The radial-velocity curve of SX Phoenicis, derived from plates taken by G. Westerhout

Woltjer, L.

interstellar velocities we find the following elements for the velocity curve of o Persei

$$P = 4^d.419175 \pm 0.000013$$
$$A = 112.0 \pm 1.0 \text{ km/sec}$$
$$\gamma = + 14.0 \pm 3.0 \text{ km/sec}$$
$$V_{Ca^+\lambda3933} = + 8.5 \pm 2.9 \text{ km/sec}$$

ζ Persei

For ζ Persei we find from 15 spectra a mean velocity + 29.2 ± 1.2 km/sec. The internal standard error of a velocity determination of unit weight was ± 3.3 km/sec, the external error ± 4.5 km/sec and when one rather strongly deviating plate is omitted ± 3.8 km/sec. It is thus clear that it will not be possible from this material to state anything about a possible duplicity of ζ Persei, except that the amplitude must be small. Such a duplicity has been announced by BOUIGUE [1]), who found an amplitude of about 6 km/sec for the radial-velocity curve of ζ Persei from a study of the He lines. In WILSON'S 'General Catalogue of radial velocities' the value + 20.6 km/sec is given for the radial velocity of ζ Persei from a rather large number of observations. Assuming the standard error of this result to be the same as in our determination we find that a systematic correction of − 8.6 ± 1.7 km/sec must be applied to our velocities to bring them in the system of the General Catalogue. For the velocity of the K line of Ca^+ in ζ Persei we find from 10 spectra + 23.3 ± 2.8 km/sec. After applying our systematic correction this becomes + 14.7 ± 3.3 km/sec. In the General Catalogue the value + 12.2 km/sec is given for this line, in good agreement with our result. From these results it is probable that this line is interstellar, as indicated already in the General Catalogue.

[1]) R. BOUIGUE, *Ann. Obs. de Toulouse* **20**, 45, 1950.

THE RADIAL-VELOCITY CURVE OF SX PHOENICIS, DERIVED FROM PLATES TAKEN BY G. WESTERHOUT

BY L. WOLTJER

The radial-velocity curve of SX Phe is derived from 110 spectra taken in 1952 by G. WESTERHOUT. It appears that the radial-velocity curve can be represented as a superposition of two sine waves with amplitudes of 20.6 km/sec and 5.4 km/sec. The skewness of the velocity curve, if present, must be very small.

1. *Introduction.*

The RR Lyrae type variable SX Phoenicis with the very short period of $0^d.05496420$ was discovered by EGGEN [1]) in 1950. An extensive series of photoelectric observations was made by WALRAVEN [2]) in 1952. It appears that the amplitude of the light-curve varies strongly with a period of $0^d.192836$. This beat period is caused by interference of the fundamental oscillation with period P_0 and another oscillation with period $P_1 = 0^d.04277268$. In the same paper WALRAVEN has shown that the light-curves can be derived very accurately from a superposition of two harmonic curves by applying a distortion in magnitude (M-distortion) and a distortion in phase (S-distortion). These distortions appear to be monotonous functions of the magnitude in the original undistorted curve.

In the same season in which WALRAVEN made the photoelectric observations, G. WESTERHOUT obtained in six nights 116 spectra for this star with the 74″ Radcliffe reflector.

[1]) O. J. EGGEN, *P.A.S.P.* **64**, 31, 1952.
[2]) TH. WALRAVEN, *B.A.N.* **12**, 223, (No. 459), 1955.

Professor P. TH. OOSTERHOFF suggested to me to measure these plates, which were kindly put at my disposal by Mr WESTERHOUT. Because of the extremely short period of the variable, the exposure times were on the average only six minutes, equivalent to $\frac{1}{13} P_0$. On most plates the spectrum was visible from 4900 Å to 3900 Å. The dispersion was 49 Å/mm at H_γ. An iron arc was used for the comparison spectrum.

2. *Reduction of the spectra.*

Almost all spectra were measured with the micrometer of the Astronomical Institute of Amsterdam, kindly loaned to us by Professor ZANSTRA, Director of the Institute. In every spectrum we measured a number of metal lines and when possible the hydrogen lines H_β, H_γ and H_δ and the K line of Ca^+. The metal lines are very weak and difficult to measure. A list of the lines used is given in Table 1. The lines were taken from a list by PETRIE [3]) for A stars, from the list by STRUVE and BLAAUW [4]) for RR Lyrae and from the list by GRATTON and LAVAGNINO [5]) for AI

[3]) R. M. PETRIE, *J.R.A.S. Canada* **42**, 213, 1948.
[4]) O. STRUVE and A. BLAAUW, *Ap. J.* **108**, 60, 1948.
[5]) L. GRATTON and C. J. LAVAGNINO, *Zs. f. Ap.* **32**, 69, 1953.

Velorum. The lines λ 4455.02 (L&G) and λ 4314.09 (S&B) were rejected, as they often showed considerable deviations, probably due to the presence of other lines nearby in the spectrum. It was not possible to measure the same lines in all spectra. When all spectra were measured we carefully investigated all lines individually and obtained systematic corrections for four lines and weights for all of them. In this way we tried to make the whole set of velocity measurements as homogeneous as possible. The corrections and weights are also given in Table 1. The hydrogen lines and the K line of Ca^+ were treated separately, as some investigators have found a different behaviour for these lines in other variables. These lines are very wide and difficult to measure. The metal lines for every spectrum were combined to one weighted average. To every velocity determination thus obtained a weight was assigned, depending on the number of weight units entering in the average. For some spectra with bad comparison lines we lowered these weights by one unit. All velocities were corrected for the earth motion. The average velocities for the metal lines and their weights are given in Table 2, together with the velocities for the three hydrogen lines and the K line of Ca^+. For two nights a systematic correction was applied to all velocities, as will be explained in the next section.

TABLE 1

Lines measured in the spectra of SX Phoenicis

λ	Elements	Correction	Weight	Source
4861.34	H_β			P [1]
4549.56	Fe^+, Ti^+	+ 7.0 km/sec	3	P
4534.02	Fe^+, Ti^+		2	P
4501.28	Ti^+		2	P
4481.24	Mg^+, Cr^+, Fe^+		3	P
4415.14	Fe		2	L&G [2]
4404.73	Fe, Fe		2	P
4394.84	Ti^+, Ti^+,		1	L&G
4340.47	H_γ			P
4325.58	Sc^+, Fe		3	L&G
4307.91	Fe		3	P
4289.97	Cr, Ti^+		2	L&G
4271.56	Fe, Fe		2	P
4254.35	Cr		1	L&G
4250.51	Fe, Fe		1	P
4233.31	Fe^+, Fe, Cr^+		3	P
4226.73	Ca	− 10.0 km/sec	3	S&B [3]
4215.60	Sr^+, Cr^+, Fe,		4	P
4202.04	Fe		1	P
4198.66	Fe, Fe		1	L&G
4181.76	Fe, Cr^+		2	L&G
4143.73	Fe		2	S&B
4101.75	H_δ			P
4077.71	Sr^+	+ 7.0 km/sec	5	S&B
4071.55	Fe, Fe, Cr^+		2	P
4063.42	Fe, Fe		4	P
4045.67	Fe, Zr^+, Mn		3	P
4030.54	Mn, Fe, Cr^+	− 10.0 km/sec	2	P
4024.78	Fe, Fe^+, Ti^+		4	P
3933.68	Ca^+			P

1) R. M. PETRIE, *l.c.*
2) L. GRATTON and C. J. LAVAGNINO, *l.c.*
3) O. STRUVE and A. BLAAUW, *l.c.*

3. *Discussion of the results.*

The periods P_0 and P_1 of the fundamental and the interfering oscillations are well known from the optical observations. With these two periods we computed phases for all spectrographic observations. The phases φ_0 and φ_1 are given in Table 2. To demonstrate the harmonic nature of the undistorted light-curves and to derive the distortion WALRAVEN [4]) grouped his observations to a set of curves, every curve giving the magnitude of the variable for a fixed value of the phase ψ as a function of the phase χ, where the phases χ and ψ are given by $\chi = \frac{\varphi_1 + \varphi_0}{2}$ and $\psi = \varphi_1 - \varphi_0$. If the two oscillations P_0 and P_1 are harmonic and undistorted these curves are given by:

$$u = A \sin 2\pi(\chi - \Delta\chi) \qquad (1)$$

with

$$A = \sqrt{a^2 + 2ab\cos 2\pi\psi + b^2} \qquad (2)$$

and

$$\tan 2\pi\Delta\chi = \frac{a+b}{a-b}\tan\pi\psi, \qquad (3)$$

where a and b are the amplitudes of the oscillations P_0 and P_1. It was shown by GRATTON [5]) that for AI Velorum the velocity curve closely resembles the light-curve except for the fact that only the S-distortion and not the M-distortion is present in the velocity curve. Therefore we will investigate the behaviour of these distortions in the velocity curve of SX Phoenicis.

First the phases χ and ψ were computed for all our velocity determinations. On investigating the velocity differences between observations with similar values for the two phases it appeared that two nights differed systematically from the other nights. Therefore we applied to the observations of the night June 9th a correction − 7.9 km/sec and to those of June 11th a correction + 4.7 km/sec. These corrections are already applied to the values given for these nights in Table 2. We then constructed a set of ψ-curves giving the velocity variation as a function of ψ for ten intervals in χ. Next we split up the observations into five groups for five intervals in ψ. In every interval we reduced all observations to a certain value for ψ, using the preliminary ψ-curves. In this way we obtained five groups of observations for respectively $\psi = 0.1$, $\psi = 0.3$, $\psi = 0.5$, $\psi = 0.7$ and $\psi = 0.9$. In every group we combined the observations into some normal points, the weight of such a normal point

4) *L.c.*
5) L. GRATTON *B.A.N.* **12**, 31, (No. 444), 1953.

TABLE 2

The radial velocity observations

No	J.D.hel. 2434 ...	φ_0	φ_1	$V_{met.}$	Weight	H_β − met.	H_γ − met.	H_δ − met.	Ca^+ − met.
1	150.5752	.074	.232	−54.1	2	+ 6.4	+15.2	+19.1	− 4.8
2	.5808	.176	.363	−65.3	1	+12.9	+21.3	+17.8	− 3.1
3	.5857	.265	.477	−47.7	1	+40.4		−16.6	
4	.5906	.354	.592	−49.6	2	−17.1	+ 7.3	− 0.7	+ 2.7
5	.5956	.445	.709	−38.4	0	− 2.1	− 5.6	+17.4	
6	.6003	.531	.819	−43.1	1	+40.8	− 0.8	− 9.8	− 3.4
7	.6052	.620	.933	−20.4	3	+10.7	− 5.0	−35.5	+ 9.4
8	.6130	.762	.116	−13.0	3	+15.2	+ 9.6	− 8.0	+ 9.1
9	.6181	.855	.235	−22.2	2	+14.9	+ 6.9	− 0.2	−15.2
10	.6231	.946	.352	−22.3	1	− 4.0	+12.1	− 5.8	−18.8
11	.6278	.031	.462	−37.7	3	− 7.5	+ 2.2	− 2.9	− 8.5
12	.6327	.120	.576	−21.0	1	− 5.4	− 6.2	− 3.0	−11.7
13	.6380	.217	.700	−35.9	2	+35.0	−15.3	− 6.8	− 9.5
14	.6430	.308	.817	−39.7	2	−31.8	− 7.7	− 4.9	− 7.3
15	.6484	.406	.943	−36.6	1	−18.3	+ 1.0	+23.8	− 1.9
16	.6550	.526	.097	−21.1	0	−26.6	+ 2.3	−21.1	−24.4
17	.6587	.593	.184	−15.0	2	−47.0	−24.0	−11.7	−14.2
18	.6630	.672	.285	− 3.0	1	+36.0	−27.6	+ 8.3	− 8.2
19	151.5712	.195	.518	−49.4	2	− 0.5	+17.3	+24.2	− 5.8
20	.5785	.328	.688	−23.8	1		−25.2		
21									
22	.5896	.530	.948	−11.2	1				
23									
24	.6000	.719	.191	− 7.2	0				
25	.6056	.821	.322	−11.4	0				
26	.6111	.921	.451	−14.2	2		− 6.0		
27	.6184	.054	.621	− 6.6	3				
28	.6231	.139	.731	−33.2	2				
29	.6289	.245	.867	−48.8	3	+13.1			
30	.6349	.354	.007	−45.3	2	+83.2	− 0.4	−14.8	−16.0
31	.6403	.452	.133	−40.2	1		+ 4.6	− 8.8	+13.3
32	.6467	.569	.283	−36.8	1		− 0.4		
33	.6514	.654	.393	−26.8	1	+88.4	+ 3.1	+18.3	
34	.6561	.740	.503	−19.1	2	+28.4	+17.4	+12.0	−48.2
35	173.5361	.817	.044	−21.8	1		+ 7.0		
36	.5451	.981	.255	−14.4	0		−41.0		
37	.5528	.121	.435	−44.1	2				+28.6
38	.5594	.241	.589	−25.2	1				+12.1
39	.5680	.397	.790	−35.9	1				
40	.5764	.550	.986	−33.3	3		+13.3	+ 6.9	
41	.5833	.676	.148	−21.5	3	+58.4	+ 6.6		
42	.5896	.790	.295	+ 0.7	2		−12.2	−24.3	
43	.5958	.903	.440	−10.2	2	− 5.2	− 3.0		
44	.6021	.018	.587	−23.7	3	+10.7	−17.2	−18.0	−28.9
45	.6083	.131	.742	−38.7	2	+13.8	+ 1.8	+17.9	−16.3
46	.6149	.251	.886	−56.5	3				− 2.1
47	.6229	.396	.073	−44.3	2		+14.2		
48	.6292	.511	.221	−52.7	2		−11.2		
49	.6358	.631	.375	−21.6	2				
50	.6427	.757	.536	− 4.9	3		−25.3		
51	175.5657	.743	.495	− 6.6	2		−14.4	−12.9	
52	.5725	.867	.654	−14.5	2		− 9.9	+39.7	
53	.5792	.989	.811	−20.4	2		+16.3	+16.3	
54	.5862	.116	.974	−31.7	1		−21.4	− 0.3	−18.0
55	.5931	.241	.136	−59.4	0		+19.8	+10.7	+ 2.6
56	.6001	.369	.299	−43.4	2		+13.9		
57	.6161	.659	.673	−19.2	1		+20.2	−21.2	
58	.6265	.849	.916	− 7.8	1		+20.6		
59	.6334	.975	.078	−20.9	2		− 0.2		−12.2
60	.6404	.102	.241	−48.0	2		− 0.2	+11.7	
61	.6473	.228	.403	−51.3	2		−17.2	− 4.5	− 6.8
62	.6542	.353	.564	−64.5	1		− 2.2	− 9.5	

TABLE 2 *(continued)*

No	J.D.hel. 2434...	φ_0	φ_1	$V_{met.}$	Weight	H_β−met.	H_γ−met.	H_δ−met.	Ca^+−met.
63	176.5325	.333	.098	−57.8	1		+28.6	+9.8	−17.0
64	.5401	.471	.276	−46.3	1		−20.1		
65	.5506	.662	.521	−19.0	0		−20.3		
67	.5648	.920	.853	−1.6	2				
68	.5710	.033	.998	−40.5	0		+26.4		
69	.5815	.224	.244	−54.7	0				+6.1
70	.5877	.337	.389	−63.5	0		−52.2	+97.8	
71	.5943	.457	.543	−33.8	2			+9.4	
72	.6009	.577	.697	−19.6	0		+0.4		
74	.6141	.817	.006	+8.0	1				
76	.6322	.146	.429				H_γ = −82.7		
77	.6367	.228	.534	−46.5	1				
79	.6454	.387	.738	−53.9	0				
80	.6502	.474	.850	−35.1	1		+2.5		
81	252.3107	.015	.748	−14.9	2		+19.3		
82	.3182	.152	.923	−42.2	3	−40.0	+9.4		
83	.3319	.401	.243	−61.7	2		+55.9		
84	.3378	.508	.381	−47.0	1				
85	.3439	.619	.524	−16.6	1				
86	.3503	.736	.674	−5.9	3	+54.4	−10.0		
87	.3892	.444	.583	−36.3	3	−43.6	−1.7	+16.3	
88	.3958	.564	.737	−27.6	3	−4.8	−7.0	+4.9	
89	.4020	.676	.882	−13.9	3	+7.7	−13.9	−18.7	
90	.4097	.817	.062	−14.8	2	+1.4	+19.1		
91	.4145	.904	.175	−6.4	3	−18.7	+7.3	−33.1	
92	.4201	.006	.306	−31.7	3	−72.1	+0.3		
93	.4541	.624	.100	−23.5	3		+9.0	+7.1	
94	.4628	.783	.304	−7.5	3	−6.0	+15.1		
95	.4690	.895	.449	−16.9	3	+50.9	+2.5	+28.8	
96	.4749	.003	.587	−19.0	1	+16.4	−3.8		
97	.4805	.105	.718	−37.5	3				
98	.4861	.207	.849	−56.2	1	+162.8	−2.1	−1.6	
99	.4958	.383	.075	−46.1	3	−7.9	−9.1		
100	.5013	.483	.204	−38.0	3	−1.7	−13.5		
101	.5069	.585	.335	−27.2	3	+4.1	−0.8	−43.3	
102	.5125	.687	.466	−17.8	2	−17.1	+3.3	−30.3	
103	.5187	.800	.611	−9.1	3		+1.3		
104	.5260	.932	.781	−20.0	2	+13.6	+13.9		
105	.5395	.178	.097	−38.1	1	+69.6			
106	.5479	.331	.293	−53.2	3	−8.0	+8.2	+17.5	−13.2
107	.5541	.444	.438	−36.5	3		+3.3		
109	.5770	.860	.974	+1.7	3	−1.1	+2.5		
110	.5840	.988	.137	−30.8	1	−39.9	−7.4		
111	.6090	.443	.722	−20.0	2	+37.1	+18.8		
112	.6156	.563	.876	−25.2	2	+54.2	+100.1		
113	.6222	.683	.031	−21.1	3	+52.5	+62.2		
114	.6316	.854	.250	−11.9	2		−2.8		
115	.6399	.005	.444	−26.4	2	+19.8	+6.6	+1.8	
116	.6472	.138	.615	−31.3	1		−26.2		

being equal to the number of weight units of the observations from which the normal point was derived. From these points we derived the five χ-curves. It appeared that the normal points can be represented by sine curves, to which only an S-distortion has been applied. Such a distorted sine curve can quite well be represented by the formula

$$v = v_0 + A \sin 2\pi \{\chi - \delta\chi - \alpha A \sin 2\pi (\chi - \delta\chi)\}, \quad (4)$$

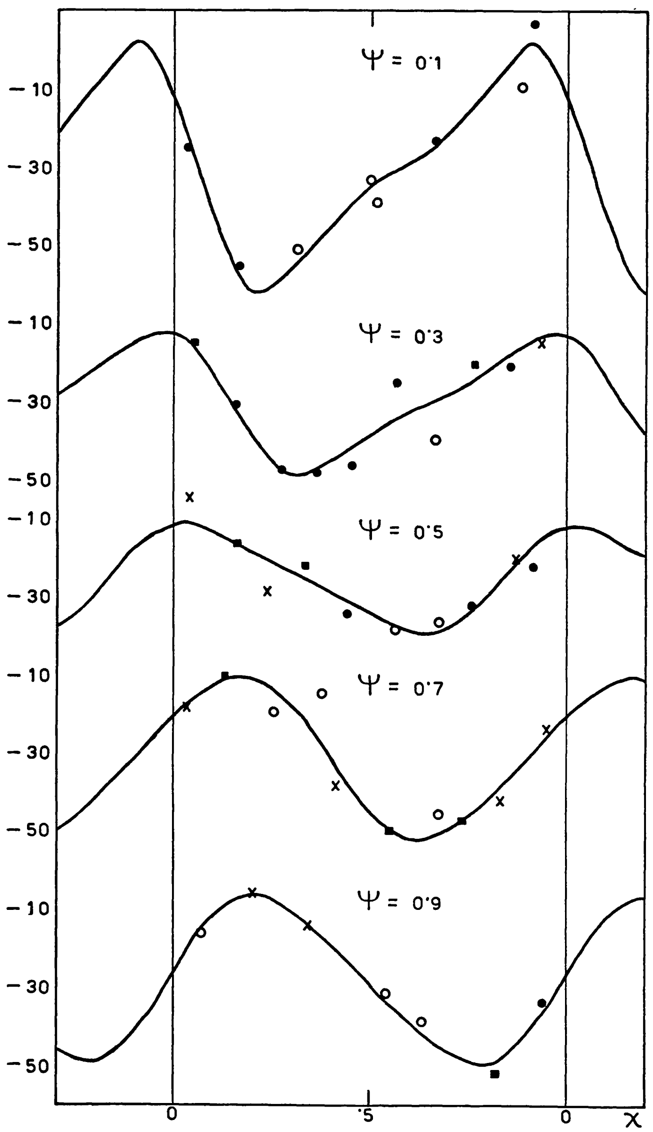
the last term in the argument causing the skewness of the curves. To the five groups of normal points we fitted by a least-squares solution curves of this form, the unknowns being in every solution v_0, A, α and $\delta\chi$. The results for the four parameters are given in Table 3. The curves are shown in Figure 1.

It appears from these solutions that the average skewness and thus the S-distortion is small. It is somewhat remarkable that the mean velocity v_0 for $\psi = 0.5$ is so high as compared to the values of the other four curves. The amplitudes of the two oscil-

TABLE 3
The values of the parameters of the χ-curves

ψ	v_0	A	α	$\delta\chi$
0.1	-30.5 ± 2.1	$+32.0 \pm 3.1$	$+0.0036 \pm 0.0010$	$+0.563 \pm 0.015$
0.3	-30.5 ± 2.0	$+19.2 \pm 3.1$	$+0.0051 \pm 0.0021$	$+0.647 \pm 0.026$
0.5	-25.4 ± 2.0	$+13.7 \pm 3.0$	-0.0047 ± 0.0030	$+0.831 \pm 0.023$
0.7	-31.2 ± 1.6	$+20.8 \pm 2.4$	$+0.0012 \pm 0.0015$	$+0.901 \pm 0.018$
0.9	-28.1 ± 1.6	$+21.7 \pm 2.0$	-0.0020 ± 0.0011	$+0.993 \pm 0.017$

FIGURE 1



Radial-velocity curves as a function of χ for different values of ψ

- ○ weight 1 — 2
- ● weight 3 — 4
- × weight 5 — 6
- ■ weight 7 — 10

lations P_0 and P_1 and the zero point of ψ were determined by a least-squares fit of the five amplitudes A to equation (2). The result is $a = 21.9$, $b = 5.8$, $\psi_0 = 0.067$, but the fit is not too good. The ratio $\frac{a+b}{a-b}$ is equal to 1.72, whereas WALRAVEN found from the optical observations the value 1.61. For ψ_0 WALRAVEN found $\psi_0 = 0.003$, but the uncertainty in our value is large. Having thus found that the M- and S-distortions are both rather insignificant for the velocity curves, we combined all our observations in one least-squares solution by fitting them to the formula

$$v = v_0 + a \sin 2\pi\,(\varphi_0 - \delta) + b \sin 2\pi\,(\varphi_1 - \delta),$$

where δ is the phase difference between the velocity curve and the light-curve. The value of δ is assumed equal for both oscillations, which seems reasonable in view of the fact that within the limits of the uncertainty we found no sensible difference between the zero points for ψ as derived from the optical and the velocity observations. We obtained the following values for the parameters

$$\begin{aligned} v_0 &= -\,29.1 \quad \pm 0.8 \text{ km/sec} \\ a &= +\,20.6 \quad \pm 1.2 \text{ km/sec} \\ b &= +\;\;5.4 \quad \pm 1.2 \text{ km/sec} \\ \delta &= +\;\;0.542 \pm 0.009 \text{ cycle} \end{aligned}$$

The curve thus obtained and the individual observations are shown in Figure 2. From this figure it is seen that no clear trace of any distortion is present.

The ratio $\frac{a+b}{a-b}$ is found to be 1.70 ± 0.22. The difference between this value and the value derived from the optical observations is less than the standard error of the determination.

It thus seems that the velocity curve is almost exactly the mirror image of the undistorted light-curve, apart from a small phase shift, retarding the velocity curve by 0.042 cycle. It is very remarkable that the skewness so clearly present in the light-curve seems to be lacking in the velocity curve. This can be understood from the consideration given in a following investigation (p. 64).

From the measurements of the hydrogen lines those of H_γ are the most reliable. We found for this line the same amplitude and mean velocity as for the metal lines.