BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS.

1926 June 28 Volume III. No. 97.

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

The distant companion C of Castor as an eclipsing variable dwarf star of determinable surface brightness, by *H. van Gent.*

1. As is well known the multiple system of α Geminorum consists of two bright components, which are both spectroscopic binaries, and of a third faint companion at a distance of somewhat more than a minute of arc. This third member of the system, *C*, was found by W. S. ADAMS and A. H. JOY (*Publ. Astr. Soc. Pacific*, **32**, 158, 1920) also to be a spectroscopic binary. The period was given as about 4 days. Both spectra are visible and the maximum separation of the lines found was 230 km/s. Professor HERTZSPRUNG therefore recommended me to examine the star for variability, because, if it proved to be an eclipsing binary, it would be possible to derive the surface brightness of the components, the parallax of Castor being fairly well known.

The visual Harvard magnitude of *C* is $9^m.03$. As the two spectra appear to be of about the same intensity, the components may provisionally be considered as equal and thus each of magnitude $9^m.03 + 2.5 \log 2 = 9^m.78$. The parallax of Castor has been frequently measured. The mean value given by SCHLESINGER with the addition of the new determination by MITCHELL (*A. J.* **857–858**) is $''.0747 \pm ''.0051$ (m. e.) This makes the absolute magnitude of each of the components of *C* equal to $9.78 + 5 \log .0747 = +4^m.15$ (parallax $= 1''$).

A maximum separation of the spectral lines of 230 km/s. by a period of about 4 days would make the system nearly a duplicate of β Aurigae. This would correspond to a mass of the components (about 2.5 ⊙) not well reconcilable with their absolute brightness ($+4^m.15$, parallax $= 1''$), and spectrum, M. As no individual observations are given by ADAMS and JOY, it could not be seen whether the period of 4 days was sufficiently founded. This doubt was communicated to Mount Wilson, from where it was then reported, that the right period was now spectroscopically found to be $^d.815$. This corrected period was very welcome. Though at that time already 5 minima had been found here, the material was not yet sufficient for an independent determination of the period. Certainly the five minima proved to be in good agreement with the period of $^d.815$, but there remained a discrepancy in the star having been found bright, at one time (table 2, Jul. Day $2424545^d.3464$ to $^d.3533$, plate not backed) when it ought to be faint. This discrepancy was only cleared up when afterwards the plate was remeasured using another comparison star nearer to the variable, the plate having been taken during cloudy weather. At another time a plate (table 2, between Jul. Day $2424525^d.3847$ and $^d.5405$) had been taken, on which all the exposures during minimum, owing to bad weather, were so faint that no determination of the brightness of the variable could be made.

The photographs have been taken with the Leiden 33 cm refractor (focallength = 524 cm, 1 mm on the plate = 39″). All the plates (with the exception of the plates beginning at J.D. $2424433^d.6498$, $440^d.5697$, $545^d.3464$, table 2) had been backed in order to avoid halo's round the image of Castor. Nevertheless several plates were rather fogged in the neighbourhood of Castor. I attribute this fog to the scattering by the atmosphere in misty weather of the light from the two chief components of Castor. This may account for the considerable errors common to all exposures on one plate occasionally met with, especially in unsteady weather for plates taken on various nights, and even on the same night when the weather conditions changed. On the other hand the fact that sometimes the observations were continued up to a great hour angle (the end of the curve in fig. 2 has been taken about 6 hours beyond the meridian) seems to have done no harm to the results.

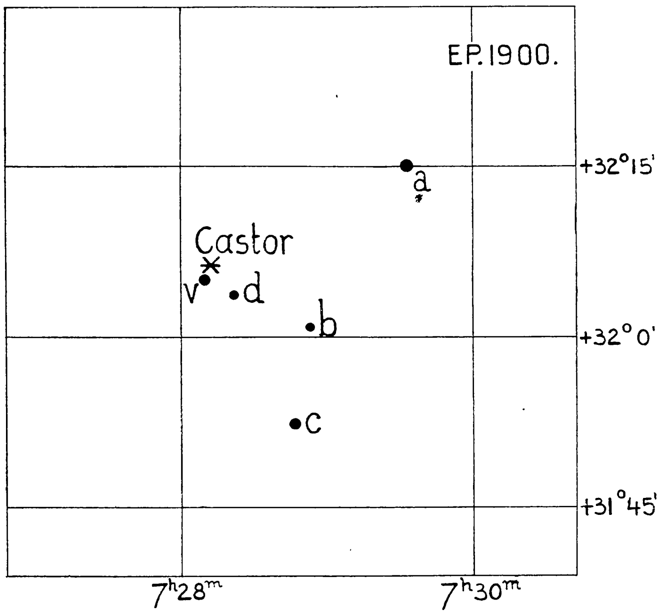
Part of the photographs, marked in the table with an asterisk, have been taken with an objective grating to determine the magnitude scale. The difference in intensity between central image and first order spectrum (grating N° 1) has been adopted to be $^m.923$, according to former measures made by SCHILT and HERTZSPRUNG. (*B. A. N.* **35**, **56** and **68**). The stars *a*, *b* and *c* (fig. 1) have been used to determine the magnitude scale from 12 grating exposures on the plate beginning at Jul. Day 2424500^d.4930. The differences thus found are:

$$m_c - m_a = {}^m.85$$
$$m_b - m_c = .69$$

Later grating exposures confirmed these values.

The intensities of the star images have been measured in the Schilt thermopile photometer (described in *B. A. N.* **60**).

Fig. 1.



2. From the 7 best defined minima, given in Table 1, the following epoch and period have been derived, by least squares:

$$\text{Min.} = \text{J. D. } 2424595^d.4105 + {}^d.81430\ (E - 200)$$
$$\pm\ .0015 \pm\ .00024\ \text{(m. e.)}$$

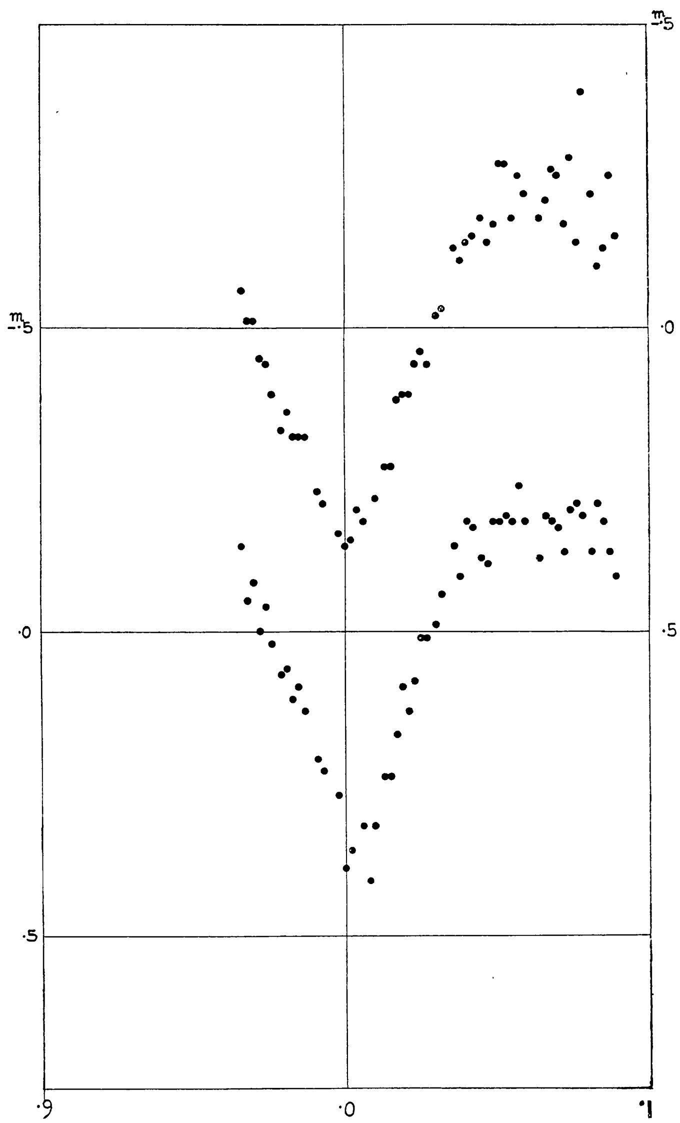
TABLE I.

Weight	Observed min.	Epoch	$O-C$
	J D 2424000 +		
	d	P	d
2	500.5471	83.5	+ .0024
1	573.4233	173.0	— .0013
1	584.4125	186.5	— .0052
1	591.3347	195.0	— .0045
0	595.4127	200.0	+ .0020
2	619.4308	229.5	— .0018
4	639.3854	254.0	+ .0025

The values $O-C$ do not show any sensible difference between the two minima.

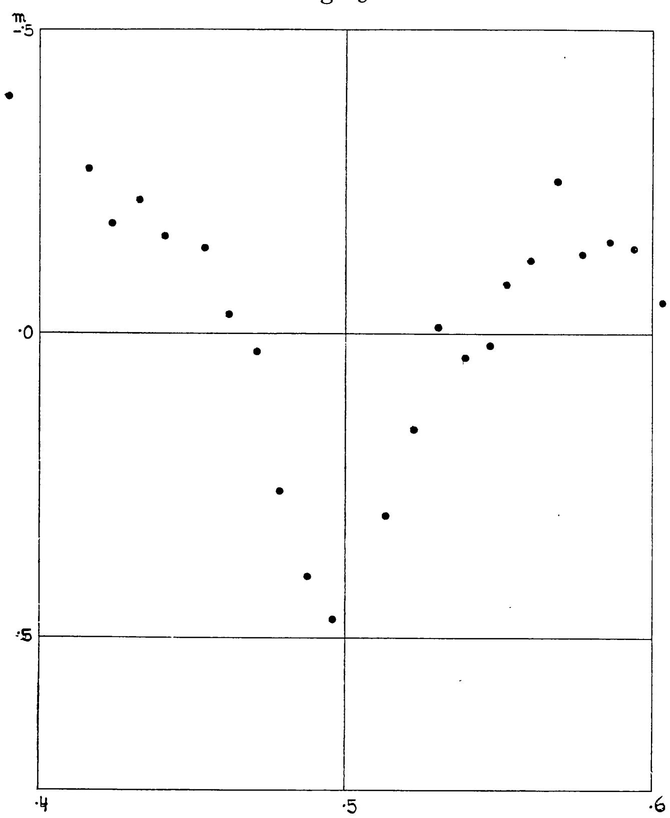
3. The last observed minimum, two lightcurves of which have been drawn (fig. 2) according to comparison of the variable with two different comparison stars, has been used for computing the relative dimensions of the system under the assumption that the two components are equal in every respect. The last minimum but one is represented in figure 3. As no sensible difference between the two minima does appear to exist, and also the spectra of both components are nearly alike in type and brightness, (*A. S. P.* **32**, 158, 1920) this supposition will not be much beside the truth. The orbit has been adopted to be circular, which is not in discordance with the values of $O-C$ in table 1. The darkening at the limb has been adopted to be the same as that of the sun for $\lambda = 450\ \mu\mu$. The two compenents are supposed to be spherical,

Fig. 2.



no reflection of light from one component on the other has been assumed. The values for the darkening towards the limb have been taken from the Gornergrat measures by MOLL, BURGER and VAN DER BILT (*B. A. N.* **91**).

Fig. 3.



If we call the projected distance of the centres of the discs c, and the radius of the disc r, the decrease of light Δm in magnitudes for a given value of c has been calculated to be:

$c = .2\,r$	$\Delta m = {}^{m}.665$
$.5\,r$	$^{m}.49$
$.9\,r$	$^{m}.29$

Taking the depth of the minimum (fig. 2) to be:

$$\Delta m = {}^{m}.538$$

the projected distance of the centres at minimum is accordingly:

$$c = .42\,r.$$

Now the duration of the eclipse will determine the radius of each component as a fraction of the orbit's radius. The points where the eclipse begins and ends are however not sharply defined on the lightcurve. Therefore the time was read off on the diagram, during which the eclipse exceeded an intermediate magnitude, viz: $^{m}.29$, corresponding, as found above, to a projected distance between the centres of the discs less than $.9\,r$. The time elapsed between the two moments, that the eclipse is $^{m}.29$ was $^{P}.040$. Consequently the radius r of each sphere in terms of the orbital radius a is computed to be:

$$\frac{r}{a} = .157.$$

From this and the apparent distance of the centres of the discs at maximum eclipse, the inclination i is found to be:

$$i = 86^\circ.2.$$

For control the corresponding duration of the whole eclipse was calculated and found to be $^{P}.112$ (or $2^{h}12^{m}$), which is in good accordance with the observations.

4. The maximum separation of the spectral lines being known to be about 230 km./s. (ADAMS and JOY, *l.c.*), the absolute dimensions of the system, the mass and the density can now be computed. I found:

radius of each component	$r = 406000$ km.
distance between the centres	$a = 2581000$ km.
mass of each component $= \frac{1}{2}(M_1 + M_2) =$	$.518\ \odot$
density	$\delta = 2.596\ \delta_\odot$

The absolute magnitude $4^{m}.15$ (parallax $= 1''$) and the mass $.518\odot$ have been compared with EDDINGTON's mass-luminosity curve (*M. N.* **84**, 311). The point corresponding tho these values (the absolute magnitude being $9^{m}.15$, parallax $= ''.1$) lies to the right of EDDINGTON's curve, the curve giving the mass $.364\odot$ for the magnitude $9^{m}.15$.

5. The parallax of Castor being known, the surface brightness of C can now be calculated. Taking for the sun's apparent magnitude the value $-26^{m}.9$, corresponding to the absolute magnitude $-{}^{m}.33$ (parallax $= 1''$), the difference in magnitude between the surface intensities of Castor C and the sun comes out $+3^{m}.31$, corresponding to a difference in $\frac{c_2}{T}$ or $\frac{14300}{T}$ of 1.67. (HERTZSPRUNG, *Zeitschr. für wiss. Phot.* Band IV, 48, table 3, 1906.) Adopting for the sun $\frac{c_2}{T} = 2.45$, the value of $\frac{c_2}{T}$ for the companion of Castor is found to be $2.45 + 1.67 = 4.12$, corresponding to an absolute temperature of 3500°, in good agreement with the spectrum M.

For comparison it is worth while to note that the components of 61 Cygni have similar absolute magnitudes and colours.

6. It may be noted that Castor C not only has the greatest parallax, but also the greatest proper motion so far known among eclipsing variable stars viz: $''/a.20$, the next two being α Cor. Bor. with $''/a.16$ and RW. Doradus with $''/a.12$.

7. An account of the observations is given in table 2.

The first column gives the heliocentric astronomical mean time Greenwich for the middle of each exposure. The beginning of a new plate has been indicated by writing the J. D. number in 3 figures before the decimal point. The observations marked with an asterisk have been taken with a grating before the objective. The third column contains the magnitude difference between the variable and the comparison star c; for the last and best plate, beginning at J. D 2424639.3550, not only this difference $m_V - m_c$ has been given, but also the magnitude difference between the variable and the comparison star d (fig. 1), increased by .29, ($m_V - m_d + \cdot 29$), ·29 being the magnitude difference $m_d - m_c$. When exposures were rejected owing to bad quality of the image, their number has been indicated in brackets between the preceding and the following exposure. A horizontal line above or under this number indicates whether the rejected observation belonged to the preceding or the following plate respectively.

8. Dr. J. Woltjer called my attention to a paper by E. A. Milne (*Phil. Trans.* A, vol. **223**, 201, 1922), in which the conclusion is arrived at that the darkening towards the limb of a star is a function of λT, where λ is the wavelength and T the surface temperature (*l. c.* 206). As the surface temperature of Castor C is about half of that of our sun, the wavelength, which for the sun gives the same darkening towards the limb as $\lambda = 450\ \mu\mu$ for Castor C, is about 220 $\mu\mu$. Therefore a new computation was made. The following values for the decrease in light in magnitudes as a function of the projected distance of the centres of the discs, c, were obtained by formula (4) (*l. c.* 206):

$c = 1{\cdot}4\,r$	$\Delta m = \cdot 08$
$\cdot 9\,r$	$\cdot 30$
$\cdot 5\,r$	$\cdot 54$
$\cdot 2\,r$	$\cdot 70$

The distance of the centres at maximum eclipse was $\cdot 50\,r$. From the lightcurve (fig. 2) I took the same value $^{P}\cdot 040$ for the time elapsed between the moments that the eclipse reached the value $^{m}\cdot 29$.

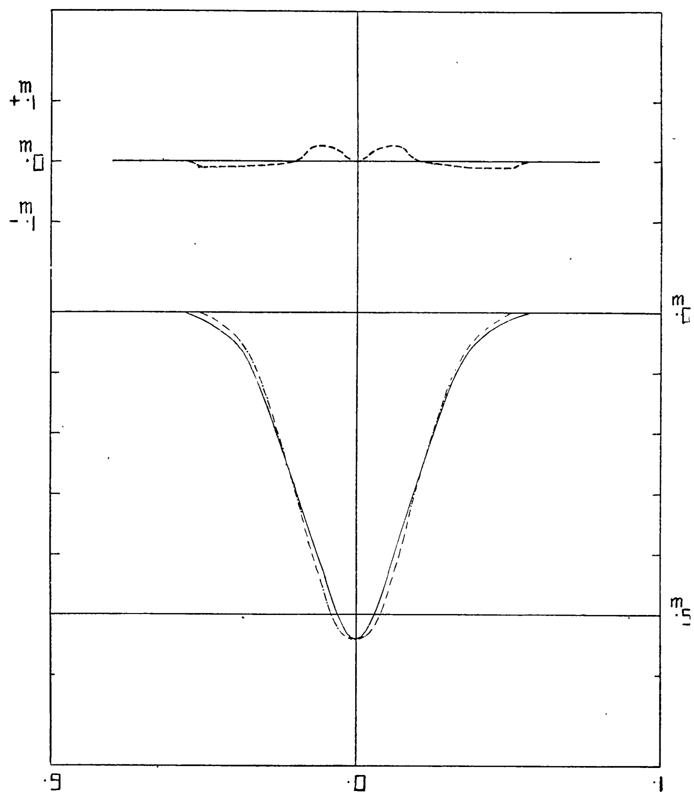
TABLE 3.

	No darkening	Darkened, $\lambda = 450\ \mu\mu$	Darkened, $\lambda = 220\ \mu\mu$
r/a	·159	·157	·162
i	86°·9	86°·2	85°·4
a	2579000 km.	2581000 km.	2582000 km.
r	411000 km.	406000 km.	418000 km.
$\frac{1}{2}(M_1 + M_2)$	$\cdot 517\ \odot$	$\cdot 518\ \odot$	$\cdot 518\ \odot$
δ	$2{\cdot}507\ \delta_{\odot}$	$2{\cdot}596\ \delta_{\odot}$	$2{\cdot}381\ \delta_{\odot}$
Surface brightness	$+\,3^{m}{\cdot}34\ \odot$	$+\,3^{m}{\cdot}31\ \odot$	$+\,3^{m}{\cdot}38\ \odot$
Surface temperature	3500°	3500°	3400°

A third solution was made, neglecting the darkening altogether, to see whether this would seriously affect the results. The distance of the centres at maximum eclipse found under this assumption from Russell's table (*Ap. J.* **35**, 333, 1912), was $\cdot 343\,r$.

The results of the three solutions, which show very little difference, are given in table 3.

Fig. 4.



The density of $2{\cdot}6\ \delta_{\odot}$ makes Castor C a rival to the dense stars of the W Ursae Majoris type.

For the two darkened solutions the theoretical lightcurves, resulting from the values in table 3, have been drawn (fig. 4). They intersect each other at $\Delta m = \cdot 29$, the lightcurve for $\lambda = 220\ \mu\mu$ (dotted line) being more U-shaped and that for $\lambda = 450\ \mu\mu$ (full line) more V-shaped. The difference between the two curves is shown in the upper part of fig. 4, the maximum difference being $^{m}\cdot 03$. The difference between the two curves is so small that the obtained lightcurve as shown in fig. 2 can satisfy both.

Fig. 5.

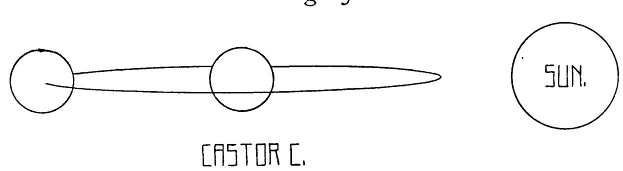


Fig. 5 gives an idea of the dimensions of the system; it has been drawn from the values given in the darkened solution, $\lambda = 450\ \mu\mu$. On the scale of fig. 5 the chief components of Castor would be situated at a distance of more than one kilometer.

TABLE 2.

Middle of exp = JD 2424000 +	Phase	$m_V - m_c$	Middle of exp = JD 2424000 +	Phase	$m_V - m_c$	Middle of exp = JD 2424000 +	Phase	$m_V - m_c$
433·6498 *	1·350	— ·23	572·4028	171·745	— ·16	591·3633	195·030	— ·30
440·5697 *	9·848	— ·22	·4063	·749	— ·14	·3668	·034	— ·51
·5815 *	·862	— ·45	·4097	·754	— ·16 (1)	·3702	·038	— ·57
·5919 *	·875	— ·22	573·3218	172·874	— ·05	·3737	·042	— ·55
·6050 *	·891	— ·53	·3252	·878	— ·06	·3771	·047	— ·43 (1)
454·5584 *	27·026	+ ·37	·3287	·882	— ·16	·3928 *	·066	— ·19
·5653 *	·035	+ ·23	·3322	·886	— ·10	591·4187	195·098	— ·38
·5722 *	·043	+ ·13	·3356	·891	+ ·03	·4222	·102	— ·42
·5791 *	·052	+ ·02	573·3689	172·932	— ·15	·4256	·106	— ·43
·5861 *	·060	+ ·02	·3723	·936	— ·06	·4291	·110	— ·53
500·4930 *	83·436	— ·03	·3758	·940	— ·15	·4325	·115	— ·50
·4999 *	·445	+ ·07	·3792	·944	— ·18	·4360	·119	— ·58
·5068 *	·453	+ ·21	·3827	·949	— ·12	·4395	·123	— ·53
·5137 *	·462	+ ·09	573·4069	172·978	+ ·28	·4429	·127	— ·58 (1)
·5207 *	·470	+ ·11	·4104	·983	+ ·40	·4586 *	·147	— ·48
·5276 *	·479	+ ·27	·4139	·987	+ ·51	·4706	·161	— ·58
·5345 *	·487	+ ·44	·4173	·991	+ ·71	·4741	·166	— ·57
·5414 *	·496	+ ·62	573·4312	173·008	+ ·54	·4776	·170	— ·63
·5484 *	·504	+ ·66	·4346	·012	+ ·51	·4810	·174	— ·47
·5553 *	·513	+ ·68	·4381	·017	+ ·34	·4845	·178	— ·59
·5622 *	·521	+ ·47	·4416	·021	+ ·46	·4879	·183	— ·46
·5691 *	·530	+ ·34	573·4693	173·055	— ·08	591·4983	195·195	— ·35
525·3432 *	113·954	— ·18	·4727	·059	— ·06	·5018	·200	— ·41
·3535 *	·966	— ·23	·4762	·063	— ·04	·5053	·204	— ·33
·3743 *	·992	+ ·04	·4797	·068	·00	594·4595	198·832	— ·27
·3847 *	114·005	+ ·05 (13)	573·4901	173·080	+ ·14	·4663	·840	— ·45 (1)
·5405 *	·196	— ·09	·4935	·085	+ ·07	5·3035	199·868	— ·21
·5509 *	·209	— ·09	·4970	·089	+ ·10	·3070	·873	— ·25
·5613 *	·222	— ·14	·5004	·093	+ ·21	·3105	·877	— ·17
·5717 *	·234	— ·04	573·5143	173·110	+ ·21	·3590	·937	— ·18
525·5932 *	114·261	·00	·5178	·114	— ·07	·3693	·949	— ·20
·6029 *	·273	— ·03	·5212	·119	— ·33	·3728	·954	— ·20
·6132 *	·285	·00	·5247	·123	— ·10	·3763	·958	— ·17
·6236 *	·298	— 06	584·3622	186·432	— ·37	595·3901	199·975	+ ·18
·6340 *	·311	— ·15	·3657	·436	— ·44	·3939	·979	+ ·28
·6444 *	·324	— ·07	·3691	·440	— ·49	·3970	·983	+ ·29
·6548 *	·336	·00	·3726	·445	— ·60	·4005	·988	+ ·27
·6652 *	·349	— ·10	·3761	·449	— ·54	·4040	·992	+ ·32
·6756 *	·362	— ·20	·3795	·453	— ·54	·4074	·996	+ ·36
·6860 *	·375	— ·02	·3830	·457	— ·53	·4109	200·000	+ ·51
·6964 *	·387	+ ·05	·3864	·462	— ·61	·4143	·005	+ ·48
·7067 *	·400	+ ·05	·3899	·466	— ·55	·4178	·009	+ ·32
·7171 *	·413	+ ·06	·3934	·470	— ·47	·4213	·013	+ ·22
543·4627	136·205	— ·20	·3968	·474	— ·28	·4247	·017	+ ·17
·4662	·210	— ·26	·4073 *	·487	+ ·06	595·4975	200·107	— ·22
·4697	·214	— ·35	584·4245	186·509	+ ·01	·5009	·111	— ·29
·4731	·218	— ·27	·4280	·513	— ·17	·5044	·115	— ·30 (1)
545·3464	138·519	— ·32	·4315	·517	— ·15	596·3492	201·153	— ·05
·3499	·523	— ·39	·4349	·521	— ·23	·3527	·157	— ·05
·3533	·527	— ·45	·4384	·526	— ·35	·3561	·161	— ·07
545·3672	138·544	— ·07	·4419	·530	— ·33	·3596	·165	— ·09
·3706	·548	— ·10	·4453	·534	— ·42	·3631	·170	— ·09
·3741	·553	— ·12	·4488	·538	— ·45	·3665	·174	— ·09
572·3107	171·632	— ·07	·4522	·543	— ·49	·3734	·182	— ·08
·3145	·637	— ·08	·4557	·547	— ·48	596·3873	201·199	+ ·22
·3177	·641	— ·04	·4626	·555	— ·43	·3908	·204	+ ·14
572·3336	171·660	+ ·14	·4661	·560	— ·49	·3942	·208	+ ·15
·3370	·664	+ .22	·4696	·564	— ·39	·3977	·212	+ ·19
·3404	·669	+ ·15	·4800 *	·577	— ·20	·4011	·216	+ ·16
572·3481	171·678	+ ·01	591·3027	194·955	— ·28	·4046	·221	+ ·19
·3516	·682	+ ·06	·3148	·970	— ·27	·4081	·225	+ ·20
·3550	·687	+ ·12	·3183	·974	— ·22	596·4185	201·238	+ ·01
572·3800	171·717	— ·12	·3217	·979	— ·21	·4219	·242	— ·03
·3834	·721	— ·13	·3252	·983	— ·04	·4254	·246	+ ·01
·3869	·726	— ·16	·3287	·987	+ ·14	·4288	·250	+ ·02
·3904	·730	— ·15	·3321	·991	+ ·08 (1)	·4323	·255	— ·01
·3938	·734	— ·16	·3478 *	195·011	+ .11	·4358	·259	+ ·03

Middle of exp = JD 2424000 +	Phase	$m_V - m_c$	Middle of exp = JD 2424000 +	Phase	$m_V - m_c$	Middle of exp = JD 2424000 +	Phase	$m_V - m_c$
596·4392	201·263	+ ·19	596·5535	201·403	— ·19	619·4295	229·496	+ ·47 (1)
596·4600	201·289	— ·08 (3)	·5570	·408	— ·23	·4434	·513	+ ·30 (3)
·4635	·293	— ·04	·5604	·412	— ·23	·4503	·522	+ ·16 (1)
·4669	·297	+ ·02	·5639	·416	— ·22	·4572	·530	— ·01 (1)
·4704	·301	+ ·16	·5674	·420	— ·23	·4642	·539	+ ·04 (1)
596·5120	201·352	— ·09	619·3430	229·390	— ·39	·4711	·547	+ ·02 (1)
·5154	·357	— ·01	·3637	·416	— ·27 (1)	·4746	·552	— ·08
·5189	·361	·00	·3707	·424	— ·18 (1)	·4815	·560	— ·12 (1)
·5223	·365	— ·06	·3776	·433	— ·22 (1)	·4884	·569	— ·25 (1)
596·5258	201·369	— ·17	·3845	·441	— ·16 (1)	·4953	·577	— ·13 (1)
·5293	·374	— ·04	·3949	·454	— ·14 (1)	·5023	·586	— ·15 (1)
·5327	·378	— ·01	·4018	·462	— ·03 (1)	·5092	·594	— ·14 (1)
596·5431	201·391	— ·11	·4088	·471	+ ·03 (1)	·5161	·603	— ·05 (1)
·5466	·395	— ·09	·4157	·479	+ ·26 (1)			
·5500	·399	— ·16	·4226	·488	+ ·40 (1)			

Middle of exp = JD 2424000 +	Phase	$m_V - m_c$	$m_V - m_d + ·29$	Middle of exp = JD 2424000 +	Phase	$m_V - m_c$	$m_V - m_d + ·29$
639·3550	253·966	— ·14	— ·06	·4069	·030	— ·01	— ·02
·3567	·968	— ·05	— ·01	·4087	·032	— ·06	— ·03
·3584	·970	— ·08	— ·01	·4121	·036	— ·14	— ·13 (1)
·3602	·972	·00	+ ·05	·4138	·038	— ·09	— ·11
·3619	·974	— ·04	+ ·06	·4156	·040	— ·18	— ·14
·3636	·976	+ ·02	+ ·11	·4173	·042	— ·17	— ·15
·3654	·979	+ ·07	+ ·17	·4190	·045	— ·12	— ·18
·3671	·981	+ ·06	+ ·14	·4208	·047	— ·11	— ·14
·3688	·983	+ ·11	+ ·18	·4225	·049	— ·18	— ·17
·3706	·985	+ ·09	+ ·18	·4242	·051	— ·18	— ·27
·3723	·987	+ ·13	+ ·18	·4260	·053	— ·19	— ·27
·3758	·991	+ ·21	+ ·27 (1)	·4277	·055	— ·18	— ·18
·3775	·993	+ ·23	+ ·29	·4294	·057	— ·24	— ·25
·3810	·998	+ ·27	+ ·29 (1)	·4312	·059	— ·18	— ·22
·3827	254·000	+ ·39	+ ·36	·4346	·064	— ·12	— ·18
·3844	·002	+ ·36	+ ·35	·4364	·066	— ·19	— ·21
·3861	·004		+ ·30	·4381	·068	— ·18	— ·26
·3879	·006	+ ·32	+ ·32	·4398	·070	— ·17	— ·25
·3896	·008	+ ·41		·4415	·072	— ·13	— ·17
·3913	·010	+ ·32	+ ·28	·4433	·074	— ·20	— ·28
·3931	·013	+ ·24	+ ·23	·4448	·076	— ·21	— ·14
·3948	·015	+ ·24	+ ·23	·4464	·078	— ·19	— ·39
·3965	·017	+ ·17	+ ·12	·4485	·081	— ·13	— ·22
·3983	·019	+ ·09	+ ·11	·4502	·083	— ·21	— ·10
·4000	·021	+ ·13	+ ·11	·4519	·085	— ·18	— ·13
·4017	·023	+ ·08	+ ·06	·4537	·087	— ·13	— ·25
·4035	·025	+ ·01	+ ·04	·4554	·089	— ·09	— ·15
·4052	·027	+ ·01	+ ·06				