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A POSSIBLE PERIOD-LUMINOSITY RELATION AMONG β CANIS MAJORIS STARS,

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Observational evidence for a possible period-luminosity relation among β Canis Majoris stars is given in the first section. Visual absolute magnitudes are derived from membership of associations and from proper motions, and lead to the relation $M = -10 - 9 \log P$; this relation is supported by W. W. MORGAN's luminosity classifications.
In the second section the theoretical period-luminosity law is derived for various opacity and energy sources, and on the assumption that these stars are homologous. Assuming a stellar model based on electron scattering and the carbon cycle, a coefficient of log P of −10.2 is predicted, which agrees with the observations within their uncertainty. Further, this model is consistent with the narrow spectral range of these stars.

Observational evidence.

The recent increase of observations on the variations in brightness and radial velocities of the β Canis Majoris stars has brought a new interest in the problem of the nature of these stars. Various explanations have been suggested, for an account of which we refer to O. STRUVE's article in *Annales d'Astrophysique* **15**, 157, 1952. In the present note we want to draw attention to the possible existence of a period-luminosity relation among these stars and its theoretical aspects.

Table 1 contains the data on the luminosities and approximate periods for the six objects for which both these quantities are known with some accuracy. The periods P in the variation of radial velocity and

TABLE 1

Observational data on β Canis Majoris stars

Star	HD	m	Spectrum	P	$\log P + 1$	Ref.	M_v	p.e.	Ref.
δ Ceti	16582	4.0	B2 IV	0.161	.208	1, 2	−2.8	± .4	
β C Ma	44743	2.0	B1 II-III	0.250	.398	3	−5.0	1.0	
σ Scorpii	147165	3.1	B1 III	0.247	.393	4, 5	−4.5	.2	13
β Cephei	205021	3.3	B2 III	0.190	.279	6	−2.8	.5	
12 Lacertae	214993	5.2	B2 III	0.193	.286	7, 8, 9	−3.4	.2	14
16 Lacertae	216916	5.5	B2 IV	0.169	.228	10, 11, 12	−3.2	.2	14

1. M. F. WALKER, *P.A.S.P.* **65**, 49, 1953.
2. F. HENROTEAU, *Pub. Dom. Obs. Ottawa* **9**, 26, 1925.
3. O. STRUVE, *Ap. J.* **112**, 520, 1950.
4. R. D. LEVÉE, *Ap. J.* **115**, 402, 1952.
5. A. R. HOGG, *M.N.* **111**, 339, 1951.
6. M. RUDKJÖBING, *Ap. J.* **109**, 331, 1949.
7. O. STRUVE, *Ap. J.* **113**, 589, 1951.
8. S. V. NEKRASOVA, *Bull. Crimean Obs.* IX, 126, 1952.
9. J. J. RUIZ, *J.R.A.S. Canada* XLVI, 203, 1952.
10. O. STRUVE *et al.*, *Ap. J.* **116**, 81, 1952.
11. G. R. MICZAIKA, *Ap. J.* **116**, 99, 1952.
12. M. F. WALKER, *Ap. J.* **116**, 106, 1952.
13. *Groningen Publ.* No. 52, Table 33, corrected for interstellar reddening according to *Mt Wilson Contr.* No. 621, 1940.
14. A. BLAAUW and W. W. MORGAN, *Ap. J.* **117**, 258, 1953.

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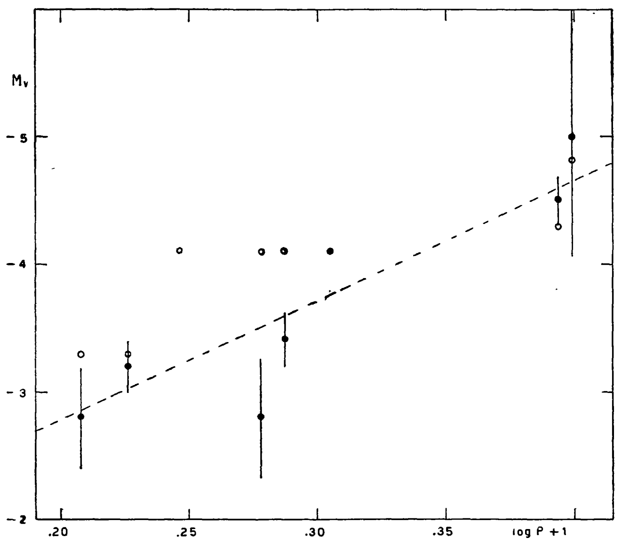
CONTENTS

FIGURE 1



Visual absolute magnitude and logarithm of period for β Canis Majoris stars. Dots: M_v derived from membership of associations or from proper motions; open circles: M_v according to W. W. MORGAN's luminosity classifications.

brightness (P_1 or P_2 in STRUVE's designation) are taken from the papers referred to next to the values of log P. As a rule, the period corresponding with the largest amplitude in radial velocity was taken. The visual absolute magnitudes, M_v, in the cases of σ Scorpii and 12 and 16 Lacertae, which are members of associations of B stars, are taken from the sources referred to next to the values of M_v. For β Canis Majoris, β Cephei and δ Ceti they were derived from the parallactic motions. The proper motions are small, and therefore care was taken to reduce them to the most reliable fundamental system, for which we adopted the average of FK3 and N30. The values of the proper motions used are those derived by H. R. MORGAN of the U.S. Naval Observatory and are part of a list of improved proper motions of early-type stars under discussion at the Leiden Observatory. For the three stars considered here, the probable error of the proper motion is below 0".001 per year. This high accuracy together with the generally very small peculiar motions of this category of stars justifies the attempt to determine individual absolute magnitudes from the υ-components. Their values (freed from precessional errors and differential galactic rotation) are: 0".0043 for β Canis Majoris, 0".0195 for β Cephei and 0".0120 for δ Ceti. The small value for β Canis Majoris is partly due to the star's nearness to the anti-apex and explains the large probable error of its M_v in the table. The uncertainty in the absolute magnitudes is due mainly to the unknown amount of the peculiar motions. In estimating the probable errors as given in the table, we have assumed an average residual motion in one component of ± 4 km/sec. For δ Ceti an additional estimate of the luminosity was obtained by assuming the star to be associated with the loose group of early B stars at high negative galactic latitudes around $l = 150°, b = -30°$. This gives $M_v = -2.2$, whereas the proper motion leads to $M_v = -3.4$. An average value was adopted.

In the accompanying figure M_v is plotted against log P (dots). A period-luminosity relation is indicated. As the M_v for β Canis Majoris is weakly determined, it contributes little weight to the suggested relation. It may be pointed out, however, that the relation is also suggested if we use the luminosities as obtained directly from W. W. MORGAN's luminosity classes, which are given in Table 1. The luminosities, as given for these luminosity classes in MORGAN and KEENAN's article in "Astrophysics" (ed. J. A. HYNEK), are plotted as open circles. These classifications are also available, together with log P, for ν Eridani, ($P = 0.174$[1])) and BW Vulpeculae ($P = 0.201$[2])). For the average relation as shown by the dotted line we adopt:

$$M_v = -10 - 9 \log P,$$

the uncertainty in the slope being of the order of 30%.

Theoretical aspects.

It is of interest to see to what extent this result can be used to suggest or limit mechanisms producing the variations. For stable stars, the requirement of hydrostatic and energy equilibrium implies the existence of two relations between the physical parameters. These will be referred to as the mass-luminosity law and the energy-generation law[3]). From the virial theorem one can show that the period-density relation is implied if gravitational forces govern the motions[4]). With the assumption that these stars are homologous (differing only in M, R, L, X, Y) these three relations can be used to express the period, P, as a function of luminosity, L, and composition (X, Y). If the variation results from internal instability, one would expect a fourth relation[5]). There is not enough information available now to predict its form.

Without detailed model calculations, we can obtain the period-luminosity relation, provided we assume that the models resemble COWLING's model. With this assumption the form of the mass-luminosity law

[1]) O. STRUVE *et al.*, *Ap. J.* **116**, 398, 1952.
[2]) R. M. PETRIE, *Pub. A.A.S.* **9**, 53, 1938; R. P. KRAFT, *P.A.S.P.* **65**, 45, 1953; V. B. NIKONOV and E. K. NIKONOVA, *Bull. Crimean Obs.* IX, 135, 1952.
[3]) H. BONDI, *M.N. R.A.S.* **110**, 595, 1950.
[4]) J. JEANS, "Astronomy and Cosmogony", Cambridge University Press, p. 376, 1928.
[5]) S. ROSSELAND, "The Pulsation Theory of Variable Stars", Oxford, Clarendon Press, p. 78, 1949.

depends upon the opacity law. We shall write this as $\varkappa \approx \varkappa_0 \rho^\beta T^\gamma$, from which one can obtain

$$L = C_1^* M^{3-\beta-\gamma} R^{3\beta+\gamma} \mu^{4-\gamma}. \qquad (1)$$

For electron scattering $\beta = \gamma = 0$, while for KRAMERS' law $\beta = 1$, $\gamma = -3.5$.

The energy-generation law usually depends on one critical process. When this is the case, we can express the energy generation per gram as $\varepsilon \approx \varepsilon_0 \rho T^\alpha$, where we include in ε_0 the abundance of the particles concerned. With this rate of energy generation, the "energy-generation law" is of the form

$$L = C_2^* M^{2+\alpha} R^{-3-\alpha} \mu^\alpha. \qquad (2)$$

The period-density relation is

$$P = C_3 M^{-\frac{1}{2}} R^{\frac{3}{2}}. \qquad (3)$$

From these, as was noted above, the mass and radius can be eliminated and we obtain

$$\Delta M = p \log P/C_3 + q \log \mu + r \log C_1^* + s \log C_2^*, \qquad (4)$$

where the asterisk denotes that the composition factors have been included in the constant. Here, p, q, r and s are found to be

$$\left.\begin{aligned} p &= -2.5\left(\frac{-9 - 3\alpha + \gamma - 3\beta - 2\alpha\beta}{3 - \alpha - \gamma}\right); \\ q &= -2.5\left(\frac{-12 + \alpha + 3\gamma}{6 - 2\alpha - 2\gamma}\right); \\ r &= +2.5\left(\frac{3 + 2\alpha}{6 - 2\alpha - 2\gamma}\right); \\ s &= -2.5\left(\frac{9 - 2\gamma}{6 - 2\alpha - 2\gamma}\right). \end{aligned}\right\} \qquad (5)$$

Numerical values of these coefficients may be found in Table 2. For the carbon cycle $\alpha \sim 20$, while for the proton-proton reaction $\alpha \sim 4$. The coefficients have been computed both for KRAMERS' opacity law and for electron-scattering opacity.

TABLE 2

Coefficients of the period-luminosity and period-temperature relations

Model	Carbon cycle		p–p reaction	
	Kramers' law	electron scattering	Kramers' law	electron scattering
α	+20	+20	+ 4	+ 4
β	+ 1	0	+ 1	0
γ	− 3.5	0	− 3.5	0
p	−21.4	−10.2	+35.5	−52.5
q	− .231	+ .588	+ 9.25	−10.0
r	− 3.98	− 3.16	+ 5.50	−13.8
s	+ 1.48	+ .662	− 8.00	+11.2
p'	+ 1.53	+ .456	− 3.45	+ 3.75
q'	+ .255	+ .176	− .575	+ 1.00
r'	+ .380	+ .301	− .450	+ 1.125
s'	− .130	− .051	+ .700	− .875

Further, on introducing

$$L = C_4 R^2 T_e^4, \qquad (6)$$

we can obtain a relation of the form

$$T_e = (P/C_3)^{p'} (\mu)^{q'} (C_1^*/C_4)^{r'} (C_2^*/C_4)^{s'}, \qquad (7)$$

where

$$\left.\begin{aligned} p' &= -\frac{11 + \alpha + \beta - 3\gamma + 2\alpha\beta}{4(3 - \alpha - \gamma)} \\ q' &= \frac{\gamma - 4 - \alpha}{8(3 - \alpha - \gamma)} \\ r' &= -\frac{1 + 2\alpha}{8(3 - \alpha - \gamma)} \\ s' &= \frac{7 - 2\gamma}{8(3 - \alpha - \gamma)}. \end{aligned}\right\} \qquad (8)$$

Numerical values of these coefficients can also be found in Table 2.

The coincidence between the coefficient p, obtained from the observations and that predicted for the carbon cycle and electron scattering, is noteworthy. As mentioned above, however, the observational uncertainty in the slope is large and one must accept its numerical value as insecure. Although the carbon cycle is most probably the major source of energy, the proper values of β and γ for the opacity law can only be obtained by computing a consistent model for these stars.

In a model for the O9 main-sequence components of Y Cyg, M. RUDKJÖBING [1]) finds that ninety per cent of the opacity is produced by electron scattering. This ratio depends on the temperature, density, and composition. For Y Cyg, RUDKJÖBING obtains a consistent model with $Z = .08$, which is much higher than that for most solar models. For the β CMa stars (with the same composition), the electron-scattering opacity will be more nearly comparable with the photoelectric opacity. If the abundance of heavy elements is reduced, however, electron scattering will again be dominant. From this discussion of Y Cyg one can also conclude that the neglect of the radiation pressure is permissible, since it never amounts to more than four per cent for the β CMa stars.

This model, based upon energy generation by means of the carbon cycle and opacity produced by electron scattering, is not inconsistent with the observed correlation between period and spectral class. The coefficient $p' = 0.456$ is equivalent to a change in

1) *Annales d'Astrophysique* **16**, 65, 1953.