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NANOSTRUCTURE PHYSICS AND FABRICATION

Proceedings of the International Symposium

College Station, Texas *March 13–15, 1989*

Edited by

Mark A. Reed

Central Research Laboratories
Texas Instruments Incorporated
Dallas, Texas

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Department of Physics
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College Station, Texas

ACADEMIC PRESS, INC.
Harcourt Brace Jovanovich, Publishers

Boston San Diego New York
Berkeley London Sydney
Tokyo Toronto

ELECTRON BEAMS AND WAVEGUIDE MODES: ASPECTS OF QUANTUM BALLISTIC TRANSPORT

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Quantum ballistic transport in a two-dimensional electron gas can ideally be explored by means of quantum point contacts. A horn-like shape or a potential barrier in the point contacts gives rise to collimation of the injected electron beam, and thereby has a dramatic effect on magneto-transport effects. The theoretical concepts underlying this new field are discussed, and a geometry for their investigation is proposed. This geometry has potential applications as a novel hot-electron transistor.

1. INTRODUCTION

The new field of quantum ballistic transport (1) in a two-dimensional electron gas (2DEG) can be approached from two complementary points of view, similar to geometrical and wave optics. One is the *trajectory picture*, in which the ballistic motion of electrons along classical trajectories is taken as a starting point. Quantum effects can subsequently be incorporated in a semi-classical approximation. The other is the *mode picture*, which focuses instead on the quantum states of electrons in a confined geometry. These states are the propagating waveguide modes of the transport problem.

Classical ballistic transport in metals is very similar to geometrical optics. It has been studied extensively after the pioneering work on point contacts by Sharvin (2). The 2DEG in a GaAs-AlGaAs heterostructure with its large Fermi wavelength (40 nm) and long transport mean free path (10 μm) offers the possibility to extend these studies to the quantum ballistic transport regime. New effects have been found, suggesting an analogy to wave optics. An example is the conductance quantization of quantum point contacts (3..5), which can be treated in terms of transmission through an electron waveguide. A description of transport as a quantum mechanical transmission problem was proposed in 1957, in a seminal paper by Landauer (6). An extension of his approach to multi-terminal measurements in the presence of a magnetic field has been developed by Büttiker (7). Analogies such as the one with optics are useful, because they can suggest new experiments. In this paper we explore in particular an analogy with Knudsen effusion (8) of a rarefied molecular gas, which is a text-book example of purely classical ballistic transport. This is followed by a consideration of transport in a geometry with point contacts in series, which mimics the physics of molecular beams.

2. QUANTUM POINT CONTACTS

2.1 Knudsen Flow and Sharvin Point Contacts

Consider the three-dimensional flow of a rarefied molecular gas through a narrow orifice (see Fig. 1a). The physics of this Knudsen (8) effusion problem is determined by the condition that the mean free path l is much larger than the linear dimension W of the orifice. The current of molecules J between two reservoirs, between which a gas density difference δn is externally maintained, is given by

$$J = \frac{1}{4}\delta n\ \bar{v}\ W^2 \tag{1}$$

for a square orifice of side W. Note that δn does not have to be small compared to the average density. Here $\bar{v} = (8kT/\pi m)^{1/2}$ is the mean thermal speed of the molecules of mass m. This purely classical result follows readily on integrating the Maxwell-Boltzmann equilibrium distribution function over velocity magnitude and direction (8).

The ballistic electron flow through a Sharvin point contact, connecting two metallic half spaces (2) is illustrated in Fig. 1b. At low temperatures, and for a small voltage difference across the contact, the electrons contributing to the transport are those which move with the Fermi velocity v_F. In the ballistic limit $l \gg W$, the electron flux through a 2D constriction expressed in terms of a density difference is

$$J = \frac{1}{2}\delta n \langle v_F \rangle W, \tag{2}$$

where the brackets denote an angular average over the half space

$$\langle v_F \rangle = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} v_F \cos\phi\ d\phi \ , \tag{3}$$

$$\rightarrow\ J = \frac{1}{\pi}\delta n\ v_F W\ ;\quad \delta n \ll n\ . \tag{4}$$

In the analogous 3D case $J = \delta n\ v_F W^2/4$, which differs from Eq. (1) because of the different statistics for the two problems (Maxwell-

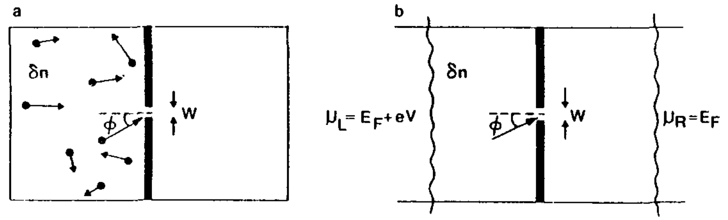

Fig. 1 (a) Knudsen effusion through a narrow orifice connecting two reservoirs with a gas density difference δn. (b) Ballistic transport through a Sharvin point contact.

Boltzmann versus Fermi-Dirac). Accordingly, Eq. (4) only applies if the density difference is small (see sec. 4). An electron density difference cannot be maintained, because of the cost in electro-static energy. Screening charges will reduce δn without changing the electro-chemical potential difference $\delta\mu = eV$, which is the thermodynamic driving force for the current. The two quantities are related by $\delta n = D(E_F)\delta\mu$, with $D(E_F) = m/\pi\hbar^2$ the 2D density of states at the Fermi level, including spin degeneracy. One thus finds (3) for the conductance $G \equiv e^2 J/\delta\mu$,

$$G = \frac{2e^2}{h}\frac{k_F W}{\pi}\ . \tag{5}$$

The 3D Sharvin point contact conductance [1] is $G = (2e^2/h)(k_F^2 W^2/4\pi)$.

Note that the derivation given here does not have to be modified for a channel of finite length connecting the two half spaces, provided the scattering of the particles from the channel boundaries is purely specular (corresponding to conservation of momentum in the channel direction). As it happens, this is a much more realistic assumption for ballistic transport in a constricted 2DEG than in the case of free molecular flow.

2.2 A Refractive Index Step And Transport Over a Barrier

The result Eq. (5) is readily extended to the case where, in addition to the geometrical constriction, a local potential barrier limits the current flow. Such a barrier is indeed present in typical 2DEG point contacts, because of a reduction in local carrier density (3). Although the electro-

[1] In the 3D case the point contact resistance for *small* mean free path ($l \ll W$) is known as the Maxwell spreading resistance which is of order ρ/W, the resistance of a cube of size W. The backscattering associated with the reduced finite mean free path for hot electrons allows one to perform point contact spectroscopy (see *e.g.* Ref. 9). In 2D the analogous spreading resistance does not dominate the sample resistance, since it is only of order ρ, the resistance of a square.

static potential is likely to have a smooth shape, we discuss for simplicity an abrupt potential step in the constriction (see Fig. 2a). Only electrons with sufficient longitudinal momentum will be transmitted, while the transverse momentum is conserved. For a potential barrier of height E_o this leads to a reduction of the *cone of acceptance* (10) of the constriction from its original value of π to $2\alpha_{\max}^{\text{barrier}} = 2\arccos(E_o/E_F)^{1/2}$. The conductance is still given by Eq. (5), but with the reduced Fermi wave vector $k_{F,\min} = (1/\hbar)\,[2m(E_F - E_o)]^{1/2}$. A reduction in carrier concentration and a reduction in width thus both reduce the point contact conductance. Note that a potential barrier in the constriction is analogous to a refractive index step in optics.

2.3 The Conical Reflector And Adiabatic Transport

The analogy of ballistic transport with optics suggests a purely geometrical effect which also limits the cone of acceptance of the point contact. The device we have in mind is the conical reflector (see Fig. 2b), although horns of somewhat different shape obey essentially the same physics (11,10). For such weakly flared constrictions (which may also contain a slowly varying potential barrier), the quantity $S = k_F W \sin\alpha$ is a constant of the motion, or *adiabatic invariant* (S is proportional to the action for motion transverse to the channel (12)). In the semi-classical approximation, the invariance of S implies that trajectories entering the point contact within a cone of opening $2\alpha_{\max}^{\text{horn}} = 2\arcsin[(W_{\min}/W_{\max})\,(1 - E_o/E_F)^{1/2}]$ are transmitted, the others being reflected. The conductance is still given by Eq. (5), but with $k_{F,\min} W_{\min}$ replacing $k_F W$. Both the flaring of the constriction and the presence of a potential barrier tend to *collimate* the injected electron beam at the exit of the constriction. It would be of interest to investigate to what extent diffraction reduces this classical effect, which may be important in geometries with point contacts in series (see below) (10).

2.4 Quantum Mechanical Aspects And Finite Temperature Averaging

So far quantum mechanical aspects other than the degeneracy of the electron gas have been ignored. The conductance quantization of the point contact can be explained semi-classically (3,13), but is most naturally treated in terms of the mode picture, using the multi-channel generalization of the Landauer formula for two-terminal conductances (14)

$$G = \frac{2e^2}{h}\, Tr\; tt^\dagger \quad . \tag{6}$$

where t denotes the transmission matrix, and the trace has to be taken

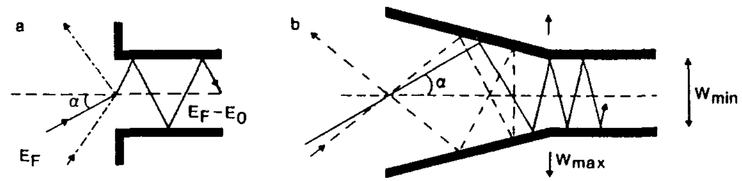

Fig. 2 (a) Ballistic transport through an abrupt constriction containing a potential barrier of height E_o. (b) Ballistic transport through a constriction flared in the form of a conical reflector.

over all populated 1D subbands, or modes, occupied in the 2DEG leads attached to the constriction. If width and potential barrier change sufficiently slowly, the constriction is adiabatic (see before), and the problem becomes remarkably simple as noted by Glazman *et al.* (15). The quantum number n labeling 1D subbands or modes is an adiabatic invariant (in fact, πn corresponds to the classical invariant $S = k_F W \sin\alpha$ mentioned above). Consequently, there is no off-diagonal mode coupling, the N lowest index modes or subbands are perfectly transmitted, and the higher index modes in the 2DEG are perfectly reflected. From this argument and Eq. (6) the conductance quantization follows directly

$$G = \frac{2e^2}{h}\, N \quad . \tag{7}$$

Eq. (5) is recovered in the limit of large quantum numbers, since N is equal to the largest integer smaller than $k_F W/\pi$. We note that contact resistances of order h/e^2 were first proposed by Imry (16).

For an abrupt constriction the physics is more complicated. Recent numerical and analytical work has shown, surprisingly, that even in that case clear plateaux in the conductance as a function of width are obtained (17..20). A mean field approximation in good agreement with the numerical results has been formulated (18), which assumes strong off-diagonal mode coupling. This is consistent with the trajectory picture: Since $\sin\alpha = n\pi/k_F W$ is invariant for motion on a straight trajectory, it follows that n increases appreciably when the width W increases abruptly. One then has highly non-adiabatic, but perfectly ballistic transport [2].

The question arises which model is most realistic. The interference effects observed at low temperatures in the absence of a magnetic field (21) are in conflict with perfect adiabatic transport, and thus would favor the second theoretical model (which predicts transmission resonances

[2] *cf.* our discussion (10) of the interesting series resistance experiment by Wharam *et al.* (22).

(17..19)). However, the single point contact conductance, which measures the total transmission probability, is not the most sensitive tool to discriminate between the two models. Experiments with two point contacts, acting as injector and collector, are much better suited to this purpose. This includes electron focusing experiments (4) and measurements on point contacts in series (22). Coherent electron focusing experiments in a weak magnetic field have unequivocally demonstrated that for extremely narrow point contacts ($W < \lambda_F$), where diffraction is important, a range of magnetic edgestates (see below) is coherently excited (4). This is a clear example of non-adiabatic transport. On the other hand, an interesting phenomenon has recently been found, which demonstrates that adiabatic transport can indeed occur. This is the anomalous quantum Hall effect, which is observed in the electron focusing geometry (23,4), for somewhat wider point contacts in a strong magnetic field. It was found that quantum Hall plateaux in that geometry are due to *selective* excitation and detection of Landau levels by the quantum point contacts. The observation of adiabatic transport between two adjacent point contacts indicates that the potential landscape was sufficiently smooth on the scale of the magnetic length $l_m = (\hbar/eB)^{1/2}$, which in a high magnetic field takes over the role of λ_F.

We pause to discuss the influence of a finite temperature. In ballistic transport, its most important effect is the smearing of the Fermi-Dirac distribution. Two regimes can be distinguished. If $kT \ll \delta E$, the energy difference between successive 1D subbands at the Fermi level, the major effect is to reduce the interference structure (21). In the opposite limit, the 1D subband structure is no longer important, and the semi-classical result (5) (and its 3D counterpart) should be adequate. A finite temperature also affects the coherence length related to inelastic scattering, which is the dominant effect in the diffusive transport regime (1). So far, no evidence has been found which shows that this is equally important for quantum ballistic transport. [Of course, at elevated temperatures (above 10 K) inelastic scattering reduces the transport mean free path, and thus induces a gradual transition to diffusive transport.] This suggests an optimum temperature for well defined plateaux in the point contact conductance, as was indeed observed experimentally (about 0.6 K) (3,5).

2.5 Skipping Orbits And Traversing Trajectories

In this section we give a brief summary of magnetic field effects (see also (24)). It is sufficient to consider electrons at the Fermi level since only they contribute to the non-equilibrium current. A magnetic field affects the nature of the trajectories of these electrons. In the bulk of the 2DEG the electrons move in cyclotron orbits with a radius $l_{cycl} = mv_F/eB$, and at the 2DEG boundary in skipping orbits with the same radius of curvature. In the narrow constriction skipping orbits confined to a single boundary, and traversing trajectories (25,24) interacting with both boundaries coexist if $2l_{cycl} > W$. The two-terminal resistance of the entire sample is essentially a bottle neck problem. In the absence of a magnetic field the dominant bottle neck is the point contact. However, once $2l_{cycl} < W$, an even narrower bottle neck is constituted by the contact between the alloyed ohmic contact, and the wide 2DEG regions. The reason is that the cyclotron orbits do not contribute to the current in ballistic transport because of their circular motion, so that the current is carried by electrons in skipping orbits localized within a distance $2l_{cycl}$ from the boundary. The role of the point contact width W is thus gradually taken over by that of the cyclotron radius. This classical argument is substantiated by a treatment in terms of depopulation of subbands (3,24), which gives for strong fields $2l_{cycl} < W$

$$G = \frac{2e^2}{h} N_L \quad , \tag{8}$$

with $N_L \approx E_F/\hbar\omega_c$ the number of occupied edge states, corresponding to an equal number of bulk Landau levels below the Fermi level. A formula describing the point contact conductance in weak and strong fields has been given elsewhere (3,4). Note that if a potential barrier is present in the constriction $N_L \approx (E_F - E_o)/\hbar\omega_c$, so that in that case the conductance is limited by the point contact, even in a strong magnetic field.

As mentioned before, adiabatic transport is at least approximately realized in a strong magnetic field. The energy of the edge states can be separated in a part labeled by the quantum number $n = 1,2,...,N_L$, corresponding to the quantized circular motion, and a part corresponding to the motion of the guiding center of the skipping orbit along the 2DEG boundary: $E_G = E_F - (n - ½)\hbar\omega_c$. Because of energy conservation, the adiabatic motion of the guiding center is thus along equipotentials. This picture replaces that of the classical trajectories, appropriate in weak magnetic fields [3].

3. ELECTRON BEAMS AND MULTI-TERMINAL MEASUREMENTS

So far we have concentrated on the two-terminal conductance of a single point contact. An important distinction between two- and four-terminal measurements of the conductance has to be made if a magnetic field is applied perpendicular to the 2DEG. A novel negative

[3] Tunneling between right- and left-moving edgestates localized at opposite boundaries can occur at the potential steps at entrance and exit of the constriction. This mechanism was invoked in Ref. 26 to explain an Aharonov-Bohm effect in the point contact resistance.

magnetoresistance effect was found in four-terminal measurements, and it was explained in terms of the reduction of backscattering by the point contact on increasing the magnetic field. This effect follows from the nature of the electron trajectories, discussed earlier (27). In geometries with more point contacts an even richer magnetotransport behavior can be expected (10), as is evident from electron focusing experiments (4). In this section we illustrate the crucial role of the measurement set-up, and the injection of ballistic electrons in a geometry with point contacts in series (see Fig. 3). The physics is similar to that of collimated molecular beams (28). The 2DEG regions between the point contacts can be equipped with ohmic contacts, and we assume that near these contacts a local equilibrium has been established as a result of inelastic scattering. The following discussion is based on the Landauer-Büttiker formalism, which treats transport as a transmission problem (this can be done equally well for classical as for quantum mechanical transport problems). The ohmic contacts (and a region of 2DEG around them) act as *reservoirs* (6,7) with well-defined electro-chemical potentials. For a two-terminal measurement current and voltage probes [4] coincide, and the obtained conductance reflects the total transmission probability. We refer the chemical potentials to a zero ground level. The terminals at ground level act as sinks for electrons which are not *directly* transmitted through the point contacts (like the pumps in a molecular beam apparatus). Another terminal, connected to a 2DEG region behind a point contact acting as injector, is maintained at a chemical potential μ_i. The resulting electron beam is detected by measuring the potential of a terminal attached to a 2DEG region behind a point contact acting as collector. Alternatively, one can measure the collector current. The transmission through the individual point contacts can also be measured separately, thereby allowing a characterization of their properties.

The symmetry properties of multi-terminal measurements in the presence of an external magnetic field have been discussed by Büttiker (7). A four-terminal measurement can be interpreted as a generalized longitudinal resistance measurement if the current flow does not intersect an imaginary line between two voltage probes, and as a generalized Hall resistance measurement otherwise. Three-terminal measurements correspond to one or the other, depending on the sign of the magnetic field. Experimentally, this has been demonstrated in an electron focusing experiment (4). A theoretical description of transport in multi-terminal geometries can be based on the Büttiker formula

[4] A current probe is defined as an ohmic contact with an externally maintained electro-chemical potential, while the electro-chemical potential on a voltage probe follows from the condition that no *net* current flows in the probe.

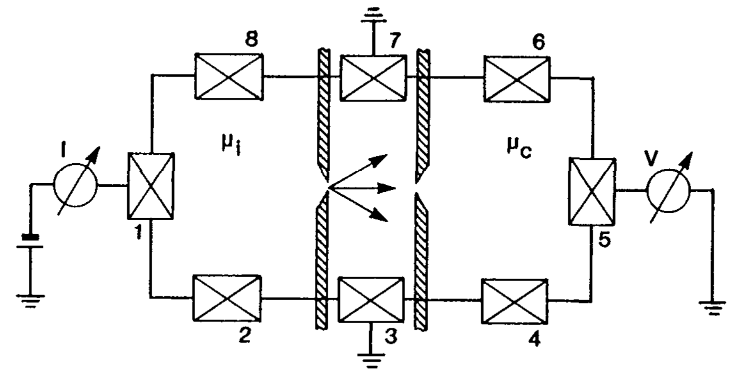

Fig. 3 A geometry with two point contacts in series. Electrons are injected in the 2DEG by the first point contact, and collected by the second. The ohmic contacts 3 and 7 act as sinks for electrons which are not directly transmitted. This structure has also potential use as a novel hot-electron transistor (see sec. 4).

$$\frac{h}{2e} I_\alpha = (N_\alpha - R_\alpha)\mu_\alpha - \sum_{\beta\neq\alpha} T_{\beta\to\alpha}\,\mu_\beta \quad , \tag{9}$$

which relates the current in lead α (with N_α quantum channels) to the chemical potentials μ_β of each of the reservoirs, via transmission and reflection probabilities $T_{\beta\to\alpha}$ (from reservoir β to α) and R_α (from reservoir α back to the same reservoir). These equations may be solved using symmetry relations, and a model (classical or quantum mechanical) for the various transmission and reflection probabilities. This approach has recently been applied to a wide range of phenomena: the quenching of the Hall effect (25), the quantum Hall effect (29,27,23,4,30), propagation around a bend (31), point contacts in series (10), and classical and coherent electron focusing (4). A detailed discussion of these phenomena is beyond the scope of this paper. As an illustration, we apply Eq. (9) to the geometry of Fig. 3.

First, we consider the injection of a total current I. According to Eq. (9), we have

$$\frac{h}{2e} I = \frac{h}{2e^2} G_i\mu_i - T_{c\to i}\,\mu_c \quad , \tag{10}$$

Here we used the definition of the conductance of the injector $G_i = (2e^2/h)(N_i - R_i)$. [Eq. (7) is recovered if $N_i - R_i = N$, the number of subbands occupied in the point contact itself]. The equation for the collector gives us

$$0 = \frac{h}{2e^2} G_c\mu_c - T_{i\to c}\,\mu_i \quad . \tag{11}$$

Only the *direct* transmission probability $T_{i\to c}$ enters here, because all other reservoirs are connected to ground. We thus find for the collector voltage (normalized by the current)

$$\frac{V_c}{I} = \frac{2e^2}{h} \frac{T_{i\to c}}{G_c G_i - \delta} \quad , \tag{12}$$

with $\delta = (2e^2/h)^2 T_{i\to c} T_{c\to i}$. Note the explicit injector-collector reciprocity (7,4). This simple but powerful result has also served as a starting point for a treatment of low field electron focusing (where $T_{i\to c}$ is an oscillating function of B for one field direction, and 0 for the other), and of the anomalous quantum Hall effect in the same geometry (where $T_{i\to c}$ follows from the assumption of adiabatic transport) (23,4). Quite generally $\delta \ll G_i G_c$, as is certainly true for electron focusing, where $T_{c\to i} \approx 0$, but also for narrow point contacts in series because $T_{i\to c} \ll N_i - R_i$. If this is the case, the collector voltage is linearly proportional to the transmission probability $T_{i\to c}$. The geometry of Fig. 3 permits a straightforward determination of the *direct* transmission probability, in contrast to the series resistance experiment of Wharam (22), where the analysis is complicated due to the contribution of indirectly transmitted electrons (10).

The direct transmission probability for the geometry of Fig. 3 can be estimated classically, along the lines of Ref. 10. In the absence of a magnetic field this is quite simple: for abrupt identical point contacts with N quantum channels, separated by a distance $L \gg W$, one finds from the geometry $T_{i\to c} = N(W/2L)$. We note that the transmission probability can be enhanced by flaring of the constrictions, and the introduction of a potential barrier in the point contacts, because of their collimating effects (10). Finally, in the opposite limit of adiabatic transport through identical point contacts with N quantum channels, one would have $T_{i\to c} = N$, which is an upper limit for the direct transmission probability (32). We expect that an external magnetic field deflects the injected electron beam away from the collector, causing a sudden reduction in $T_{i\to c}$ (10). It would be of interest to study experimentally the effects of a magnetic field on the collector voltage in the geometry of Fig. 3, and in different four- or two-terminal configurations (10). Such a study could provide direct experimental evidence for the horn and barrier related collimation effects originally proposed in Ref. 10. Very recently, it has been argued (33) that horn collimation effects are the primary cause for the quenching of the Hall effect in multi-probe electron wave guides (34), by the enhancement of the transmission probability along the channel, at the expense of that into the side probes (7,25). Barrier collimation effects were not invoked in Refs. (33), but may well be important. The quenching of the Hall effect in the experimental systems thus seems to be in essence a classical and not a quantum mechanical effect (25). The role of collimation in coherent electron focusing is discussed in (35).

4. NON-LINEAR TRANSPORT

The current-voltage characteristic of a semi-classical point contact is expected to be non-linear if the voltage drop across the point contact is no longer small compared to the Fermi energy. This is a consequence of the Fermi-Dirac statistics, which shows up as an interdependence of the Fermi velocity and the electron gas density ($mv_F = \hbar k_F = \hbar(2\pi n)^{1/2}$ in 2D, with k_F the Fermi wave vector). For a finite voltage drop across the point contact, electrons in an energy interval eV contribute to the current, and one has to integrate over velocity magnitude and direction

$$J = \frac{1}{2} D(E_F) W \int_{E_F - eV}^{E_F} \left(\frac{2E}{m}\right)^{1/2} dE \; \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos\phi \, d\phi \quad . \tag{13}$$

The resulting current is no longer linear in the applied voltage

$$I = eD(E_F) \frac{W}{\pi} \frac{2}{3} \left(\frac{2}{m}\right)^{1/2} \left[E_F^{3/2} - (E_F - eV)^{3/2}\right] \quad . \tag{14}$$

This result applies for $eV \leq E_F$. For $eV \ll E_F$ Eq. (5) is recovered. For $eV \geq E_F$ this argument predicts a current limited by a saturation value reached at $E_F = eV$. This saturation is a consequence of our assumption that the entire voltage drop is across the point contact, with neglect of any accelerating fields outside the point contact region. At large voltages this is probably not a realistic assumption, and the actual electric field distribution should be determined self-consistently. However, the fact that non-linear transport can arise, as a consequence of the degeneracy of the electron gas, is true regardless of the sophistication of the model used, and is a distinguishing feature between the physics of Sharvin point contacts and classical Knudsen flow.

Non-linear transport through metallic point contacts has been widely investigated because of the possibility to observe phonon related structure in the second derivative of the I-V characteristics. This is known as point contact spectroscopy (9). Since typical phonon energies are of the order of 30 meV and the Fermi energy in a metal is several eV, the non-linearity of Eq. (14) does not play a role in metals. In a 2DEG, where E_F is typically 10 meV the situation is reversed, and these effects have so far obscured possible structure due to inelastic scattering processes. The arguments of this section (still neglecting self-consistency) are readily generalized to the quantum case. The associated breakdown of the conductance quantization of point contacts has been discussed elsewhere (36,37).

The effect of an abrupt potential barrier of height E_o in the constriction can be taken into account heuristically, by replacing E_F by $E_F - E_o$ in Eq. (14). A realistic description would have to include the detailed shape of the barrier, and its dependence on the applied bias voltage. Also, it is important to note that the constriction is electro-statically defined by means of an external gate voltage, referred to the potential of one of the ohmic contacts. An appreciable voltage drop across the sample will therefore also affect the effective gate potential, and thereby the shape and barrier height of the constriction .

Due to the presence of a barrier in the constriction, the point contacts can be used as hot-electron injectors. The non-equilibrium velocity distribution of the injected electrons has been studied in an electron focusing experiment (35,38) One could exploit this by using the geometry of Fig. 5 as a novel *hot-electron transistor*. Williamson and Molenkamp [39] have proposed, as an example, that current amplification can be realized by arranging that the point contact spacing is of the order of a few interaction lengths. Avalanche electron multiplication can then occur in the region between the point contacts. If the second point contact is just pinched-off, the hot electrons created in that proces are transmitted to region 4..6, causing an excess replenishing current in region 3-7.

5. CONCLUDING REMARKS

As we have tried to show, both the trajectory and the mode picture give valuable insight in the physics of quantum ballistic transport Specifically, we have proposed (10) that the presence of a reduced carrier density in narrow constrictions, and the flaring of their opening into a horn, has a dramatic effect on ballistic magneto-transport effects, because of the associated collimation of the injected electron beam. An exploitation of these effects, and of hot-electron injection by means of quantum point contacts, is likely to lead to a new class of ballistic electron devices. We conclude by listing a few areas deserving further attention It would be of interest to model the shape of the potential landscape around the point contact, and at the 2DEG boundary, on the basis of a self-consistent solution of Poisson's equation. Charge-transfer effects between the narrow constriction and the wide 2DEG regions can be expected to play a role in strong magnetic fields. The role of spin has not been discussed in this paper, but there are significant indications that it causes interesting anomalies in high magnetic fields, possibly due to a discrepancy in the value of the g-factor in the point contacts and in the wide 2DEG regions (4) Non-linear transport clearly deserves further experimental and theoretical study. It is obvious that the physics of hot-carrier transport in layered structures ("vertical" transport) and the device concepts developed in that field (such as hot-electron spectroscopy, resonant tunneling hot-electron devices) (40) may be readily translated into similar concepts and devices in the parallel ballistic transport regime discussed here Finally, we note that quantum ballistic transport is not a branch of "mesoscopic" physics, even though it deals with sub-micron structures (16) The experimental realization of a device with *predictable* ballistic quantum interference effects therefore remains an important challenge.

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