TABLE 6 (*Continued.*)

$\beta > 50°$

$d < 5''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	1	0	0	1
9.0	0	1	0	0	0	0	0	0	1
10.0	0	2	0	0	1	1	0	0	0
11.0	2	0	0	1	0	1	1	0	1
12.0	1	1	2	2					
13.0 >	3								

$d = 5'' - 10''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	3
9.0	0	1	0	0	1	0	0	1	0
10.0	2	1	0	0	0	1	0	0	1
11.0	1	1	1	0	0	2	0	0	0
12.0	0	2	1						
13.0 >	1								

$d = 10'' - 15''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	1
8.0	0	0	0	0	0	0	0	0	2
9.0	0	1	0	0	1	0	0	1	0
10.0	2	1	0	0	0	1	0	0	0
11.0	1	1	0	0	1	1	0	0	0
12.0	0	2	1						
13.0 >	1								

The Double stars in the Greenwich Astrographic Catalogue, fourth paper,
by Dr. *E. A. Kreiken.*

Dr. GROOT has computed his magnitudes from the general formula

$$\text{magn} = a - b\sqrt{d}.$$

In the declinations $< 72°$ these magnitudes were chequed at the Greenwich Observatory and their values were more accurately determined.

For the declinations $> 72°$ we have only at our disposal the magnitudes derived from the diameters of the star images.

We may be certain, that there will be differences between Dr. GROOT's magnitude scale and the one used at the Greenwich Observatory.

Therefore I have tried in the first place to find the relation between the two magnitude scales.

This was done by comparing Dr. GROOT's magnitudes in the declinations 64°—72° with the ones determined at Greenwich. I have plotted Dr. GROOT's magnitudes against the Greenwich ones and through the points obtained in this way a smooth curve was drawn.

From this curve we read the magnitudes in Dr. GROOT's system which correspond to the magnitudes 7.0, 8.0 ... 13.0 on the Greenwich scale. Table 1 contains the results. The statistical international magnitudes, which have also been entered in this same table were taken from table 3 of the preceding article.

The galactic latitudes of the declination zones considered here $D = 73°—87°$, are chiefly confined to the latitudes 20°—40°. Only a relative small portion of the area falls in the latitudes 0°—20° and 40°—60°. I have not included the data of the declinations > 87°, where a separate magnitude curve was used. The number of stars contained in those declinations is rather small. So we have no possibility to give an accurate reduction of this magnitude scale to the international photographic one.

The observed distribution of double stars contained in table 2 is found by direct counts in Dr. GROOT's Catalogue. As the m and Δm scales used here are the same ones as used in our preceding article, the results of both areas are immediately comparable.

As before, the material was devided into three groups, according to the galactic latitude of the considered area.

To find the numbers of optical pairs $D_o(m)$ in the regions treated here we simply multiply the numbers of optical pairs derived in our preceding article by a fraction, which for the different latitudes will be equal to

$$\frac{\text{Surface of area in square degrees between the declinat. } 73° \text{ and } 87°}{\text{Surface of area in square degree between the declinat. } 64° \text{ and } 73°}$$

TABLE 1 Dr. GROOT's magnitudes compared with the Greenwich and international magnitudes (photographic) in the different latitudes.

Magn. Dr. Groot	Magn. Greenw.	Intern. Magn. 0—20	20—40	40—60
6.4	7.0	7.15	7.37	7.45
7.7	8.0	8.17	8.43	8.62
8.6	9.0	9.24	9.42	9.75
9.4	10.0	10.26	10.50	10.56
10.2	11.0	11.27	11.62	11.53
11.0	12.0	12.41	12.60	12.51
11.7	13.0	13.20	13.38	13.33

TABLE 2 Observed Distribution of double stars over m and Δm (m and Δm on the Greenwich scale)

$\beta = 0° - 20°$ surface 157 □ degrees

$d < 5''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
	0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0	0
9.0	0	0	1	0	0	1	0	0	2	4
10.0	1	1	0	3	0	0	1	0	0	6
11.0	1	4	2	3	4	1				15
12.0	5	4	4	1						14
13.0 >	8	2								10

$d = 5'' - 10''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
	0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	2	2
8.0	0	1	0	1	0	1	0	0	0	3
9.0	0	1	1	0	0	2	0	3	2	9
10.0	1	0	0	1	0	0	3	2	1	8
11.0	2	2	3	3	2	1	0			13
12.0	3	5	3	1						12
13.0 >	7	1								8

$d = 10'' - 15''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
	0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	2	2
8.0	0	0	0	0	9	0	0	0	1	1
9.0	1	0	0	0	0	1	0	0	2	4
10.0	2	0	0	1	0	2	4	1	0	10
11.0	3	0	1	1	2	1	0			8
12.0	4	3	5	2						14
13.0 >	10	1								11

$d = 15'' - 30''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	2.8	2.1	2.4 >	Total
	0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	1	1	0	2	4
8.0	0	0	0	0	0	0	1	0	1	2
9.0	1	0	0	0	2	2	0	1	10	16
10.0	2	1	1	2	3	1	6	2	1	19
11.0	6	7	2	13	7	2	3			40
12.0	18	17	12	3						50
13.0 >	29	4								33

TABLE 2 (*Continued.*)

$\beta = 20° - 40°$ surface 616 □ degrees.

$d < 5''$

$m \backslash \Delta m$	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
	0	0	0	0	0	0	0	0	0	0
7.0										
	0	0	0	0	0	0	0	0	1	1
8.0										
	0	0	0	0	0	0	0	0	0	0
9.0										
	1	0	0	2	0	0	0	1	2	6
10.0										
	5	7	3	1	7	3	3	2	1	32
11.0										
	9	7	4	2	7	2				31
12.0										
	13	7	6	1	1					28
13.0										
>	21	3								24

$d = 5'' - 10''$

$m \backslash \Delta m$	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
	0	0	0	0	0	0	0	0	0	0
7.0										
	1	0	0	0	0	0	0	0	3	4
8.0										
	0	0	0	0	0	0	0	1	7	8
9.0										
	1	0	2	1	2	2	1	3	9	21
10.0										
	1	5	0	4	4	7	4	7	1	33
11.0										
	3	6	8	5	1	5				28
12.0										
	16	16	7							39
13.0										
>	31	6	2							39

$d = 10'' - 15''$

$m \backslash \Delta m$	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
	0	0	0	0	0	0	0	0	0	0
7.0										
	0	2	2	0	0	0	0	0	8	12
8.0										
	1	0	0	0	1	0	1	0	4	7
9.0										
	0	0	0	0	1	2	0	0	9	12
10.0										
	1	1	0	0	4	7	5	2	2	22
11.0										
	1	6	9	6	7	2	3			34
12.0										
	11	15	9	1	2					38
13.0										
>	19	3								22

$d = 15'' - 30''$

$m \backslash \Delta m$	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
	1	1	0	0	0	0	0	0	0	2
7.0										
	0	0	0	0	2	0	1	1	9	13
8.0										
	1	0	2	0	2	0	0	0	9	14
9.0										
	0	0	3	2	6	2	5	4	24	46
10.0										
	7	4	2	6	13	15	19	12	5	83
11.0										
	7	7	18	28	31	24	5	1		121
12.0										
	22	20	55	35	6	1				139
13.0										
>	79	11								90

$\beta = 40° - 60°$ surface 157 □ degrees.

$d < 5''$

$m \backslash \Delta m$	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
	0	0	0	0	0	0	0	0	0	0
7.0										
	0	0	0	0	0	0	0	0	0	0
8.0										
	0	0	0	0	0	0	0	0	0	0
9.0										
	1	0	0	1	1	0	0	0	1	4
10.0										
	4	0	1	0	1	0	0	1	0	7
11.0										
	0	2	0	1	1					4
12.0										
	1	1	2	1						5
13.0										
>	5									5

$d = 5'' - 10''$

$m \backslash \Delta m$	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
	0	0	0	0	0	0	0	0	0	0
7.0										
	0	0	0	0	0	1	0	0	1	2
8.0										
	0	0	0	0	0	1	0	0	0	1
9.0										
	0	0	0	0	0	0	0	1	0	1
10.0										
	0	0	0	1	0	0	1	1	0	3
11.0										
	2	0	1	0	2	1	0	0	0	6
12.0										
	2	4	2	0	0					8
13.0										
>	2	1	1							4

TABLE 2 (*Continued.*)

$d = 10'' - 15''$

$m \backslash \Delta m$	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
7.0	0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0	0
9.0	0	0	0	0	0	0	0	0	1	1
10.0	0	0	0	0	0	0	0	0	1	1
11.0	1	1	0	1	0	2	1	2	0	8
12.0	0	1	2	0	2	1	0	—	—	6
13.0	3	1	1	1						6
>	2									2

$d = 15'' - 30''$

$m \backslash \Delta m$	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
7.0	0	0	0	0	0	0	0	0	1	1
8.0	0	1	0	1	0	0	0	0	1	3
9.0	0	0	0	0	0	0	0	1	4	5
10.0	0	0	0	0	2	0	1	0	3	6
11.0	2	1	1	0	1	1	4	3	—	13
12.0	1	0	2	3	7	4	2	—		19
13.0	4	4	4	1	1					14
>	10	3								13

In table 3 the numbers of optical pairs derived in this way are compared with the observed numbers of double stars.

The differences obs. — comp. give the numbers of physical pairs between definite limits of magnitude. For each group of d the percentages of the observed numbers, which are physical, have been computed.

In our preceding articles we have found, that the double stars in the group $d = 15'' - 30''$ are largely optical ones.

If it is argued that the double stars contained in this group are optical ones, the difference between the observed numbers and the computed ones should be nearly equal to zero.

From our table 3 we find this to be the case when m is smaller than 12[th] magnitude. For the fainter stars large differences occur. This is probably due to the fact, that the probable error of the magnitudes computed from the formula

$$m = a - b \sqrt{d}$$

is very large for the small values of the diameter d, *i. e.* for the fainter stars.

In the final results which were derived from the four papers published until now I have therefore omitted alltogether the values obtained in this paper for the double stars with magnitudes $m_{\text{Greenw.}} > 12.0$.

This was done because we must also expect that there will be a defect of the optical numbers of these same magnitudes in the other groups of distance d.

The fractions indicating the distribution of the optical pairs over m and Δm were given in table 5 (p. 225) of our third article. The numbers of optical double stars $D_o(m, \Delta m)$ between definite limits of magnitude and with a magnitude difference of the components between the limits Δm_1 and Δm_2 etc. are found by multiplying those fractions with the numbers of optical pairs in table 3.

The $m, \Delta m$ distribution of the physical pairs is found by substracting these computed numbers from the observed numbers in table 1.

The results are given in table 4.

TABLE 3. Numbers of optical and physical double stars, percentages of physical pairs.

$\beta = 0° - 20°$

Magn. Greenw.	$d < 5''$ $D_o(m)$ Obs.	$D_o(m)$ Comp.	Phys. $O-C$	% Phys.	$d = 5'' - 10''$ $D_o(m)$ Obs.	$D_c(m)$ Comp.	Phys. $O-C$	% Phys.	$d = 10'' - 15''$ $D_o(m)$ Obs.	$D_c(m)$ Comp.	Phys. $O-C$	% Phys.	$d = 15'' - 30''$ $D_o(m)$ Obs.	$D_o(m)$ Comp.	Phys. $O-C$	% Phys.
7.0	0	0	—	—	0	0	—	—	0	0	0	—	0	1	(— 1)	—
8.0	0	0	—	—	2	0	2	100	2	0	2	100	4	1	3	75
9.0	0	0	—	—	3	0	3	100	1	0	1	100	2	4	(— 2)	—
10.0	4	0	4	100	9	1	8	89	4	2	2	50	16	11	5	31
11.0	6	1	5	83	8	2	6	75	10	4	6	60	19	21	(— 2)	0
12.0	15	1	14	93	13	3	10	77	8	9	2	25	40	32	8	20
13.0	14	1	13	93	12	3	9	75	14	5	9	64	50	24	26	52
>	10	0	10	100	8	0	8	100	11	0	11	100	33	2	31	94

TABLE 3 (*Continued.*)

$\beta = 20° - 40°$

Magn. Greenw.	$d < 5''$				$d = 5'' - 10''$				$d = 10'' - 15''$				$d = 15'' - 30''$			
	$D_o(m)$ Obs.	$D_o(m)$ Comp.	Phys. $O-C$	% Phys.	$D_o(m)$ Obs.	$D_o(m)$ Comp.	Phys. $O-C$	% Phys.	$D_o(m)$ Obs.	$D_o(m)$ Comp.	Phys. $O-C$	% Phys.	$D_o(m)$ Obs.	$D_o(m)$ Comp.	Phys. $O-C$	% Phys.
	0	0	—	—	0	0	0	—	0	0	0	—	2	2	0	0
7.0	1	0	1	100	4	0	4	100	12	1	11	92	13	4	9	60
8.0	0	0	—	—	8	1	7	88	7	2	5	71	14	10	4	29
9.0	6	1	5	83	21	3	18	86	12	5	7	58	46	25	21	46
10.0	32	3	29	91	33	8	25	76	22	14	8	36	83	73	10	12
11.0	31	4	27	87	28	12	16	57	34	20	14	41	121	108	13	12
12.0	28	3	25	89	39	8	31	80	38	14	24	63	139	73	66	47
13.0 >	24	0	24	100	39	1	38	97	22	1	21	95	90	6	84	93

$\beta = 40° - 60°$

Magn. Greenw.	$d < 5''$				$d = 5'' - 10''$				$d = 10'' - 15''$				$d = 15'' - 30''$			
	$D_o(m)$ Obs.	$D_o(m)$ Comp.	Phys. $O-C$	% Phys.	$D_o(m)$ Obs.	$D_o(m)$ Comp.	Phys. $O-C$	% Phys.	$D_o(m)$ Obs.	$D_o(m)$ Comp.	Phys. $O-C$	% Phys.	$D_o(m)$ Obs.	$D_o(m)$ Comp	Phys. $O-C$	% Phys.
	0	0	—	—	0	0	—	—	0	0	0	—	1	0	1	100
7.0	0	0	—	—	2	0	2	100	0	0	0	—	3	1	2	67
8.0	0	0	—	—	1	0	1	100	1	0	1	100	5	2	3	60
9.0	4	0	4	100	1	0	1	100	1	1	0	0	6	3	3	50
10.0	7	0	7	100	3	1	2	67	8	2	6	75	19	8	11	58
11.0	4	1	3	75	6	2	4	67	6	3	3	50	19	15	4	21
11.0	5	1	4	80	8	2	6	75	6	2	4	67	14	14	0	0
13.0 >	5	0	5	100	4	0	4	100	2	0	2	100	13	2	11	85

TABLE 4 Distribution of physical pairs, m and Δm on the scale of the Greenwich Astrographic Catalogue.

$\beta = 0° - 20°$ surface 157 square degrees.

$d < 5''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0
9.0	0	0	1	0	0	1	0	0	2
10.0	1	1	0	3	0	0	0	0	0
11.0	1	4	2	2	4	1			
12.0	4	4	4	1					
13.0 >	8	2							

$d = 5'' - 10''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	2
8.0	0	1	0	1	0	1	0	0	0
9.0	0	1	1	0	0	2	0	3	1
10.0	1	0	0	0	0	0	2	2	1
11.0	2	1	3	2	1	1			
12.0	2	4	2	1					
13.0 >	7	1							

TABLE 4 (*Continued.*)

$d = 10'' - 15''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	24 >
	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	2
8.0	0	0	0	0	0	0	0	0	1
9.0	1	0	0	0	0	0	0	0	1
10.0	2	0	0	0	(—1)	0	4	1	0
11.0	2	(—1)	1	0	0	0			
12.0	2	1	4	2					
13.0 >	10	1							

$\beta = 20° - 40°$ surface 616 □ degrees.

m \ Δm	$d < 5''$ 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	$d = 5'' - 10''$ 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	3
8.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	6
9.0	1	0	0	2	0	0	0	1	1	1	0	2	1	2	2	1	3	6
10.0	5	7	3	1	6	2	2	2	1	1	4	0	2	3	5	3	6	1
11.0	9	7	3	1	6	1				2	5	6	2	1	1			
12.0	13	7	4	0	1					14	14	4						
13.0 >	21	3								30	6	2						

$d = 10'' - 15''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	3.1	2.4 >
	0	0	0	0	0	0	0	0	0
7.0	0	2	2	0	0	0	0	0	7
8.0	1	0	0	0	1	0	1	0	2
9.0	0	0	0	0	1	1	0	(—1)	6
10.0	0	0	0	0	1	4	1	1	1
11.0	0	5	6	1	2	(—2)	2		
12.0	9	13	3	0	0				
13.0 >	18	3							

TABLE 4 (*Continued.*)

$\beta = 40 - 60°$ surface 157 square degrees.

	$d < 5''$									$d = 5'' - 10''$								
m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
7.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
9.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
10.0	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	1	0
11.0	4	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0
12.0	0	1	0	1	1					1	0	0	0	2	1			
13.0	0	1	2	1						1	2	2	0					
>	5									2	1	1						

	$d = 10'' - 15''$								
m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0
9.0	0	0	0	0	0	0	0	0	1
10.0	0	0	0	0	0	0	0	0	0
11.0	1	1	0	1	0	1	1	1	0
12.0	0	0	1	0	1	1	0		
13.0	2	0	1	1					
>	2								