a}$$

$$V(\phi) = -\mu|\phi|^2 + \lambda|\phi|^4, \qquad \mu, \lambda > 0, \tag{2b}$$

which is invariant under the $U(1)$ gauge transformations with *local* parameter $\Lambda(x)$ in inifinitesimal form

$$A_\mu(x) \to A_\mu(x) + \frac{1}{g}\,\partial_\mu\Lambda(x)$$

$$\phi(x) \to (1 - i\Lambda(x))\phi(x).$$

Rewriting L in fields $v = \phi - \bar{\phi}$ around the asymmetric ground state of the potential $|\bar{\phi}|^2 = \mu/2\lambda$ leads to massive gauge fields (the second term of L giving $\frac{1}{2}g^2|\bar{\phi}|^2 A_\mu^2$) with mass$^2 = g^2\mu/2\lambda$ and after a gauge rotation we have

$$-L = \frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\,g^2\,\frac{\mu}{2\lambda}\,A_\mu^2 + \frac{1}{2}\,|D_\mu v_1|^2 + 2\mu v_1^2 - L_{\rm interaction}(v_1, A_\mu). \tag{3}$$

Thus after spontaneous symmetry breaking we have a gauge invariant theory with massive vector fields A_μ and no massless scalars v_2 (the Higgs miracle). The precise form of $L_{\rm interaction}$ in Eq. (3) is crucial to keep gauge invariance, necessary for renormalizability. Note that we do have a massive scalar v_1 and for realistic G similar ones will turn out to be essential for the baryon number generation in the early Universe.

Presently it is thought that Nature uses the spontaneous breaking of

$$G_{\rm unified} \xrightarrow{M_U} SU(3)_{\rm colour} \times \{SU(2) \times U(1)\}_{\rm electroweak}$$

$$\xrightarrow{M_{WS}} SU(3)_{\rm colour} \times U(1)_{\rm electromagnetic}$$

(at low energies the 8 gluons and photon are still massless) at a hierarchy of energies $M_U \sim 10^{15}$ GeV and $M_{WS} \sim 10^2$ GeV (see Ellis, 1980). Finite temperature effects change the potential of Eq. (2b): $V(T, \phi_{c1}) = -\mu^2\phi_{c1}^2 + \lambda\phi_{c1}^4 + cT^2\phi_{c1}^2$, c is a constant (Weinberg, 1974). The non-zero minimum disappears at a critical temperature $T_c = c^{-1/2}\mu$. These symmetry restorations at high temperature resemble phase transitions (for a review see Linde, 1979), but for a small range around T_c the perturbation expansion used, giving the $cT^2\phi_{c1}^2$ term, breaks down.

The symmetry breaking will lead to a different energy density of the vacuum before and after the transition at $T_c(\Delta\varrho \sim T_c^4)$. The presently observed vanishing of the cosmological constant Λ and the relation $\Lambda_{\rm eff} = 8\pi M_{Pl}^2\varrho_v$ imply $\varrho_v(T=0) \lesssim 10^{-29}$ g cm$^{-3} \sim (10^{-2}$ eV$)^4$. Thus the gravitating vacuum energy density is $\varrho_v \sim T_c^4$ and $\varrho_v \sim 0$ before and after the transition, respectively (Kolb and Wolfram, 1980a), which can considerably change the expansion of the Universe (cf. Fig. 1). We will consider the baryon number generation as compared to that of the standard Big Bang model (i.e. $a \sim 1/T \propto t^{1/2}$).

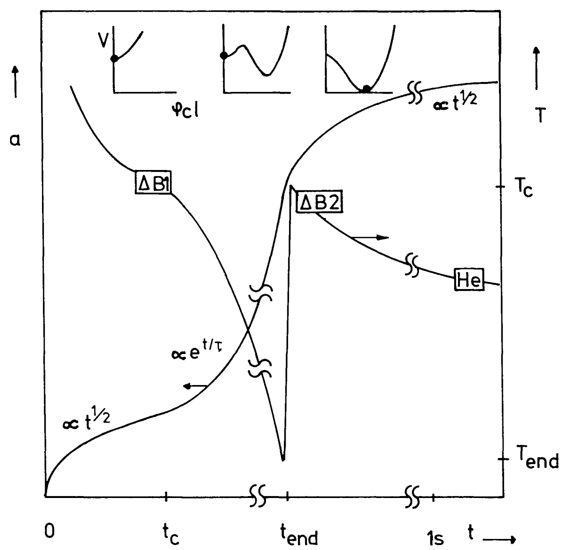

Fig. 1. Sketch of the history of the Universe during a first order phase transition: a is the scalefactor and T the equilibrium temperature (for adiabatic expansion $T\propto a^{-1}$). Included representative forms of the effective potential V, whose minima are possible vacuum state (see text). Twice baryon number (B) nonconserving reactions could have generated a baryon-anti baryon asymmetry, but in the symmetric vacuum state there will be no net B generation during the $\Delta B1$ period. The observed B should thus be created after the reheating ($T_R \sim T_c$), which must be smooth in order to preserve a homogeneous Helium synthesis (see text)

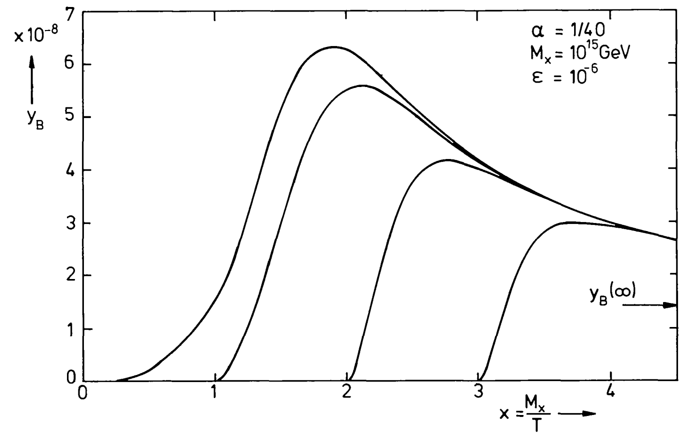

Fig. 2. The generation of the baryon number to photon ratio $Y_B(x; x_0)$, where $x = M_X/T$ and M_X the mass of the B-violating boson, is calculated for a simple model, with CP violation parameter ε. Earlier calculations (Kolb and Wolfram, 1980b) use symmetric starting conditions at $x_0 = 0$. Finite temperature effects lead to phase transitions (*PT*), *which give the boson a mass* M_X only for $T < T_c \sim M_X/g$. The $Y_B(x; x_0)$ evolution is calculated for realistic starting values after a second order PT ($x_0 \sim g \sim \frac{1}{2}$) or after the smooth reheating ending the period of supercooling of a first order PT ($x_0 \sim 1$). The final Y_B values are given in Fig. 3

When the Universe cools to $T \leqq T_c$ a phase transition will take place from the symmetric vacuum $\langle\phi\rangle = 0$ (expectation values for a Gibbs ensemble with temperatur T) to the broken state $\langle\phi\rangle = v(T)$. If the transition occurs smoothly at T_c due to the onset of instability of the symmetric state in $V(T_c, \phi_{c1})$, we speak of a second order phase transition ($2PT$): $v(T) \sim \{1 - T^2/T_c^2\}^{1/2} v(T=0)$. The expansion rate of the Universe $H^2 = (8\pi G/3)\,(\varrho_v + \pi^2 NT^4/30)$ will hardly be changed by the $2PT$: the energy density is dominated by the particles ($\varrho \sim NT^4 \gg \varrho_v \sim T_c^4$). But even in this unspectacular case ($2PT$) the standard calculations of Sect. 1 (Kolb and Wolfram, 1980b) are not correct for $T > T_c$ [Screening corrections to the cross-sections ($\sigma \sim \alpha^2/T^2$ instead of $\sim \alpha^2/M_X^2$) appear to be quite unimportant for the final n_B, Harvey et al. (1981)]. The same Boltzmann equations can be applied to the cosmological context but with starting conditions of thermal equilibrium at $T = T_c$ instead of at $T = \infty$ (or T_{P1}).

To illustrate the effects this modification, we consider a simple model (Kolb and Wolfram, 1980b) and solve the differential equations, describing the baryon number changing processes, for a range of initial conditions (details can be found in Appendix A). In Fig. 2 the evolution of the ratio $Y_B = n_B/n_\gamma$ of the net baryon number density to the photon number density is plotted: $Y_B\,(x;\, x_0)$ gives Y_B at temperature $T = M_X/x$ under the initial condition of thermal equilibrium at $x_0 = M_X/T_0$. Thus $Y_B(x_0;\, x_0) \equiv 0$ and $Y_B(\infty;\, x_0)$ is the final baryon to photon ratio for $T \to 0$, neglecting a factor O (10^{-1}) from later annihilation heating (e.g. $e^+e^- \to \gamma$).

Figure 2 shows how the baryon production peaks around $x = 1$ ($T = M_X$), followed by substantial damping (quantitative details depend on several parameters, as discussed in Appendix A). Starting at $x_0 \neq 0$ lowers the maximally attainable Y_B, but relaxes to nearly the same final value, due to a balance of production and damping at large x. Thus it is clear that the standard calculations, as discussed in Sect. 1, hold true in the case of a $2PT$ ($x_0 = M_X/T_c \sim g \sim \frac{1}{2}$), to high precision (final deviations $\ll 0.01\,\%$ in the model of Fig. 2).

In the case of a first order phase transition ($1PT$) the transition of the symmetric metastable state to the energetically favourable broken state is blocked by a barrier, either (i) from a mass² term in V: $T^2 c\phi_{c1}^2$ with c a constant depending on g^2 or λ, or (ii) from temperature independent quantum corrections to the tree potential if $\lambda < g^4$. The Universe cools below T_c with the vacuum remaining in the symmetric state (hence called a false vacuum) and its *constant* energy density $\varrho_v \sim T_c^4$ dominates over that of the particles ($\sim NT^4$, $T \propto 1/a$). This leads to an effective cosmological constant $\Lambda_{\rm eff} = 8\pi G\varrho_v \sim 8\pi T_c^4/M_{P1}^2$ which results in exponential expansion ($a \propto \exp t/\tau$, $\tau \equiv (8\pi/3)^{-1/2} M_{P1}/T_c^2$) with rapid supercooling ($T \propto a^{-1} \ll T_c$). This is illustrated in Fig. 1.

There are several alternatives to end the period of supercooling. If the false vacuum remains metastable (confined by a barrier) down to $T = 0$, the Universe can only have reached the presently observed broken state by nucleation of bubbles of true vacuum, which by expansion (velocity $\sim c$) conquer the false vacuum (Coleman, 1977; Callan and Coleman, 1977). There are two types of nucleation: 1. Thermal nucleation rates have a maximum just below T_c and either the bubble density gets high enough and they quickly fill the Universe (little supercooling) or else these few bubbles cannot catch up with the rest of the Universe which expands too rapidly, being continuously accelerated (Sato, 1981; Guth and Weinberg, 1981). 2. Nucleation by barrier penetration (a non-perturbative effect, cf. Coleman, 1979) has a constant rate per space volume. Obviously barrier penetration effects are small and a large supercooling results. This originally was the motivation for

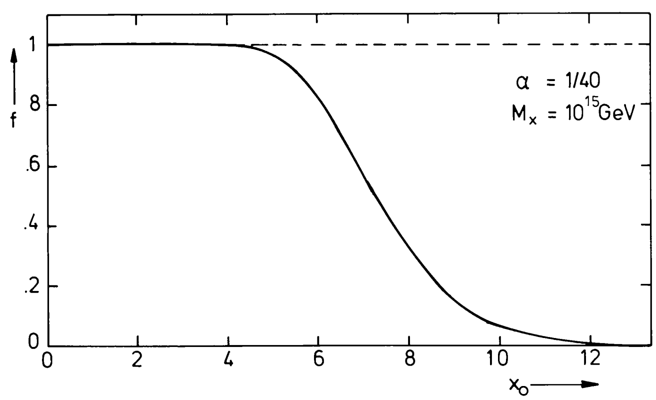

Fig. 3. Final baryon-number to photon ratio $Y_B(\infty, x_0)$ for realistic starting conditions ($x_0 \sim 1$) as compared to $x_0 = 0$: $f \equiv Y_B(\infty, x_0)/Y_B(\infty, 0)$. In this case $Y_B(\infty, 0) = 1.43\ 10^{-8}$

considering 1 *PT*'s, where the stretched horizons ($d_{\rm H} = a(t) \int_0^t dt'/a(t') \sim \tau e^{t/\tau}$) may reduce the monopole density, one per $d_{\rm H}(t_{\rm end})^3$ typically (Einhorn et al., 1980; Guth and Tye, 1980; see below). If the tunneling nucleation rate is small, leading to a cooling to $T \ll T_c$, it is not clear how at a certain temperature the *whole* Universe can have made the transition to the true vacuum (cf. Guth, 1981): although at every fixed point the probability that a transition false→true has taken place approaches unity, the ongoing exponential expansion in the false vacuum region prohibits a complete vaccum conversion. But the isotropy of the cosmic background radiation excludes the presence of bubble walls in the presently observable Universe (Zel'dovich et al., 1974). If the 1 *PT* ends with huge bubbles, of galaxy size, say, filling the Universe two further problems arise: 1. Thermalisation of the energy in the bubble walls before He production, which is to give $\sim 25\%$ *everywhere*, seems impossible (bubble sizes$\gg$available travel time of one light-second; see Fig. 1) and 2. It is not clear that the reheating is instantaneous and that $\sim T_c$ will be attained; this could jeopardize the standard baryon number creation results of Sect. 1 (see below).

Another way to end the period of supercooling (evading the above problems) is that the false vacuum becomes unstable at $T_1 < T_c$. This may happen if the barrier vanishes [in the $U(1)$ Higgs model if $3g^4/16\pi^2 < \lambda < g^4$ (Linde, 1979); in $G = {\rm SU}(5)$ region d of Guth and Weinberg (1981)]. At $\sim T_1$ the *whole* of the Universe still in the symmetric state will shift to the broken one, with a diverging nucleation rate when the confining barrier vanishes. The latent heat ($\varrho_v(>T_1) - \varrho_v(<T_1) \sim T_c^4$) relased in the many small bubbles from the last "flash" of nucleation probably will be thermalised, quickly reheating the Universe to just below T_c.

Which scenario the Universe follows (2 *PT*; 1 *PT* thermal or tunneling nucleation, or T_1 shift) depends on the yet unknown coupling constants in the GUT Lagrangian. Daniel and Vayonakis (1981) find for SU(5) a weak 1 *PT* typically. If the Higgs potential has scalars with *bare* masses vanishing ($V_{\rm tree} = \lambda \phi_{c1}^4/4!$), quantum corrections from the gauge bosons still may give symmetry breaking, for $U(1)$: one loop $V^{(1)} = (3g^4/64\pi^2)\phi_{c1}^4$ (ln $(\phi_{c1}^2/\langle\phi\rangle^2) - 1/2$) and $\lambda = (33/8\pi^2)g^4$ (Coleman and Weinberg, 1973). There are two strong arguments to expect radiatively broken SU(5): 1. this might explain the hierarchy problems $M_{WS} \ll M_U \ll M_{P1}$, and 2. this SU(5) GUT with massless scalars and 3 generations of $(\bar{5} + 10)$ fermions (as observed) might be the remnant of the $N = 8$ extended supergravity theory (references in Hut and Klinkhamer, 1981). The phase transition for the radiatively broken SU(5) GUT is strongly 1 PT, and supercooling goes to ~ 1 GeV when the nucleation rate equals the expansion rate (Billoire and Tamvakis, 1981; Daniel, 1981). Recently it was realised that non-perturbative effects may reduce the period of supercooling, destabilising the vacuum perhaps at $T = {\rm O}$ (10^6 GeV) (Tamvakis and Vayonakis, 1981).

Let us mention another possible origin for a general shift of the vacuum (Hut and Klinkhamer, 1981). Until now local fields on a flat background were used, but global space-time effects probably are important at low enough temperatures and might induce a shift to the broken state at $T_1 \sim T_c^2/T_{Pl} \sim 10^{11}$ GeV [for radiatively broken SU(5)]. Basically thermal radiation at this temperature will have wavelengths of the order of the event horizon $D_H \sim T_{Pl}/T_c^2$, thus invalidating the usual flat space-time description of the barrier that should stabilize the symmetric state.

Now we consider the baryon number generation after the reheating in a strong 1 *PT*. We assume that the supercooling ends at T_1 by destabilisation of the false vacuum. For definiteness we will consider the radiatively broken SU(5) theory. Unfortunately nothing is known about the thermalisation involving bubble-bubble and bubble-particle interactions. We estimate the typical bubble size, nucleated when the barrier became vanishingly small, to be of order $v^{-1} \sim T_c^{-1}$ and we *assume* the thermalisation to take place in the same time scale. In the cosmological context this means instantaneous ($\ll H^{-1}(T_c) \sim N^{-1/2} T_{p1} T_c^{-2}$) and smooth [$\ll$ effective horizon after $T_R \sim H^{-1}_{\rm standard}(T_R)$] reheating.

The Universe passes twice through temperature regimes where baryon number violating reactions are important (Fig. 1). At the first occasion, however, the Universe cools below T_c in the symmetric state, in which the gauge bosons are still massless and the Higgs bosons have a mass (defined by $\delta^2 V/\delta\phi_a \delta\phi_b$) of order $m_{\rm H} \sim gT$ (Linde, 1979). The crucial ingredient of baryon number generation, i.e. deviation from thermal equilibrium when the Universe cools below the boson mass (cf. Kolb and Wolfram, 1980b), is thus missing since $m_X = 0$, cf. Toussaint et al. (1979). Another important modification is that any truly primordial baryon number from the quantum gravity epoch will be even more strongly diminished than in the standard scenario. Both gauge boson X (inverse) decays and X exchanging $2-2$ reactions will now be in equilibrium (rates $\Gamma > H$) for 10^{16} GeV $\gtrsim T \geqq T_1$, for example $(\Gamma/H)_{2-2} \sim \alpha_{\rm GUT}^2 (T_{Pl}/T)$ (cf. Ellis et al., 1980b). Hence any primordial baryon asymmetry is damped by a factor $\sim$exp $\{-\alpha_{\rm GUT}^2 N^{-1/2} T_{Pl}/T_1\}$ [cf. Kolb and Wolfram, 1980b, Eq. (4.2)]. Thus for $T_1 \lesssim 10^{-2}\ T_c$ the damping is huge ($\lesssim 10^{-40}$). The latent heat released at the transition reduces this primordial n_B/s even more by an extra factor $\sim (T_1/T_c)^3$.

With no surviving baryon number it is all the more important to have sufficient reheating after the period of supercooling. Comparing the number of degrees of freedom (Einhorn and Sato, 1980) before the transition at T_1 (N_1) and after reheating to T_R (N_R) we find

$$T_R = T_c \{30/(N_R \pi^2) + (N_1/N_R)\ (T_1/T_c)^4\}^{1/4} \sim 0.4\ T_c$$

for $\{{\rm SU}(3) \times {\rm SU}(2) \times U(1)\}_R$, 6 quark flavours and one 5 of SU(5) Higgs giving one Weinberg-Salam doublet, $N_R = 106.75$. Note that if the SU(5) theory is a remnant of SU(8)/E(7) superunification the many superheavies ($\sim M_{Pl}$) will not count in estimating T_R, because of densities $\propto$exp $(-M/T_1)$ from the period of equilibrium unified interactions as discussed above.

Now we will discuss the baryon asymmetry generated for $T \leqq T_R$. First we assume that the thermalisation process for the small (T_c^{-1}) bubbles produces no asymmetry itself. Again, earlier

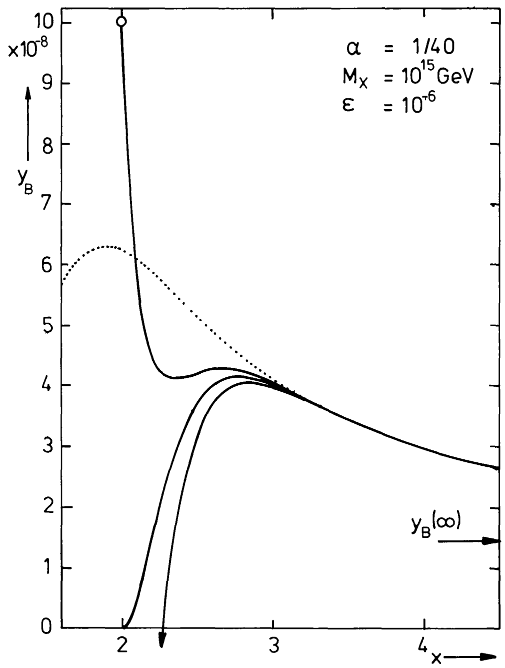

Fig. 4. The generation of the final baryon-antibaryon asymmetry for a first order phase transition, if the thermalisation of the small bubbles of true vacuum at the end of the supercooling epoch itself gives a net baryon number $Y_B^{\text{bubbles}}(x_0=2)=\pm 10^{-7}$. For comparison the curves for $Y_B(x_0=0)=0$ (dotted) and $Y_B(x_0=2)=0$ (Fig. 2) are given

calculations have to be modified only by taking more realistic initial conditions in solving the same rate equations. For the simple model of Kolb and Wolfram (1980b), we can extend the previous discussion of 2 *PT*'s, and use Fig. 2 in case of smooth reheating after the supercooling, with starting values $x_0 \sim M_X/0.4T_c \sim 1$. In Fig. 3 the dependence of the finally generated baryonnumber $Y_B(\infty; x_0)$ on the reheating temperature $T_R = M_X/x_0$ is plotted in units of $Y_B(\infty; 0)$. Nearly the same baryon number is produced in the range $x_0 = 0 \sim 5$, followed by a sharp drop around $x_0 = 7$. Thus we conclude that details of the initial conditions are unimportant for this mode as long as smooth reheating takes place to $T_R \gtrsim \frac{1}{5}M_X$, which holds true for all realistic unification models. But the X considered here was taken to be a gauge boson ($M_X \sim gT_C \sim 10^{15}$ GeV) and the light Higgs bosons (for radiatively broken GUTs: $M_H \sim \alpha^{1/2}M_X$) will give the major contribution to $Y_B(\infty)$ from delayed decay at temperatures lower than $T_R \sim 0.4T_c$. This Higgs contribution will be larger because 1. the CP violating diagrams are of lower order than for the gauge boson X, and 2. dilution of the baryon number density from the delayed Higgs decay will be less ($\alpha_{\text{eff}} < \alpha$; see Sect. 1).

Finally the assumption of baryon number conserving thermalisation processes after the reheating can be dropped: in Fig. 4 it is shown that a sizeable asymmetry $Y_B(x_0; x_0) = \pm 10^{-7}$ will be damped out for moderate x_0-values (in Fig. 4 we chose $x_0 = 2$).

3. Discussion

We have presented a consistent scenario of baryon number generation in the early Universe, taking into account the effects of phase transitions of gauge symmetries. Previous analytical estimates and numerical calculations are invalid at $T > T_c$. After a second order phase transition the final n_B/s turns out to be the same since the final result depends mainly on processes operative at $T \lesssim M_X$. A strongly first order phase transition will drastically change the behaviour of the Universe, leading to a period of accelerated expansion. But even in this case, after reheating following by thermalization, the gauge bosons will produce roughly the same baryon number density as in the standard zero temperature calculations! For a simple model, quantitative details are presented in the Appendix A. But the major contribution to the final n_B/s will come from the simple delayed decay of the lighter Higgs bosons, which have larger CP violating amplitudes i.e. $\varepsilon^{\text{Higgs}} \sim \alpha^{-1}\varepsilon^{\text{Gauge}}$) and less dilution.

To complete our discussion of the realistic baryon number creation we discuss two further problems: 1. The generation of density inhomogeneities leading to the formation of the present structure, e.g. galaxies. In Appendix 2 we argue against an origin for the required inhomogeneities from 1 *PT*'s only; most plausibly the origin lies in the quantum gravity epoch. 2. Directly related to phase transitions and vacuum structure is the problem of monopole creation in the early Universe.

These monopoles are classical finite energy solutions of the Yang-Mills equations of motion with boundary solutions of $\phi(\theta,\varphi,r=\infty)$ that minimize $V(\phi)$ with a non-trivial mapping of S_2 (i.e. the sphere at infinity) on the remaining symmetry G/H, if H is the little group of the V minima (for a review see Actor, 1979). For the 't Hooft-Polyakov monopole [$G=\text{SU}(2)$, $H=U(1)$ which gives the monopole-like long distance force, $V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4$ for triplet ϕ_a, $a=1,2,3$] the asymptotic value at $r\to\infty$ $\phi_a = \tilde{r}_a\{m\lambda^{-1/2} + (\text{const}/gr)\exp(-2^{1/2}mr)\} \sim \tilde{r}_a m\lambda^{-1/2}$ (note the mixing of space and group indices) is not continuously deformable to $\phi_a = n_a m\lambda^{-1/2}$ with the group space vector n_a fixed at $r\to\infty$. Recently multimonopole, i.e. static, localized, non-singular, finite energy, solutions of the classical SU(2) theory with Higgs triplet ($\lambda, m\to 0$, $m^2/\lambda = D^2$ fixed) in 4-dimensional Minkowski space have been found (for roads to quantization see Jackiw, 1977). These are axisymmetric solutions with topological charge n (i.e. the homotopy class of the mapping $S_2 \to G/H$, which is linked to the magnetic charge $g_m = n/e$) equal to 2 (Ward, 1981) and generally $n \geqq 1$ (Prasad, 1981; Prasad and Rossi, 1980). Even more generally the existence of multi monopole solutions of arbitrary charge and separations has been proved (Jaffe and Taubes, 1980). The axial $n \geqq 2$ solutions, being self-dual, obviously have $E(n) = nE(1) = n(4\pi/g^2)gD$ [note that for $m^2, \lambda \neq 0$ 't Hooft found the same $E(1)$ up to a constant $1 \leqq C(\lambda/g^2) \leqq 1.787$]. If this (for n equal, centered charges) also holds for the most general multi-monopole solutions this seems to imply no interactions; hopefully the attraction between two differently charged monopoles remains (cf. Manton, 1977), so that the annihilation calculated by Preskill (1979) is correct, although not even sufficient yet (see below).

If after the phase transition the distribution of ϕ directions in group space is "random" one expects typically p monopoles per volume l^3, where $p \sim 0.1$ a combinatorial factor and l the ϕ direction correlation length, which will be $l \lesssim 2$ ct from causality. Because $M_{\text{mon}} \sim \alpha^{-1}M_X$ and $t(T_c)$ is very small, this leads to an enormous mass density, invalidating the standard result ^{4}He $\sim 25\%$. Before proceeding we remark that the "if" above is not at all trivial and that a correct gauge invariant (cf. Jackiw, 1980) calculation of the eventual monopole creation at a phase transition $G \to G' \times U(1)$, with G and G' simple groups, has not been done yet.

If we stick to the naive "if" the trick of a 1 PT is to have a larger l at symmetry breaking than in the standard model (Einhorn et al., 1980; Guth and Tye, 1980). There are two possible estimates of the $l_{1\,\mathrm{PT}}$: a) $l_{1\,\mathrm{PT}}$ as the stretched up particle horizon from the period of supercooling ($T_c - T_{\mathrm{end}}$), and b) first as the field correlation length $\xi \sim (2/g)|T^2 - T_{\mathrm{end}}^2|^{-1/2}$, but then with limited growth $d\xi/dt \leqq 1 + \xi'/a \sim 1 + \xi/\tau$ (Einhorn and Sato, 1981). In Sect. 2 we discussed the two alternatives for the transition.

1. Ending by the filling with bubbles if the barrier between false and true vacuum remains. For all bubbles nucleated at T_c (each with independent mean ϕ directions) one might use estimate a) and then $T_{\mathrm{end}} < 10^{12}$ GeV and of course a lower T_{end} if there are also smaller bubbles, but anyway estimate b) requires $T_{\mathrm{end}} < 10^8$ GeV.

2. If the barrier disappears at T_1 we cannot use a), because there will be a very great number of small bubbles, but again b) requires $T_{\mathrm{end}} < 10^8$ GeV. Of course the numberical values (Einhorn and Sato, 1981) of these limits are quite uncertain.

Thus the apparent monopole problem might be alleviated by a strongly first order phase transition ending with smooth reheating, which also guarantees the successes of the Friedmann model of the Universe: baryon number creation and Helium-synthesis.

Note

Soft CP breaking, i.e. complex phases arising in spontaneous symmetry breaking, has been evoked to get an overall $B=0$ (Senjanovic and Stecker, 1980). To counter the standard objections (e.g. Ellis et al., 1980b) of annihilation and separation these authors invoke a period of exponential expansion to stretch the very small domains, each with $n_B/s \sim \pm 10^{-10}$ where the sign of the breaking is correlated, to at least galaxy cluster sizes (Sato, 1981b). Apart from the unnaturalness of requiring non-zero expectation values of some Higgses for all energies [otherwise our Universe would contain domain walls, Zel'dovich et al. (1974) and Vilenkin (1981)] and *assuming* supercooling to occur, we note that for the required supercooling to $T = O(100$ eV$)$ very small renormalised Higgs masses are needed whereas for a Coleman-Weinberg potential (bare mass zero) the cooling goes only (!) to $O(1$ GeV$)$ (Billoire and Tamvakis, 1981), or even less low because of non-perturbative effects. As discussed in the Sect. 2 the reheating has to be very smooth. Two further arguments might be that with the larger Higgs sector needed 1. $\delta\theta_{\mathrm{weak}}$ perhaps is too large (Ellis and Gaillard, 1979) and 2. the calculated proton lifetime and SU(2) $-U(1)$ mixing angle $\sin\theta_W$ go down and up, respectively, for more 5's of Higgs. It is clear that we disfavour Senjanovic and Stecker's (1980) suggestion and, of course, Occam's razor on the observations (Steigman, 1976) suggests a clean shaven Universe: $n_B/s \sim +10^{-10}$ *everywhere*.

Appendix A: A Model for Baryon Number Creation

All calculations, illustrating baryon number generation after phase transitions, are done for the simple model introduced by Kolb and Wolfram (1980b). First we will summarize the model and the reaction rate equations. After that we will discuss the modifications if we take phase transitions into account.

The model consists of two types of particles: nearly massless particles b and $\bar{b}$ carrying baryon numbers $B=\frac{1}{2}$ and $B=-\frac{1}{2}$ respectively, and massive bosons X and $\bar{X}$ mediating baryon-number violating reactions. The decay amplitudes M of these massive bosons are parametrized as

$$\begin{aligned}
|M(X\to bb)|^2 &= (1+\eta)\tfrac{1}{2}|M_0|^2,\\
|M(X\to \bar{b}\bar{b})|^2 &= (1-\eta)\tfrac{1}{2}|M_0|^2\\
|M(\bar{X}\to \bar{b}\bar{b})|^2 &= (1+\bar{\eta})\tfrac{1}{2}|M_0|^2,\\
|M(\bar{X}\to bb)|^2 &= (1-\bar{\eta})\tfrac{1}{2}|M_0|^2,
\end{aligned} \tag{A1}$$

with $|M_0|^2$ of the order of a small coupling constant α. Because of unitarity and CPT invariance only two free parameters $\eta, \bar{\eta}$ are left, where $\eta - \bar{\eta} = O(\alpha)$ measures the amount of CP breaking. Thus a state initially containing an equal number of X and $\bar{X}$ $(n_X^0 = n_{\bar{X}}^0)$ will decay, in the absence of back reactions, to a system with a net baryon number $n_B = (\eta - \bar{\eta})\frac{1}{2}(n_X^0 + n_{\bar{X}}^0)$. For simplicity all particles are given only one spin degree of freedom, and obey Maxwell-Boltzmann distributions. Because in the expanding Universe all densities drop quickly, a convenient type of variable is

$$Y_A \equiv n_A/n_\gamma,$$

the relative number density of particle $A(=b, \bar{b}, X$ or $\bar{X})$ with respect to photons. Finally an "effective Planck mass" M_P is defined as

$$M_P = (\pi/8)^{1/2}\,G^{-1/2}\,N^{-1/2} \simeq 7.5(N)^{-1/2}\;10^{18}\ \mathrm{GeV},$$

where G is the gravitational constant and N is the number of massless particle species (which is temperature dependent).

The reaction-rate equations gouverning the time evolution of n_b, $n_{\bar{b}}$, n_X, $n_{\bar{X}}$ can be simplified by the choice of a dimensionless "time" parameter $x = M_X/T$, for which

$$\frac{dY_A}{dx} = \frac{x}{M_X x_P}\,\frac{dY_A}{dt},$$

where $x_P = M_X/M_P$ is a constant. Assuming the b, $\bar{b}$ to undergo many baryon-number conserving reactions $(\gamma b \to \gamma b)$ with the other particles in the Universe, b and $\bar{b}$ must have exactly opposite chemical potentials, leaving only one degree of freedom for

$$Y_B = Y_b - Y_{\bar{b}}$$

since $Y_b + Y_{\bar{b}} = 2$. Defining

$$Y_\pm = \tfrac{1}{2}(Y_X \pm Y_{\bar{X}})$$

the rate equations read [Kolb and Wolfram, 1980b, Eq. (3.1.2)]:

$$\begin{aligned}
\frac{dY_+}{dx} &= -A(x)\left[(Y_+ - Y_+^{eq}) + \left(\frac{\eta-\bar{\eta}}{2}\right) Y_B Y_+^{eq}\right]\\
\frac{dY_-}{dx} &= -A(x)\left[Y_- - \left(\frac{\eta+\bar{\eta}}{2}\right) Y_B Y_+^{eq}\right]\\
\frac{dY_B}{dx} &= A(x)\Big[\{(\eta-\bar{\eta})\,(Y_+ - Y_+^{eq}) + (\eta+\bar{\eta})\,Y_-\}\\
&\quad -2\,Y_B\Big\{Y_+^{eq} + \frac{n_\gamma}{\langle\Gamma_x\rangle}\,\langle v\{\sigma'(bb\to\bar{b}\bar{b})\\
&\quad +\sigma'(\bar{b}\bar{b}\to bb)\}\rangle\Big\}\Big].
\end{aligned} \tag{A2}$$

Here $A(x)$ determines the overall reaction rate and is given by

$$A(x) = \frac{x}{x_P}\,\frac{\langle\Gamma_X\rangle}{M_X}.$$

The relative number density of X, $\bar{X}$ in thermal equilibrium is given by

$$Y_+^{eq}(x) = \tfrac{1}{2}x^2 K_2(x)$$

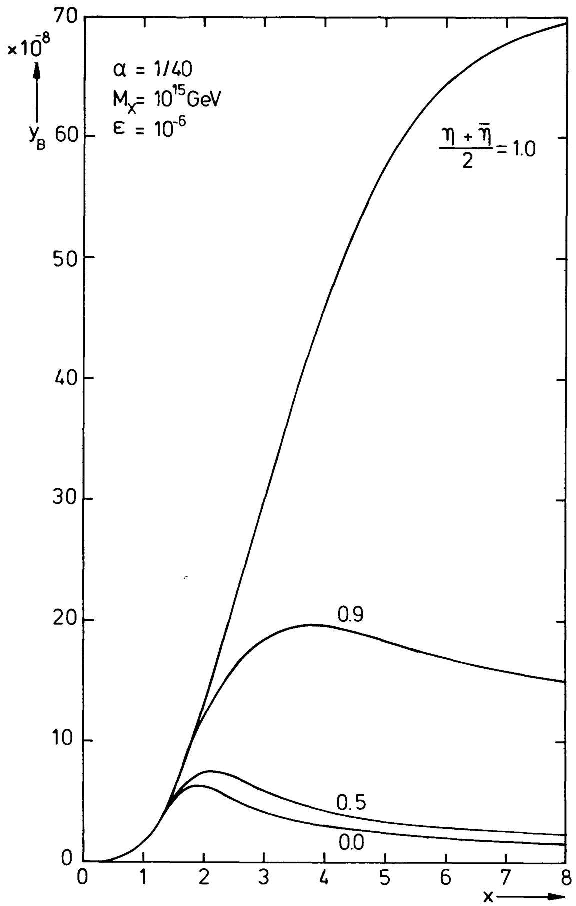

Fig. 5. $Y_B(x; x_0=0)$ for different branching ratios of X, $\bar{X}$ decays (see Appendix A). For $(\eta+\bar\eta)/2=1$ there is a nearly total suppression of the B damping processes

where $K_2(x)$ is the modified Bessel function (using the notation of Gradshteyn and Ryzhik, 1965).

Finally

$$\langle\Gamma_X\rangle=\frac{K_1(x)}{K_2(x)}\Gamma_X=\frac{1}{4}\frac{K_1(x)}{K_2(x)}M_X\alpha$$

and in the low-energy approximation for the cross sections $\sigma'(bb\to\bar{b}\bar{b})$ and $\sigma'(\bar{b}\bar{b}\to bb)$

$$\langle v\sigma'\rangle\sim\frac{18\pi\alpha^2T^2}{M_X^4},$$

or

$$\frac{n_\gamma}{\langle\Gamma_X\rangle}\langle v\sigma'\rangle=\frac{72\alpha}{\pi}\cdot\frac{1}{x^5}\cdot\frac{K_2(x)}{K_1(x)}.$$

For a discussion of the approximations made to derive Eqs. (A2), see Kolb and Wolfram (1980b).

In order to integrate the three coupled rate Eqs. (A2), we need initial conditions and numerical values of the parameters α, η, $\bar\eta$, and x_P. In all calculations presented in Figs. 2–5 we use $\alpha=1/40$, $M_X=10^{15}$ GeV, and $N=100$, leading to $x_P=0.00131$. For the combination $\varepsilon=\eta-\bar\eta$, measuring the amount of CP violation, we use $\varepsilon=10^{-6}$. These four values, which are reasonable for GUTs as discussed in Sect. 1, are the same as used by Kolb and Wolfram (1980b). However we disagree with their statement that the calculations are insensitive to the value of $\eta+\bar\eta$. As can be seen from the X, $\bar{X}$ decay amplitudes (A1), $\eta\sim\bar\eta\sim1$ would imply a strong preference for the decay modes $X\to bb$ and $\bar{X}\to\bar{b}\bar{b}$, and therefore decays and inverse decays of X, $\bar{X}$ would be inefficient in the damping of a net baryon number. On the other hand, $\eta\sim\bar\eta\sim0$ would imply a maximal damping of any net baryon number. Since $\eta-\bar\eta=\varepsilon=10^{-6}$, $\eta\sim\bar\eta$ and we expect the finally produced baryon number density to increase monotonically with the absolute value $|\eta+\bar\eta|$. In Fig. 5 are plotted a few sample calculations, showing this effect very clearly. Initial conditions used thermal equilibrium at very early times: $Y_+=1$ and $Y_-=Y_B=0$ at $x_0=0$. As expected, for moderate values of $\eta+\bar\eta$ between 0 and 1, say, the calculations are insensitive to the precise value. Only just below $\eta+\bar\eta=2$ are half the decay channels of X, $\bar{X}$ nearly blocked, damping becomes less effective and much more baryon number will be produced. (For $\eta+\bar\eta=2$ the maximum Y_B is reached at $x=12$ as $Y_B=7.05\ 10^{-7}$, relaxing for $x\to\infty$ to $Y_B\to7.04\ 10^{-7}$.)

In all other calculations, except those of Fig. 5, we used $\eta+\bar\eta=0$. This implies equal decay amplitudes for $X\to bb$ and $\bar{X}\to bb$, as well as for $X\to\bar{b}\bar{b}$ and $\bar{X}\to\bar{b}\bar{b}$. Therefore X can be taken to be its own antiparticle $\bar{X}$, which implies $Y_-=0$. Indeed the second rate Eq. (A2) guarantees $Y_-(x)=0$ if $Y_-(x_0)=0$, for $\eta+\bar\eta=0$. We are left with only two equations for Y_+ and Y_B. Of course in realistic GUTs X has an electric charge (Nanopoulos and Weinberg, 1979) and cannot be its own antiparticle, but in the simple model under consideration Fig. 5 shows how a sizeable mixing between X, $\bar{X}$ decay channels produces comparable results.

From the last Eq. (A2) it can be seen that baryon number generation is proportional to the deviation from equilibrium of the X-paricles, $Y_+-Y_+^{eq}$. Choosing this as a new variable

$$Y_\Delta=Y_+-Y_+^{eq}$$

we have, with $\eta+\bar\eta=0$, the following two rate equations:

$$\frac{dY_\Delta}{dx}=\tfrac{1}{2}x^2K_1(x)-\frac{\alpha}{4x_P}\left\{x\frac{K_1(x)}{K_2(x)}Y_\Delta(x)+\tfrac{1}{4}\varepsilon x^3K_1(x)Y_B(x)\right\}\tag{A3}$$

$$\frac{dY_B}{dx}=\frac{\alpha}{4x_P}\left\{\varepsilon x\frac{K_1(x)}{K_2(x)}Y_\Delta(x)-x^3K_1(x)Y_B(x)-\frac{288\alpha}{\pi}x^{-4}Y_B(x)\right\}.$$

In the first equation it is clear how the first RHS terms determines the production of Y_Δ, independent of already existing Y_Δ and Y_B, as a function of temperature only ($T=M_X/x$). The second term destroys the deviation from equilibrium of the X's, and is therefore proportional to α/x_p or $\alpha G^{-1/2}$ (G is the gravitational constant): the magnitude of the deviation is governed by a competition between particle reaction rates and Universe expansion. The third term is typically ten orders of magnitude smaller than the second one in our calculations ($\varepsilon=10^{-6}$; $Y_B\lesssim\varepsilon$). Therefore the rate equation for Y_Δ is nearly completely Y_B-independent, and $Y_\Delta(x; x_0)$ is fixed by specifying the initial condition $Y_\Delta(x_0; x_0)$ independent of Y_B.

The rate equation for Y_B shows a production term $\propto Y_\Delta$, with the same reaction vs. expansion factor α/x_P, but also the small CP violation parameter ε. The next two terms $\propto Y_B$ determine the damping of Y_B, by means of all baryon number changing processes, independent of ε, which already indicates that $Y_{B\,\max}\lessapprox\varepsilon$. The first of these two terms describes inverse decays of X,

$\bar{X}(bb \to X$, etc.), and drops off quickly for high $x\,(K_1(x) \propto x^{-1/2}e^{-x}$ for $x \gg 1$). The last term is the Fermi approximation for $2 \to 2$ scattering processes ($bb \to \overline{bb}$, etc.), which is the dominating, but still rather unimportant, term for $x \gtrsim 20$. For high energies this term is unimportant, since the large contribution to $2 \to 2$ scattering by the exchange of on-shell intermediate X's is already included in the previous term (see Kolb and Wolfram, 1980b, Sect. 2.3.2). Therefore we just replaced x^{-4} by 1 for $x < 1$, since a detailed treatment of the $2 \to 2$ processes would be unnecessary in this regime.

To investigate the effects of phase transitions on baryon number creation, we solved Eq. (A3) for various initial conditions. Starting from thermal equilibrium, $Y_\Delta(x_0) = Y_B(x_0) = 0$, Fig. 2 shows $Y_B(x; x_0)$ for the values $x_0 = 0, 1, 2, 3$. Figure 3 shows how $Y_B(\infty; x_0)$ drops quickly for $x_0 \geqq 7$. Finally we investigate the effects of an initial baryon number density, as a possible result of the thermalisation after a 1 PT: we choose at the starting value $x_0 = 2$, $Y_\Delta(2) = 0$, and $Y_B(2) = \pm 10^{-7}$. From Fig. 4 it is clear that already at $x = 3$ the two curves nearly coincide with a third one, starting from thermal equilibrium ($Y_B(2) = 0$). Note how the Y_B curve starting at $Y_B = +10^{-7}$ drops to a minimum. The short rise is a direct effect of the steep increase of Y_Δ, starting from $Y_\Delta = 0$ at $x_0 = 2$, and reaching its maximum value around $x = 2.4$.

Appendix B: Galaxy Formation

Several authors (e.g. Guth and Tye, 1980) have suggested that strong first order phase transitions (1 PT) *might* generate the required small density perturbations that later, after growth by their selfgravity, give the observed structure: galaxies to (super) clusters ($\sim 10^{12}$–$10^{16}\,M_\odot$; cf. Peebles, 1980). This optimism was based on the fact that 1 PT have two valuable ingredients:

1. During the period of supercooling (from t_c to $t_{\rm end}$) the exponential expansion of the Universe ($a \sim a(t_c)\exp\,(t - t_c)/\tau)$) greatly enlarges the particle horizon $d_H^{1\,\rm PT} = a(t_{\rm end}) \int_{tc}^{t_{\rm end}} dt'/a(t')$ $\sim \tau \exp\,(t_{\rm end} - t_c)/\tau)$ so that after the instantaneous reheating to T_c at $t_{\rm end}$ the stretched up horizon $d_h^{1\,\rm PT}$ can be much larger (roughly by a factor $T_c/T_{\rm end}$) than in the standard scenario $d_h^{\rm stand} = 2\,t_c$, where $\tau \sim T_{Pl} T_c^{-2}$ and $t_c \sim N^{-1/2}\,T_{Pl} T_c^{-2}$. This might (partially) resolve the standard problem of the generation of perturbations on a galaxy scale for which $\lambda \gg$ the causally connected region $d_h^{\rm stand}$.

2. Nucleation of true vacuum might very well lead to density differences [note that for bubbles nucleated by barrier penetration all the released latent energy of the false vacuum is put in the acceleration of the bubble walls; Coleman (1977)].

Our pessimism is based on the following points:

a) To profit from the stretching some bubbles must have been nucleated before the transition ($T \gg T_{\rm end}$). But on the other hand the corresponding strong (?) inhomogeneities cannot be on scales $> (a(t)/a(1\,{\rm s}))$ 1 s if we want to preserve the homogeneous He production which requires a sufficient thermalisation (causal process) of the bubble wall inhomogeneities. How does this arise naturally, i.e. what determines the precise parameters in the Lagrangian, which give the required $T_{\rm end}/T_c$ and the nucleation rates?

b) What is the form of the wall-particle and the wall-wall interactions? How does thermalisation take place? What is the precise nature of the inhomogeneities formed after thermalisation? Which kind of density enhancements arise from an effective pressure of moving walls? Also gravitational collapse must be avoided, or at least for masses $> 10^{15}$ g, which cannot have evaporated by now.

c) Pending the problems mentioned above, let us consider the optimal case: $(\delta\varrho/\varrho)_{t_{\rm end}} \sim 1$ on a size $\lambda_{\rm bubble} \sim \{a(t_{\rm end})/a(1\,{\rm s})\}$ 1 s or $\sim 1\,M_\odot$ of baryons. Thermalisation of the false vacuum will produce a homogeneous energy density outside the rare bubbles of true vacuum, which remain nearly empty. Denoting the mean distance between these cavities by L, the smeared out density contrast in a volume L^3 is $\overline{\delta\varrho/\varrho} \sim (\lambda_{\rm bubble}/L)^3$. On larger scales, containing N cavities, we have

$$\overline{\delta\varrho/\varrho} \sim \left(\frac{\lambda_{\rm bubble}}{L}\right)^3 N^{-1/2}. \qquad \text{(B1)}$$

Contrary to the usual exponent $-7/6$, which arises if density fluctuations are made by reshuffling particles, i.e. with energy-momentum conservation (Zel'dovich, 1965), we have here a really statistical exponent $-1/2$, since the nucleation centers are distributed at random (excluded, of course, the interior regions of existing bubbles). We then expect the large scale inhomogeneities to grow as $\delta \equiv \delta\varrho/\varrho \propto t$, so that to arrive at the required $\delta_{\rm hor}$ (galaxy) $\sim 10^{-3}$ the bubble density must be very low at $t_{\rm end}$; which can easily be provided by a small thermal nucleation rate at $\sim T_c$ (Guth and Weinberg, 1981). The scenario would be: very few thermal bubbles; nearly the whole Universe smoothly shifting to the real vacuum at T_1 ($\lesssim 10^8$ GeV?); the cavities remaining after the shift from the few thermal bubbles giving a $\delta \propto N^{-1/2}$ spectrum. There might thus be a positive ring, because *a priori* large enough $\delta_{\rm hor}$ (galaxy) *could* be made.

We conclude that, pending new information on GUTs, strongly first order phase transitions do not provide *naturally* the required density perturbations for galaxy formation. We are already happy that the homogeneous ΔB and ^{4}He production survives phase transition complications. In conclusion we would like to mention an argument in favour of a quantum gravity origin of the density perturbations (cf. Klinkhamer, 1981). The only continuous density spectrum which avoids large metric perturbations on *any* scale (pro Friedmann, contra primordial black holes) is of the Zel'dovich type; $(\delta\varrho/\varrho)_{\rm horizon} = {\rm constant} \ll 1$. The corresponding small constant metric perturbations (cf. Peebles, 1980) are suggestive of a (quantum) gravitational origin. After the epoch of baryon number generation the resulting density perturbations will be of the adiabatic type. If isothermal perturbations are demanded by observations these could be generated from large scale shear at $T \sim M_X$ (Bond et al., 1981), but the (quantum) gravitational origin of this shear again is an open question.

References

Actor, A.: 1979, *Rev. Mod. Phys.* **51**, 461

Barbieri, R., Masiero, A., Nanopoulos, D.V.: 1981, *Phys. Letters* **98B**, 191

Barr, S., Segrè, G., Weldon, A.: 1979, *Phys. Rev.* **D20**, 2494

Billoire, A., Tamvakis, K.: 1981 (preprint CERN-TH 3019)

Bond, J.R., Kolb, E.W., Silk, J.: 1981 (preprint)

Callan, C.G., Coleman, S.: 1977, *Phys. Rev.* **D16**, 1762

Coleman, S., Weinberg, E.: 1973, *Phys. Rev.* **D7**, 1888

Coleman, S.: 1977, *Phys. Rev.* **D15**, 2929

Coleman, S.: 1979, in *The Whys of Subnuclear Physics*, eds. A. Zichichi, Plenum Press, New York

Daniel, M.: 1981, *Phys. Letters* **98B**, 371

Daniel, M., Vayonakis, C.E.: 1981, *Nucl. Phys.* **B180**, 301

Einhorn, M.B., Stein, D.L., Toussaint, D.: 1980, *Phys. Rev.* **D21**, 3295
Einhorn, M.B., Sato, K.: 1981, *Nucl. Phys.* **B180**, 385
Ellis, J.: 1980, 21st Scottish Universities Summer School in Physics (preprint CERN-TH 2942)
Ellis, J., Gaillard, M.K.: 1979, *Nucl. Phys.* **B150**, 141
Ellis, J., Gaillard, M.K., Nanopoulos, D.V.: 1979, *Phys. Letters* **88B**, 320
Ellis, J., Gaillard, M.K., Nanopoulos, D.V., Rudaz, S.: 1980a, *Nucl. Phys.* **B176**, 61
Ellis, J., Gaillard, M.K., Nanopoulos, D.V.: 1980b, in *Unification of Fundamental Particles Interactions*, eds. S. Ferrara, J. Ellis, P. van Nieuwenhuizen, Plenum Press, New York
Ellis, J., Gaillard, M.K., Nanopoulos, D.V., Rudaz, S.: 1981a, *Phys. Letters* **99B**, 101
Ellis, J., Gaillard, M.K., Nanopoulos, D.V., Rudaz, S.: 1981b, *Nature* **293**, 41
Fry, J.N., Olive, K.A., Turner, M.S.: 1980, *Phys. Rev.* **D22**, 2953, 2977
Gradshteyn, I.S., Ryzhik, I.M.: 1965, Table of Integrals, Series, and Products, 4th ed., Academic Press, New York
Guth, A.H.: 1981, *Phys. Rev.* **D23**, 347
Guth, A.H., Tye, S.H.: 1980, *Phys. Rev. Letters* **44**, 631, 963
Guth, A.H., Weinberg, E.: 1981, *Phys. Rev.* **D23**, 876
Harvey, J.A., Kolb, E.W., Reiss, D.B., Wolfram, S.: 1981, *Phys. Rev. Letters* **47**, 391
Hut, P., Klinkhamer, F.R.: 1981, *Phys. Letters* **104B**, 439
Jackiw, R.: 1977, *Rev. Mod. Phys.* **49**, 681
Jackiw, R.: 1980, *Rev. Mod. Phys.* **52**, 661
Jaffe, A., Taubes, C.: 1980, *Vortices and Monopoles* (Birkhäuser)
Kirzhnits, D.A., Linde, A.D.: 1976, *Ann. Phys.* **101**, 195
Klinkhamer, F.R.: 1981 (to be published in *Astron. Astrophys.*)
Kolb, E.W., Wolfram, S.: 1980a, *Astrophys. J.* **239**, 428
Kolb, E.W., Wolfram, S.: 1980b, *Nucl. Phys.* **B172**, 224
Linde, A.D.: 1979, *Rep. Prog. Phys.* **42**, 389
Manton, N.S.: 1977, *Nucl. Phys.* **B126**, 525
Nanopoulos, D.V., Weinberg, S.: 1979, *Phys. Rev.* **D20**, 2484
O'Raifeartaigh, L.: 1979, *Rep. Prog. Phys.* **42**, 159
Peebles, P.J.E.: 1980, The Large Scale Structure of the Universe, Princeton, Princeton University Press
Peccei, R.D., Quinn, H.R.: 1977, *Phys. Rev.* **D16**, 1791
Prasad, M,K.: 1981, *Commun. Math. Phys.* **80**, 137
Prasad, M.K.: 1980 (to be publ. *Commun. Math. Phys.*) (MIT preprint)
Prasad, M.K., Rossi, P.: 1980, *Phys. Rev. Letters* **46**, 806
Preskill, J.P.: 1979, *Phys. Rev. Letters* **43**, 1365
Sato, K.: 1981a, *Monthly Notices Roy. Astron. Soc.* **195**, 467
Sato, K.: 1981b, *Phys. Letters* **99B**, 66
Senjanovic, G., Stecker, F.W.: 1980, *Phys. Letters* **96B**, 285
Steigman, G.: 1976, *Ann. Rev. Astron. Astrophys.* **14**, 339
Tamvakis, K., Vayonakis, E.E.: 1981 preprint CERN-TH 3108)
Taylor, J.C.: 1976, Gauge Theories of Weak Interactions, Cambridge, Cambridge University Press
Toussaint, D., Treiman, S.B., Wilczek, F., Zee, A.: 1979, *Phys. Rev.* **D19**, 1036
Vilenkin, A.: 1981, *Phys. Rev.* **D23**, 852
Ward, R.S.: 1981, *Commun. Math. Phys.* **79**, 317
Weinberg, S.: 1972, Gravitation and Cosmology, Wiley
Weinberg, S.: 1974, *Phys. Rev.* **D9**, 3357
Weinberg, S.: 1979, *Phys. Rev. Letters* **42**, 850
Yildiz, A., Cox, P.H.: 1980, *Phys. Rev.* **D21**, 906
Zel'dovich, Ya.B.: 1965, *Adv. Astron. Astrophys.* **3**, 241
Zel'dovich, Ya.B., Kobzarev, I.Yu., Okun, L.B.: 1975, *Soviet Phys.* JETP **40**, 1