

INTEGRATED MASSES OF GALAXIES

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ABSTRACT

Twenty-one centimeter hydrogen line velocities, half-widths, and fluxes are presented for 112 spiral and irregular galaxies, the majority in groups. Comparison of our velocities with values listed in the de Vaucouleurs Second Reference Catalog (RC2) suggests that the mean errors of RC2 velocities may be underestimated typically by a factor of 1.5. Integrated Brandt masses (M_T) are derived from the half-widths, taking into account the increased knowledge of the properties of galactic rotation curves obtained by Huchtmeier. Masses derived by our technique are, on the average, within a factor of 1.4 of those determined by detailed analysis of individual rotation curves. Holmberg luminosities corrected for internal and Galactic extinction (L_{pg}) are determined. Our sample of galaxies has an average integrated mass-to-light ratio $\langle M_T/L_{pg} \rangle = 11 \pm 1$ solar units (when $H = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$), and $\langle M_T/L_{pg} \rangle$ is independent of morphological type. The mass-to-light ratio of the region within a Holmberg radius, however, tends to decrease toward later Hubble types.

Subject headings: galaxies: clusters of — galaxies: general — galaxies: structure — line profiles — radio sources: 21 cm radiation

I. INTRODUCTION

Nearly a decade has passed since Roberts (1969) derived the mass-to-luminosity ratios of a large sample of spiral and irregular galaxies from early optical and 21 cm rotation curves. In the 1970s, the sensitivity and resolution of radio telescopes have improved dramatically, and accurate rotation curves extending well beyond a Holmberg radius have been observed for a moderate sample of galaxies. Huchtmeier (1975) examined in detail the rotation curves for 17 galaxies. He found that eight of these galaxies show a well defined peak in rotational velocity as a function of distance from the center of the galaxy, with the rotational velocity definitely decreasing far from the center. Six galaxies maintain a high velocity in their outer parts, and three lack sufficient information for a decision. Subsequent analyses with more data (Roberts 1975; Wakamatsu 1976; Huchtmeier, private communication) give similar conclusions.

To provide a basis for analysis of other galaxies for which full rotation curves are not available, the observed portion of all the rotation curves can be fitted to a Brandt (1960) curve of the form

$$V_c = \frac{uV_m \csc i}{\left(\frac{1}{3} + \frac{2}{3}u^n\right)^{3/2n}}, \quad (1)$$

where V_m is the maximum line-of-sight velocity at angular radius R_m , V_c is the circular velocity at position u , which is the angular radius in units of R_m , i is

the inclination of the orbital plane of neutral hydrogen to the plane of the sky, and n is a parameter which sets the shape of the rotation curve.

The total mass of a galaxy with a Brandt rotation curve is given by

$$M_T = \left(\frac{3}{2}\right)^{3/n} \csc^2 i V_m^2 R_m d / G, \quad (2)$$

where d is the distance and G is the gravitational constant.

Clearly the adoption of a mean Brandt curve to represent the rotation of all galaxies is subject to considerable uncertainty. Not only are there differences with galactic type (for which a correction can be applied), but the function extrapolates the distribution of material beyond the observed regions and may also be dependent upon smearing in the inner portions of the galaxy where the curve is determined. Other schemes of determining the mass, however, such as choosing an arbitrary cutoff at some limiting radius which is either a small multiple of the Holmberg radius (Fisher and Tully 1975; Balkowsky 1973) or determined by an arbitrary sensitivity limit (Dean and Davies 1975), clearly miss some of the mass in these galaxies. Therefore in the present study we shall adopt the Brandt masses to represent the total masses of the galaxies.

From the limited number of detailed data on the rotation curves available in the 1960s, Roberts (1969) adopted $n = 3$ and $R_m = \frac{1}{3}R_H$ (where R_H is half of the

TABLE 1
LINE PROFILE PARAMETERS

GALAXY	v	$\Delta v/2$	$fT_A dv$	$fSdv$	NOTES	GALAXY	v	$\Delta v/2$	$fT_A dv$	$fSdv$	NOTES
	km/s	km/s	K km/s	Jy km/s			km/s	km/s	K km/s	Jy km/s	
N255	1594	120	20.4	22.9	a	N3623	818	255	18.5	17.9	b
N275	1749	140	24.4	26.5		N3627	716	193	46.8	45.3	
N278	640	78	31.8	29.8		N3628	847	245	186.7	180.3	
N488	2268	228	13.9	14.0	b	N3672	1855	207	49.5	54.6	a
N520	2162	111	25.0	25.4	c	N3675	767	221	46.7	43.5	
N615	1857	232	16.2	17.6	b	N3684	1173	130	36.5	34.8	b
N672	408	137	154.9	144.0	i	N3686	1142	100	15	14.3	b,c
N681	1757	193	16.4	18.3	b	N3718	992	238	97.9	93.3	
N772	2459	257	67.9	64.6	d	N3726	863	145	80.6	75.6	
N779	1386	189	15.8	17.1		N3810	995	136	40.7	39.6	
N803	2110	135	31.3	30.0		N3813	1464	158	31.8	29.4	
N871	3728	121	17.3	16.7	c	N3893	970	154	75.1	70.6	
N877	3909	201	25.2	24.3		N3898	1174	247	38.2	36.8	a
N891	525	242	144.8	134.2		N3938	809	57	63.7	59.4	
N895	2286	138	38.1	41.0		N3992	1047	242	55.3	52.7	
N925	550	111	210.5	194.5		N4051	706	134	39.8	37.1	
N1055	992	207	83.7	86.5		N4062	774	149	18.7	17.3	
N1058	518	26	64.1	59.2		N4096	575	173	58.4	54.8	e
N1068	1144	152	19.4	20.1	b	N4145	1019	116	52.0	48.1	
N1084	1410	190	55.9	60.8	b	N4151	999	78	46.0	42.6	n
N1087	1523	120	27.2	28.3	a	N4236	-3	90	286.4	296.1	
N1140	1507	120	29.3	32.5		N4303	1566	86	68.4	69.0	
N1156	372	67	61.5	57.4		N4369	1052	82	9.4	8.7	c
N1309	2137	65	17.5	20.3		N4395	315	67	191.4	176.8	
N1337	1235	135	67.4	73.8	c	N4490	572	128	229.3	212.5	g,1
N2268	2221	212	20.2	23.5		N4536	1804	172	73.9	75.4	
N2273B	2112	136	15.2	14.9		N4631	617	160	350.0	323.6	f
N2276	2418	103	16.2	19.0	e	N4656	645	97	197.2	182.3	m
N2336	2202	240	37.8	42.5		N4670	1073	106	10.3	9.6	e
N2366	100	58	188.7	194.7		N4712	4376	218	11.9	11.1	b
N2460	1452	192	42.0	41.2		N4725	1212	209	83.9	78.2	
N2500	515	61	30.3	28.6		N4747	1197	95	26.6	24.8	
N2537	443	51	18.1	16.9		N4939	3110	234	35.4	39.3	
N2541	554	106	102.4	96.4		N5033	872	225	142.4	131.7	
N2552	517	71	23.2	21.9		N5055	511	203	186.3	172.7	
N2654	1295	198	24	23.5	b	N5147	1093	78	13.2	13.5	b
N2742	1296	175	22.2	21.8	b	N5194	463	101	127.6	119.7	k
N2805	1733	67	73.6	73.6		N5204	210	69	65.0	63.3	
N2841	635	302	110.2	104.3		N5457	249	100	314.8	301.7	
N3003	1479	150	78.0	72.1	o	N5474	288	33	85.2	81.4	
N3031	-38	220	258.5	266.8	o	N5585	317	82	86.8	83.9	
N3162	1302	96	27.9	26.2		N5660	2323	77	15	14.1	
N3184	593	74	87.7	81.3		N5676	2122	246	22.8	21.5	c
N3198	663	160	147.9	138.1		N5701	1505	72	39.7	39.8	
N3227	1284	117	10.5	9.9	b	N5713	1878	128	49.1	50.8	c
N3239	751	96	76.9	73.4		N5850	2541	106	18.2	18.6	b
N3254	1366	224	41.0	38.0	a	N5879	769	144	24.8	24.0	
N3319	743	119	65.3	60.5		N5907	666	246	167.7	161.8	f
N3338	1298	178	94.0	90.7		N5970	1964	168	16.1	15.6	h
N3351	776	137	42.4	41.2		N5985	2516	268	25.5	24.9	b
N3368	891	176	62.8	61.1		N6015	830	155	75.1	74.4	
N3430	1594	170	45.4	41.9	j	N6217	1370	117	46.7	51.7	
N3432	615	136	112.2	103.7	f	N6412	1333	70	16.5	17.9	o
N3512	1388	135	10.8	10.0	b	N6643	1501	171	27.7	29.7	o
N3596	1202	73	28.0	26.9		I4182	321	35	50.4	46.6	
						A814	158	38	157.6	164.1	
						A936	141	28	37.9	39.7	

NOTES TO TABLE 1

- a) Velocity uncertain. Profile near edge of search range.
- b) Low signal-to-noise ratio.
- c) Uncertain baseline.
- d) Uncertain baseline and/or asymmetric profile.
- e) Asymmetric profile.
- f) Extended and flat profile.
- g) Extended profile.
- h) v could be 200 km s^{-1} larger, $\Delta v/2$ could be 100 km s^{-1} larger.
- i) Possibly confused with I1727.
- j) Possibly confused with N3424.
- k) Possibly confused with N5195.
- l) Possibly confused with N4485.
- m) Possibly confused with N4657.
- n) Possibly confused with N4156.
- o) Some interference on profile.

Holmberg diameter D_H) for all spiral and irregular galaxies. Huchtmeier's (1975) study, however, reveals that generally n is smaller than 3 and R_m/R_H is larger than $\frac{1}{3}$, with the ratio increasing toward later Hubble types.

The present study was undertaken to derive a homogeneous and accurate set of mass-to-light ratios for individual galaxies in groups. We have obtained accurate line profiles for 112 galaxies. Combining the values of V_m derived from the profile widths with values of n and R_m estimated statistically from Huchtmeier's (1975) results, we derive integrated (Brandt) masses. Holmberg luminosities corrected for Galactic and internal extinctions, L_{pg} , are also derived. Adopting a Hubble constant $H = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ throughout our study, we find that $M_T/L_{pg} = 11$ solar units, which is about 3.5 times larger than the early value (reduced to $H = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$) obtained by Roberts (1969).

II. THE DATA

The observations were obtained with the 91 m radio telescope at the National Radio Astronomy Observatory in the autumns of 1974 and 1975. The 21 cm hydrogen line emission of each galaxy was measured during transit of the galaxy at the position given in the *Reference Catalogue of Bright Galaxies* (RC1, de Vaucouleurs and de Vaucouleurs 1964). The equipment was described by Dickel and Rood (1975), and the adopted dependence of beam efficiency on declination used to convert antenna temperature to flux density was similar to that derived by Fisher and Tully (1975). Because the telescope, auxiliary equipment, and reduction procedures used in our study are identical or very similar to those used by Fisher and Tully (1975) and Shostak (1975), the uncertainties in observed parameters of all these studies are about the same. In general, the signal-to-noise ratio was sufficient so that the greatest uncertainty was caused by fluctuations in the spectrometer baseline. Usually we used the autocorrelation spectrometer of 192 channels, each with a width of 6.6 km s^{-1} and separated by 5.5 km s^{-1} to cover a velocity range of 1064 km s^{-1} centered on the adopted optical velocity of each galaxy. A small number of observations were obtained with a channel width of 13.2 km s^{-1} and a range of 2128 km s^{-1} .

From several independent measurements on the same galaxy, we found that the average uncertainty in v is 9 km s^{-1} (Dickel and Rood 1975).

The parameters of the line profiles for all definitely detected galaxies are presented in Table 1 and the profiles are shown in Figure 1. The definitive values for the observations obtained in the autumn of 1974 listed in this table supersede the more preliminary values given by Rood and Dickel (1976). The measured profile parameters include the heliocentric line-of-sight velocity¹ midway between points at a level of one-quarter of the peak intensity (v), the half-width ($\Delta v/2$) between these quarter-intensity points corrected for resolution (usually 6.6 km s^{-1}), and the line flux in K km s^{-1} ($\int T_A dv$) and in Jy km s^{-1} ($\int S dv$).

III. RATIOS OF MASS TO LUMINOSITY

a) Preliminary Properties

In order to derive values of M_T/L_{pg} for galaxies, it is necessary to make decisions regarding many basic parameters of each galaxy.

a) The very accurate Holmberg (1958) angular diameters D_H , axial ratios (major diameter divided by minor diameter, r_H), and photographic magnitudes m_{pg} are used in our calculations when available. Otherwise, Holmberg values are derived statistically from parameters in other catalogs through regression-line transformation relations. The regression lines for a given catalog are derived from data for the galaxies common to it and the Holmberg (1958) catalog. We have derived transformation relations as a function of morphological type for parameters in the Uppsala General Catalogue (UGC, Nilson 1973), the Reference Catalogue (RC1, de Vaucouleurs and de Vaucouleurs 1964), the Second Reference Catalogue (RC2, de Vaucouleurs, de Vaucouleurs, and Corwin 1976), and the Zwicky catalog (Zwicky *et al.* 1960–1968). The transformation relations are contained in Tables 2 and 3. The standard errors of estimate in these tables contain some interesting information. They reveal that Nilson's UGC diameters and axial ratios, although derived only from measurements on paper-print

¹ Throughout this paper we define all velocities as done by optical observers such that $v = C(\lambda - \lambda_0)/\lambda_0 = C(v_0 - v)/v_0$.

TABLE 2
ANGULAR DIAMETER AND AXIAL RATIO TRANSFORMATION RELATIONS

TYPE	$\log D_H = a + b \log x$					$\log r_H = c + d \log y$				
	<i>a</i>	<i>b</i>	<i>x</i>	<i>N</i>	s.e.	<i>c</i>	<i>d</i>	<i>y</i>	<i>N</i>	s.e.
E.....	0.45	1.01	D_{RC1}	18	0.073	0.00	0.60	r_{RC1}	18	0.050
S0, S0a.....	0.30	0.99		31	0.078	0.00	0.68		31	0.072
Sa-Sbc.....	0.30	0.86		91	0.070	0.02	0.74		91	0.073
Sc-Sdm.....	0.26	0.88		102	0.068	0.02	0.76		102	0.073
Sm, I, P.....	0.42	0.75		31	0.085	-0.03	0.81		31	0.097
E.....	0.26	0.86	D_{RC2}	20	0.085	-0.01	0.83	r_{RC2}	20	0.037
S0, S0a.....	0.22	0.88		31	0.068	-0.01	0.82		31	0.053
Sa-Sbc.....	0.23	0.89		92	0.055	0.02	0.87		92	0.064
Sc-Sdm.....	0.22	0.91		108	0.055	0.00	0.87		108	0.068
Sm, I, P.....	0.28	0.86		38	0.071	-0.02	0.95		38	0.070
E.....	0.36	0.72	D_{UGC}	19	0.091	-0.01	0.92	r_{UGC}	19	0.036
S0, S0a.....	0.33	0.70		30	0.056	0.02	0.65		30	0.063
Sa-Sbc.....	0.25	0.84		87	0.065	0.04	0.73		87	0.068
Sc-Sdm.....	0.24	0.84		95	0.058	0.02	0.74		95	0.080
Sm, I, P.....	0.30	0.74		34	0.070	0.01	0.75		34	0.082

NOTE.—*N* = number of sample galaxies in Holmberg (1958) catalog. Angular diameters are in arc minutes.

reproductions of the Palomar Sky Survey, are remarkably homogeneous and accurate. They also reveal that the corrected Harvard magnitudes of the galaxies listed in Holmberg's (1958) catalog are more accurate than the Zwicky magnitudes. This is because the Schraffier technique used by Zwicky *et al.* (1960–1968) does not work well on bright galaxies. The magnitudes of fainter galaxies in the Zwicky catalog are more accurate (Huchra 1976).

b) The Holmberg angular diameter D_H is the major axis of a standard isophote. It follows that a flat galaxy with small or moderate internal absorption seen edge-on would have a larger Holmberg diameter than if it were viewed face-on. The angular size of a galaxy is therefore better represented by a parameter that is independent of axial ratio, such as the face-on angular diameter $D_H(0)$. For convenience, it is generally assumed that $D_H \propto D_H(0)r^n$ (e.g., see Heidmann, Heidmann, and de Vaucouleurs 1972). Determinations of *n* in the literature from studies of the dependence of the angular diameter or the surface density of neutral hydrogen upon axial ratio generally range from 0.2 to 0.4. But these determinations suffer from selection effects and small-number statistics (Heidmann, Heidmann, and de Vaucouleurs 1972; Tully 1972). In the present study, we adopt $n = 0.3$ from the following analysis: The brightest galaxy in a group, brightest two galaxies, brightest five galaxies, etc., probably represent homogeneous classes of similar objects. Therefore the slope of the regression line of \log (linear isophotal diameter) versus $\log r_H$ for these galaxies should be a value of *n* which is

relatively free of selection effects. Results are presented in Table 4. The groups selected are those listed by de Vaucouleurs (1975) except for de Vaucouleurs groups 5, 9, and 11 which are replaced by the M101 group, M51 group, and Leo group following Sandage and Tammann (1975). The linear diameters D_H^L were derived from Holmberg angular diameters and group distances computed from the average line-of-sight velocity of member galaxies using a Hubble constant of $50 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The luminosity rank among the brightest several galaxies in a group was determined from magnitudes both uncorrected and corrected for Galactic and internal extinction as discussed below. For the corrected case, the galaxies were treated by individual ranks; for the uncorrected case, we used cumulative ranks.

c) The surface brightness of a galaxy in magnitudes per square arcsecond, given by $S = m_{pg} + 5 \log D_H(0)$, depends theoretically on the inclination of the galaxy, *i*, and a parameter S_x related to the face-on surface brightness, by the formula $S = S_x + \alpha \sec i$ (*i* not "too large"), where $\alpha \sec i$ is the internal extinction of the galaxy. The secant of *i* is given in terms of the axial ratio by a relation derived by Holmberg (1958, p. 39). We assume that, in this relation, the ratio of intrinsic minor to intrinsic major axis of a galaxy is 0.2. The value of $\sec i$ is insensitive to this choice for all but the largest inclinations. We have derived the regression coefficient α as a function of morphological type for (i) the galaxies in the Holmberg (1958) catalog, (ii) all galaxies listed in the de Vaucouleurs (or Sandage-Tammann) groups, and (iii) the five brightest

FIG. 1.—Original 21 cm hydrogen line profiles of 112 spiral and irregular galaxies. The NGC number of each galaxy is given in the upper right, just below the smaller scan number (all numbers refer to the NGC number, whether preceded by either NGC or N or whether no letters precede the number). The antenna temperature and velocity scales are as given. These are the original profiles with only a linear baseline fit to each one and no editing. In several instances, higher order baselines were fitted, and editing of Galactic or other interference was performed before derivation of the parameters given in Table 1.

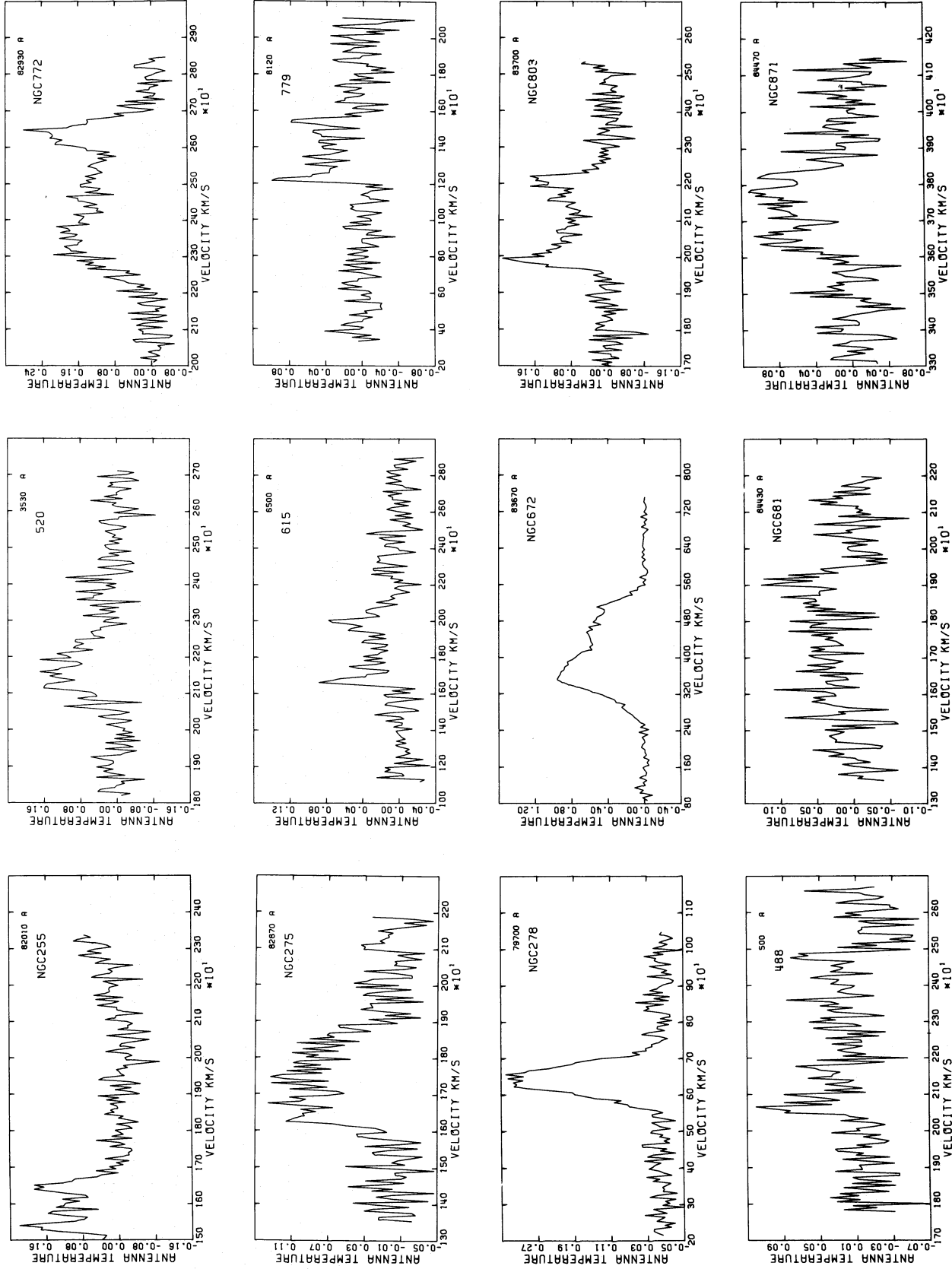


FIG. 1a

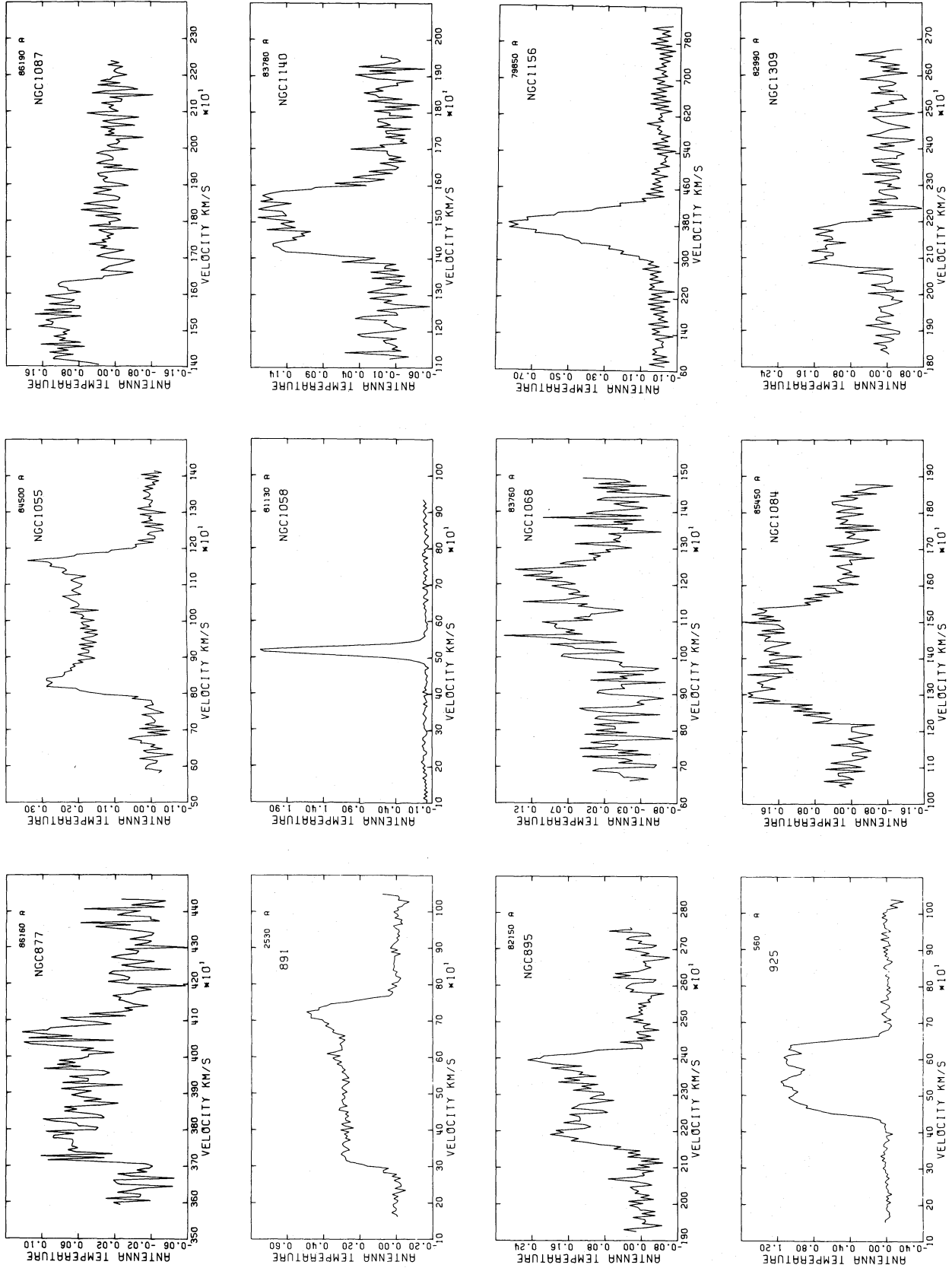


FIG. 1b

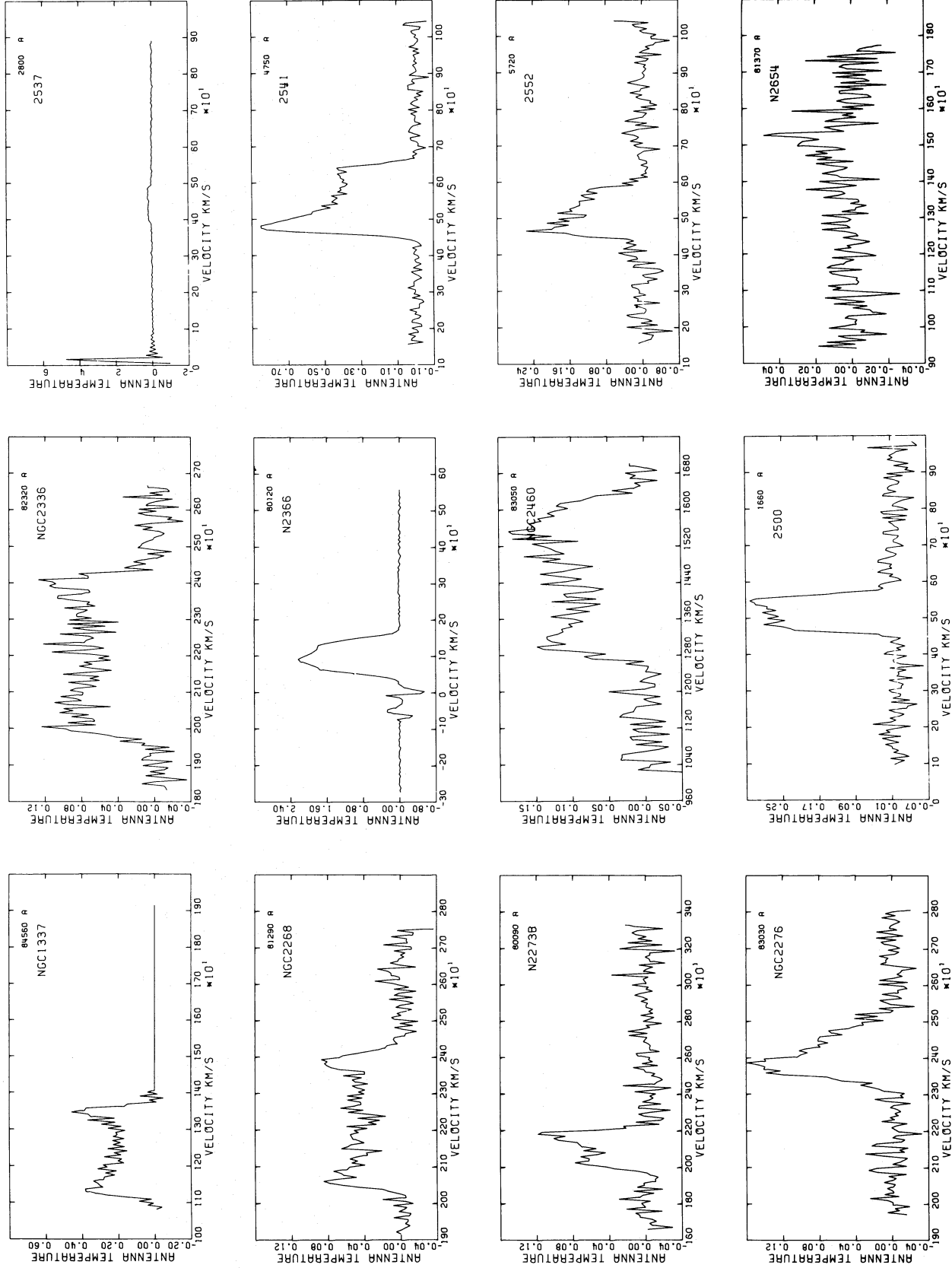


Fig. 1c

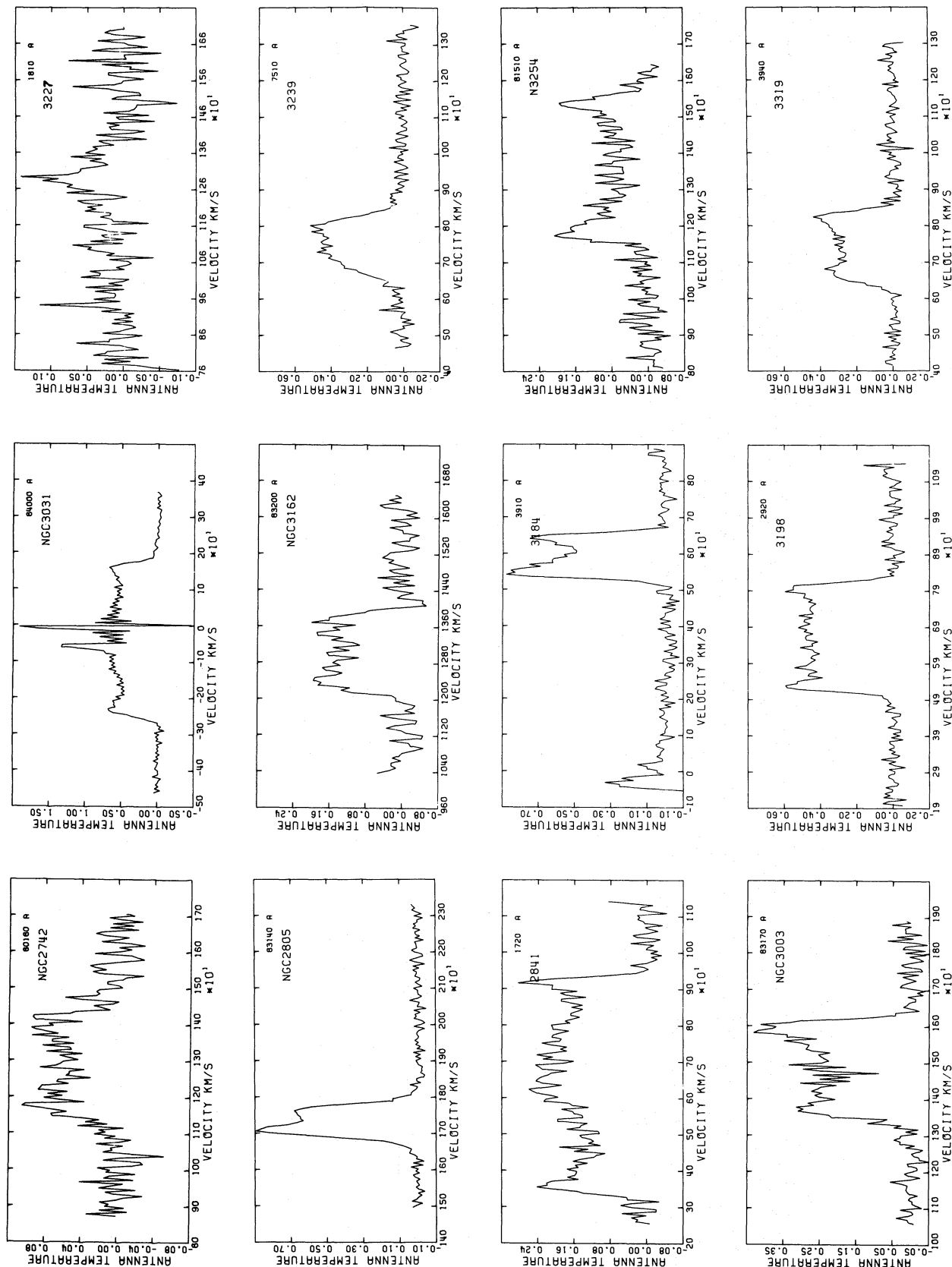


Fig. 1d

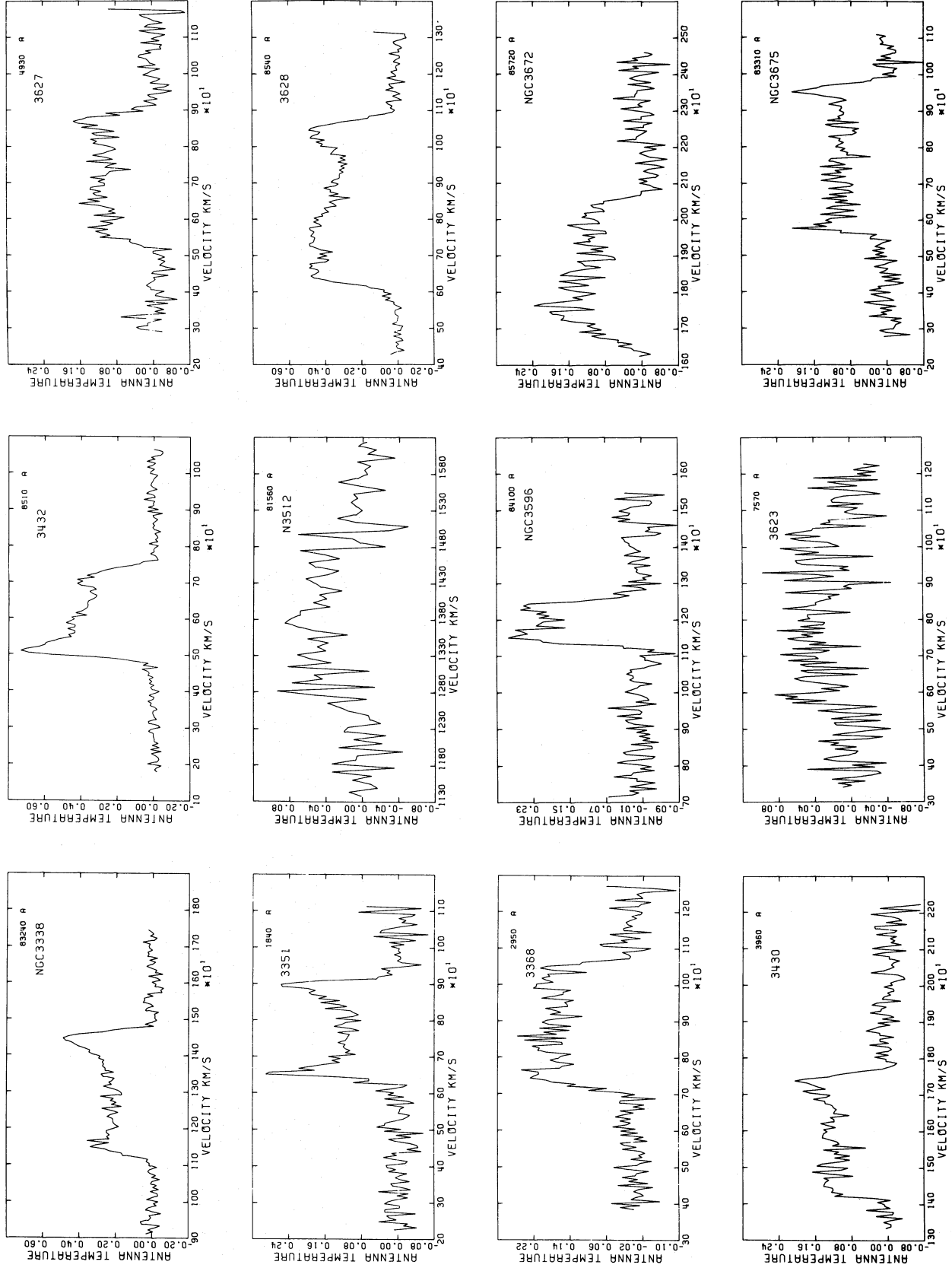


Fig. 1e

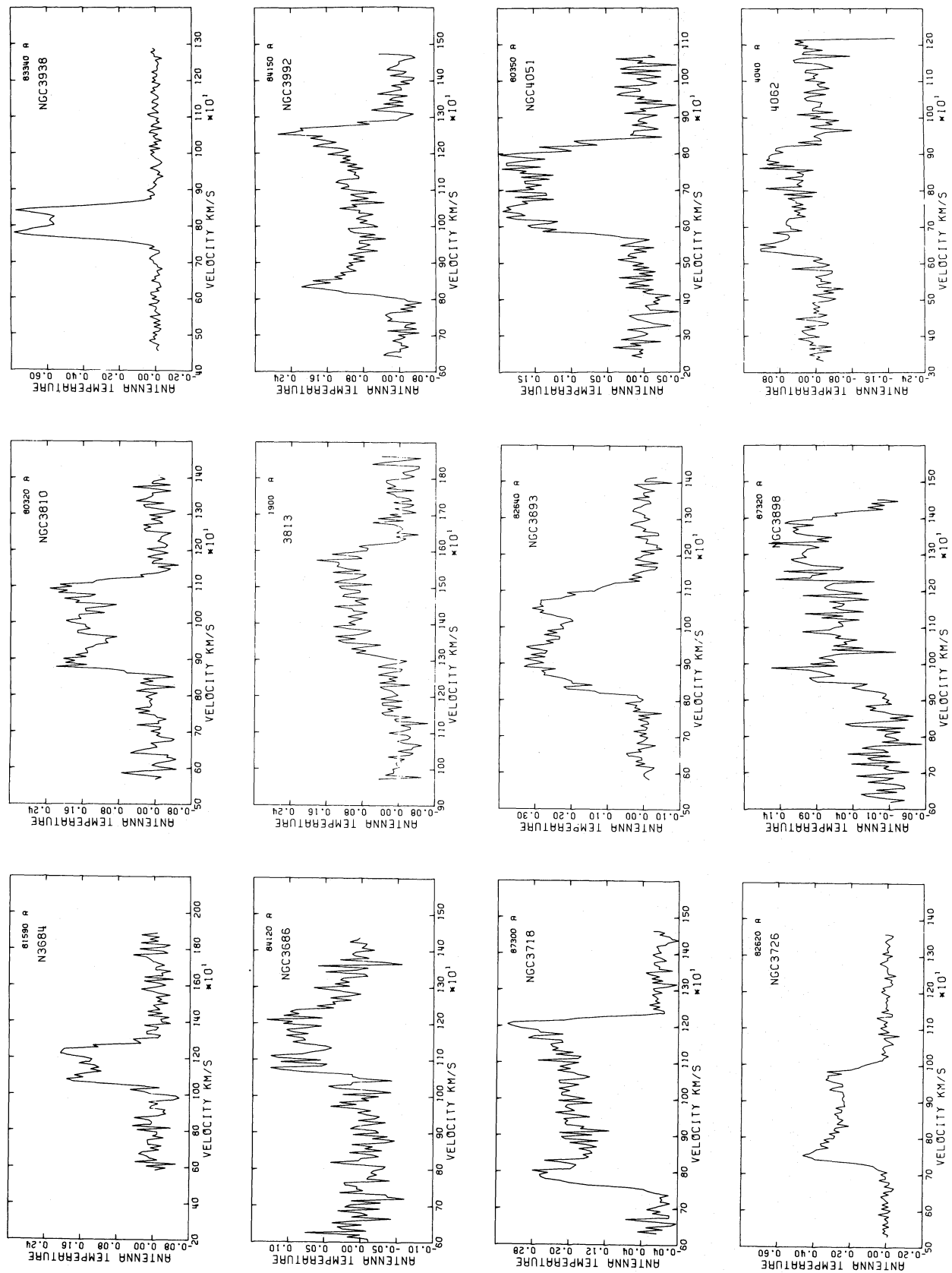


Fig. 1f

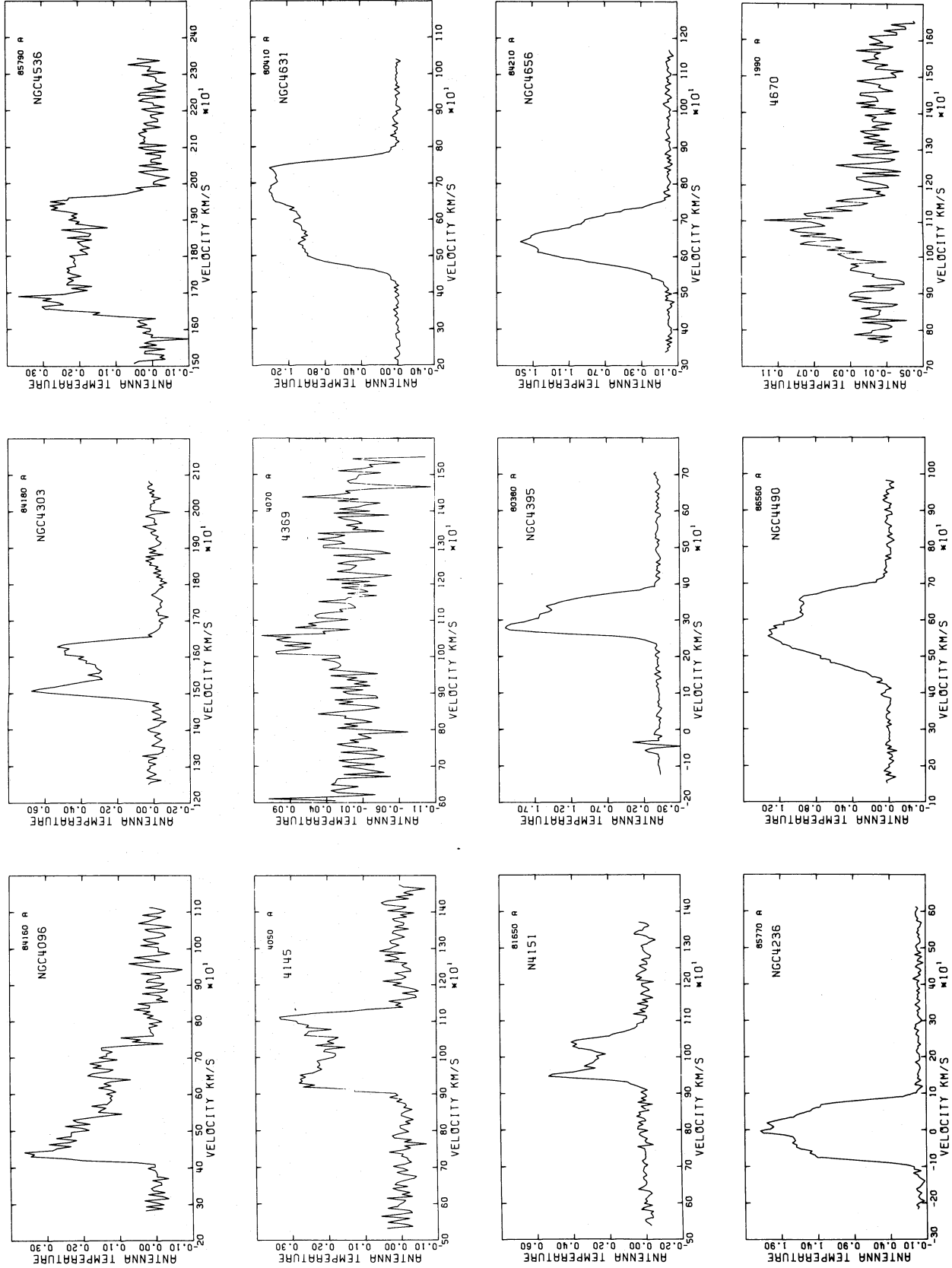


Fig. 1g

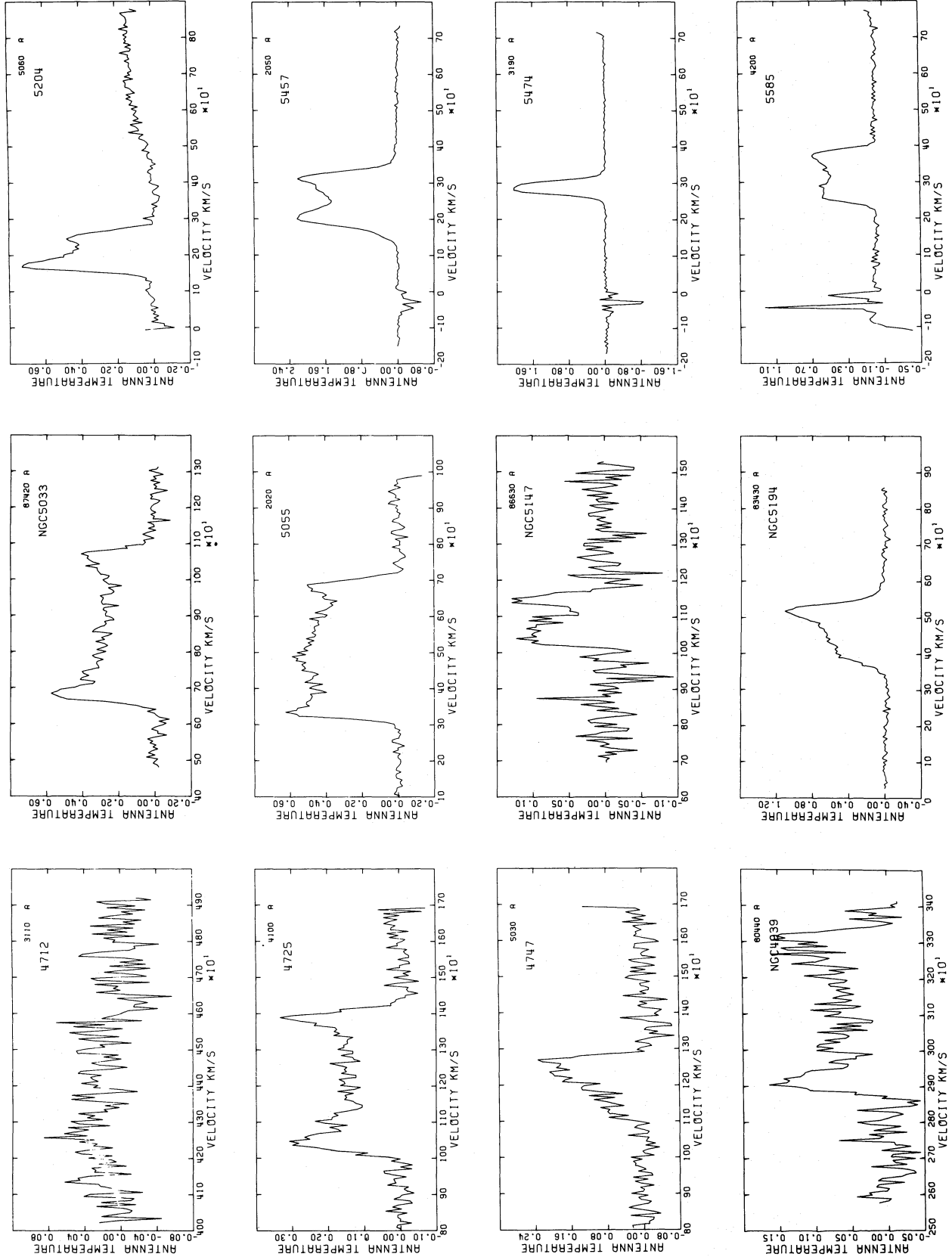


Fig. 1h

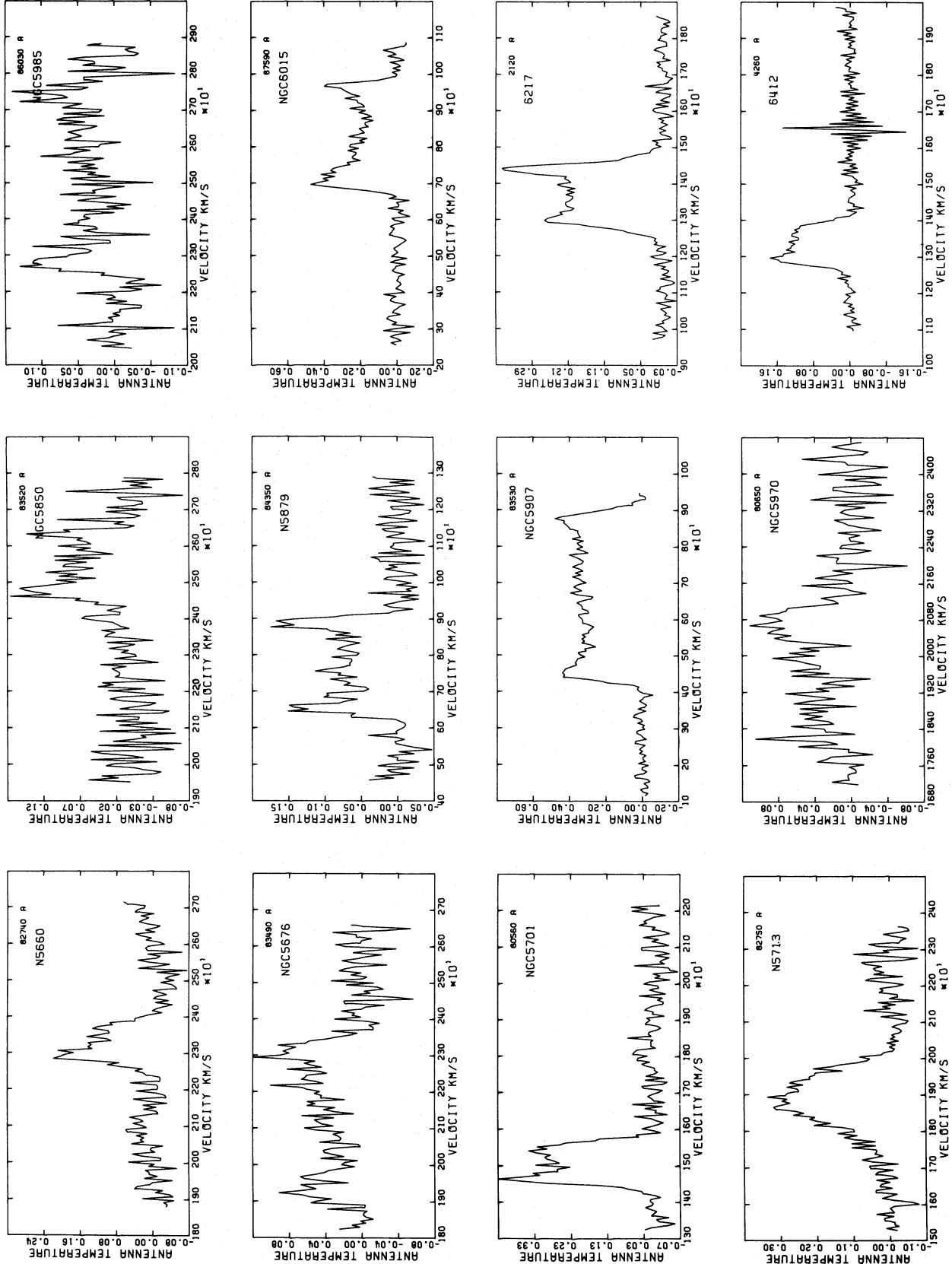


FIG. 1i

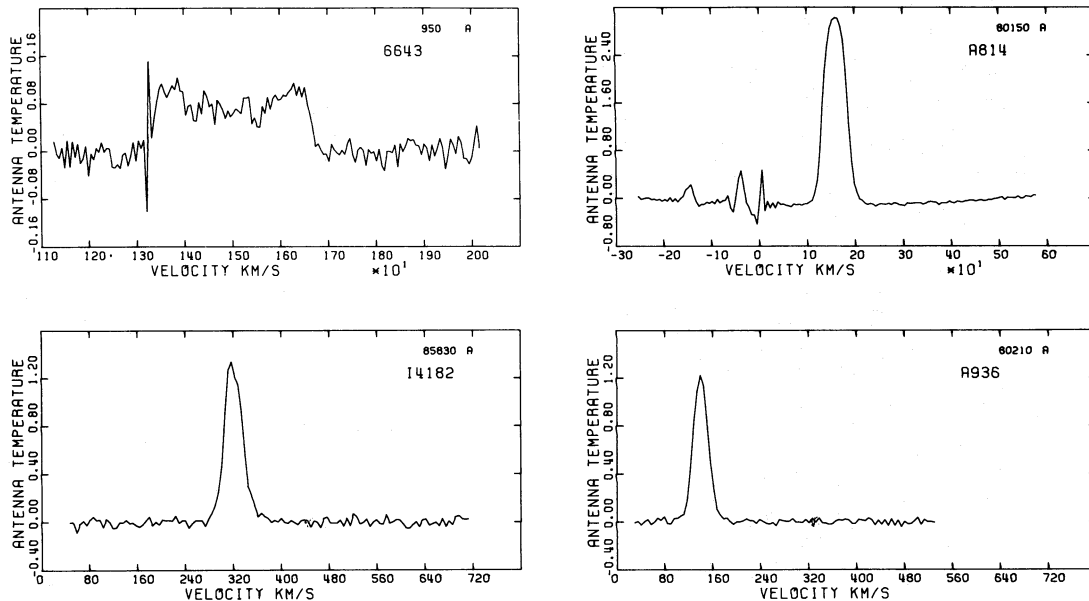


FIG. 1j

galaxies in the groups (Table 5). Because (iii) may be the most homogeneous set of data, we have given it the greatest weight in selecting adopted values. Our values for the internal extinction coefficient α are approximately half of those derived by Holmberg (1958), primarily because we used $D_H(0)$ whereas Holmberg used D_H in the expression for surface brightness. Because D_H is also a function of the inclination as discussed above, we feel that $D_H(0)$ is a more consistent parameter to use in determining the internal extinction. Recent work by Rubin *et al.* (1976) on a large homogeneous sample of Sc galaxies also suggests that the internal extinction coefficient may be smaller than that originally derived by Holmberg (1958).

Examination of the mean errors in Tables 4 and 5 reveals how very uncertain our adopted diameter and internal extinction corrections actually are. Values in the literature contain similar uncertainties. The situation is unsatisfactory and frustrating. A significant improvement is unlikely to be forthcoming in the near future. Therefore, the values for n and α that we have adopted are little more than formal values which indicate that certain physical effects have not been neglected.

b) Masses and Luminosities

To derive the galactic mass M_T from equation (2) with our data, we need values for n , i , V_m , R_m , and d . The determination of i from the axial ratios and d from the mean radial velocities of the groups was already discussed. The maximum rotational velocity, V_m , is given by $\Delta v/2$ in Table 1. For n , we adopt a value of 1.3 from the following consideration: Substitution of the masses determined by Huchtmeier (1975) into equation (2) gives a mean value for the expression

containing n of $\langle (3/2)^{3/n} \rangle = (3/2)^{3/1.3}$; without knowing individual values, we adopt that which gives the correct average mass. R_m is estimated statistically from the face-on Holmberg angular radius multiplied by the ratio $2R_m/D_H(0)$, which depends on morphological type in the manner found by Brosche (1973) and Huchtmeier (1975) (Table 6). The exponent n is also a function of morphological type (Brosche and Reinhardt 1977), but the adopted correction for radius should somewhat compensate for this effect.

To derive the galactic luminosities, we adopt Holmberg magnitudes, a solar magnitude $M_{pg\odot} = 5.16$ to obtain the luminosities in solar units, internal extinction corrections from Table 5, and a Galactic extinction correction given by $0.22 \text{ mag csc } b$, where b is the Galactic latitude (Holmberg 1958, 1974). It is interesting that the north polar Galactic extinction, 0.22 mag, is about the same as the adopted average face-on extinction for spiral galaxies.

The integrated masses, mass-to-light ratios, and other properties of the galaxies are listed in Table 7. Note that the values of some of the parameters are given to 1–2 more significant figures than is justified by observational uncertainties and assumptions. These were the values used in our calculations; and by listing them, we make it possible for an interested reader to check our quantitative results precisely. In § IV below we investigate the actual uncertainties in the statistical masses derived herein by comparison with the masses of individual galaxies determined by analysis of detailed rotation curves (Huchtmeier 1975).

IV. RESULTS AND DISCUSSIONS

We first compare our line-of-sight velocities with those listed in the RC2. There are 108 galaxies that are common to both data sets. None of the velocities are

TABLE 3
MAGNITUDE TRANSFORMATION RELATIONS
 $m_{pg} = a + bx$ ($m_{pg} \geq 9.0$ mag)

Type	a	b	Number of Galaxies	s.e.
$x = B(0)$				
E.....	-1.30	1.07	17	0.19
S0, S0a.....	0.18	0.96	30	0.17
Sa-Sbc.....	-0.15	0.99	86	0.18
Sc-Sdm.....	-0.34	1.00	99	0.13
Sm, I, P.....	0.03	0.96	28	0.18
$x = B_T$				
E.....	-0.81	1.08	18	0.13
S0, S0a.....	0.48	0.96	30	0.11
Sa-Sbc.....	-0.22	1.02	87	0.18
Sc-Sdm.....	0.22	0.97	104	0.09
Sm, I, P.....	0.11	0.98	36	0.11
$x = m_p$				
E.....	-1.61	1.10	18	0.42
S0, S0a.....	1.49	0.87	30	0.40
Sa-Sbc.....	2.08	0.79	82	0.52
Sc-Sdm.....	2.02	0.80	93	0.38
Sm, I, P.....	3.68	0.66	34	0.48
$x = m_c(\text{RC1})$				
E.....	-3.93	1.29	16	0.23
S0, S0a.....	-2.60	1.20	24	0.25
Sa-Sbc.....	-0.76	1.04	78	0.29
Sc-Sdm.....	-0.70	1.03	79	0.27
Sm, I, P.....	0.14	0.95	22	0.29
$x = m_c(\text{RC2})$				
E.....	-0.96	1.09	16	0.22
S0, S0a.....	-0.27	1.03	24	0.17
Sa-Sbc.....	-0.08	1.00	78	0.25
Sc-Sdm.....	0.20	0.97	80	0.25
Sm, I, P.....	0.26	0.96	23	0.34

NOTES TO TABLE 3.— $B(0)$ = photoelectric magnitude in RC1; B_T = photoelectric magnitude in RC2; m_p = magnitude in Zwicky *et al.* (1960–1968) catalog; m_c = corrected Harvard magnitude; m_{pg} = Holmberg photographic magnitude.

discordant in the sense of differing by several hundred km s^{-1} . The mean difference is $\langle V_{\text{RC2}} - V_{21} \rangle = 3 \pm 7 \text{ km s}^{-1}$. The mean absolute difference is $\langle |V_{\text{RC2}} - V_{21}| \rangle = 43 \pm 5 \text{ km s}^{-1}$. If the errors have a Gaussian frequency distribution, then the combined mean error is 25% larger than the mean absolute difference. Adopting a 9 km s^{-1} average mean error in our own velocities, we conclude that the average mean error in the RC2 velocities is about 53 km s^{-1} . This compares with the average of tabulated RC2 errors of 35 km s^{-1} . Hence, the actual mean errors may be up to 50% larger than those tabulated in the RC2, but in addition the 21 cm velocities probably do not always truly represent the systemic velocities of the galaxies because of asymptotic hydrogen distribu-

TABLE 4
DEPENDENCE OF OBSERVED LINEAR DIAMETER
ON AXIAL RATIO

Galaxy Rank	Number in Sample	η	Mean Error
Galaxy Rank by m_{pg} (uncorrected)			
1.....	54	0.17	0.16
1-2.....	106	0.35	0.11
1-3.....	159	0.28	0.09
1-4.....	204	0.33	0.08
1-5.....	225	0.34	0.08
Galaxy Rank by m_{pg} (corrected for Galactic and internal extinction)			
1.....	54	0.29	0.10
2.....	54	0.27	0.14
3.....	54	0.25	0.19
4.....	53	0.24	0.18
5.....	47	0.14	0.17

tions, noncircular rotation, confusion in the telescope beam, or other causes.

The average M_T/L_{pg} of the 67 galaxies in our sample included among the five brightest galaxies in groups is $\langle M_T/L_{pg} \rangle = 9.9 \pm 0.7$ solar units. The average M_T/L_{pg} of all 101 galaxies in our sample is $\langle M_T/L_{pg} \rangle = 11.2 \pm 0.7$ solar units. The average mass-to-luminosity ratio as a function of morphological type is plotted in Figures 2 and 3. Within the observational uncertainties, $\langle M_T/L_{pg} \rangle$ is independent of morphological type among spiral and irregular galaxies.

In the present study, the masses M_T are derived from the Brandt relation (eq. [2]) under the assumptions that $n = 1.3$ and that R_m can be estimated statistically from the Holmberg face-on radius and the adopted ratios in Table 6. What is the uncertainty in our estimate of Brandt masses due to these assumptions? We can answer this question by comparing our values with those derived for a number of individual galaxies in Huchtmeier's (1975) sample for which values of n and R_m are known. These "true" Brandt masses will be designated M_B . We find that $\langle \log M_T/M_B \rangle = -0.06 \pm 0.06$, which tells us that the adopted ratios in Table 6 are adequate. We also find that $\langle |\log M_T/M_B| \rangle = 0.15 \pm 0.04$, which means that the individual values of M_T are accurate, on the average, to within a factor of 1.4 ± 0.1 .

It should also be possible to obtain estimates of the Brandt masses directly from galactic luminosities, L_{pg} , multiplied by a mean mass-to-luminosity ratio, say $f = 10.2$ solar units. The data for the Huchtmeier galaxies then reveal that $\langle \log L_{pg}f/M_B \rangle = 0.10 \pm 0.09$, and $\langle |\log L_{pg}f/M_B| \rangle = 0.25 \pm 0.06$, which means that the luminosity technique provides mass estimates accurate, on the average, to within a factor of 1.8 ± 0.2 .

Our derived value of M_T/L_{pg} is about 3.5 times larger than that derived by Roberts (1969) for the following reasons: (i) The rotation curves are flatter

TABLE 5
INTERNAL EXTINCTION PARAMETER
 $S = S_* + \alpha \sec i$

Galaxy Type	Holmberg Galaxies	All Galaxies in Groups	Five Brightest Galaxies in Groups	Adopted
S0, S0a:				
Number of galaxies.....	17	71	49	
α	0.19	0.11	0.11	0.11
Mean error.....	0.11	0.11	0.14	
Sa-Sbc:				
Number of galaxies.....	53	99	81	
α	0.13	0.13	0.17	0.17
Mean error.....	0.08	0.08	0.09	
Sc-Sdm:				
Number of galaxies.....	53	82	64	
α	0.11	0.18	0.23	0.23
Mean error.....	0.06	0.11	0.14	
Ir:				
Number of galaxies.....	24	24	13	
α	0.32	0.15	0.57	0.3
Mean error.....	0.21	0.28	0.18	

NOTE.—We adopt an internal extinction equal to 4α , when $\sec i \geq 4$.

than Roberts assumed. We adopted $n = 1.3$ whereas Roberts adopted $n = 3$. This difference amounts to a factor of 1.7 in mass. (ii) Roberts adopted the ratio $R_m/R_H = 0.33$; our adopted ratios are larger by an average factor of about 2, which amounts to a factor of 2 difference in mass. (iii) Our internal extinction correction is about half of that adopted by Roberts, which amounts to a factor of about 1.4 difference in luminosity. Combination of these factors predicts that our $\langle M_T/L_{pg} \rangle$ should be a factor of 4.8 larger than Roberts's. Presumably the difference is explicable in terms of differences in the samples and adopted transformation relations.

The mass of a galaxy contained within linear radius $uR_m d$ is given approximately by

$$M(r) = V_c^2 u R_m d / G. \quad (3)$$

Using $n = 1.3$ and the values for $R_m/R_H(0)$ in Table 6, it is easy to show from equations (1) and (3) that the fraction of mass within a Holmberg radius is equal to $M(R_H)/M_T = 0.72, 0.53, 0.48, 0.39,$ and 0.24 for S0-Sb, Sc, Scd, Sd-Sdm, and Sm-Im galaxies, respec-

tively. Since the *integrated* mass-to-light ratio is independent of type, it follows that the mass-to-light ratio of the matter *within a Holmberg radius* decreases along the Hubble sequence of spiral-irregular types. This is consistent with our knowledge of the variation of stellar content with morphological type. Since $R_m/R_H(0)$ increases along the Hubble spiral-irregular sequence, it is reasonable that the fraction of dark matter beyond a Holmberg radius also increases along this sequence.

Finally, we repeat a word of caution. Although a Brandt curve provides good fits to rotation curves observed to ~ 3 Holmberg radii, there is no physical requirement that it apply at greater radii. Integrated masses derived by extrapolating Brandt curves to infinite radii, such as those in the present paper, are accordingly subject to possibly large unknown systematic errors.

We thank Dr. Morton Roberts for helpful advice concerning the detection threshold of hydrogen-line profiles of galaxies. We thank Dr. Peter Nilson for providing us with a copy of his very useful *Uppsala*

TABLE 6
RADIUS OF MAXIMUM IN ROTATION CURVE
IN UNITS OF HOLMBERG RADIUS

TYPE	HUCHTMEIER		BROSCHÉ $\langle 2R_m/D_H \rangle$	ADOPTED $\langle 2R_m/D_H(0) \rangle$
	$\langle 2R_m/D_H \rangle$	$\langle 2R_m/D_H(0) \rangle$		
S0-Sb.....	0.43	0.55	0.32	0.4
Sbc-Sc.....	0.68	0.75	0.57	0.7
Scd.....	0.84	1.12	0.89	0.8
Sd-Sdm.....	1.04	1.34	1.10	1.0
Sm-Im.....	1.6	2.2	1.44	1.5

NOTE.—Mean errors are 0.1–0.2.

TABLE 7
PROPERTIES OF THE GALAXIES

NGC	TYPE	d	csc i	$D_H(0)$	V_m^c	m_{pg}	A_b	A_i	M_T	$\frac{M_T}{L_{pg}}$	GROUP
		Mpc		arc min	km/s	mag	mag	mag	$10^{10} M_\odot$	PG solar units	
N255	Sbc	33.4	1.7707	4.25	212.5	12.36	.23	.21	38.7	17.6	
N275	SBcdp	36.3	1.4400	2.39	226.1	13.03	.23	.32	30.6	19.7	
N278	Sb	17.5	2.3928	4.11	186.6	11.58	.83	.19	8.64	4.08	
N488	Sb	47.6	1.3818	7.02	315.1	11.10	.26	.25	114.5	7.51	40
N520	P	47.6	1.0832	5.27	120.2	12.35	.26	.78	12.5	1.59	40
N615	Sb	36.5	1.1185	4.31	259.5	12.25	.24	.38	36.5	10.6	33
N672	SBcd	11.3	1.0514	8.34	144.0	11.31	.40	.75	13.5	10.6	
N681	Sab	35.6	1.2519	3.01	241.6	12.75	.24	.28	21.6	11.5	
N772	Sb	51.6	1.2859	9.22	330.5	11.10	.34	.27	179.3	9.12	
N779	Sb	36.5	1.0661	4.14	201.5	11.93	.25	.49	21.2	4.11	33
N803	(Sc)	44.3	1.1314	3.74	185.5	12.90	.32	.49	34.4	10.4	
N871	SBc:	76.4	1.0740	1.44	130.0	13.96	.32	.63	11.2	2.65	
N877	Sbc	80.0	1.5146	3.30	304.4	12.48	.32	.23	147.5	11.8	
N891	Sb?	15.5	1.0128	9.94	245.1	10.85	.73	.68	31.9	6.84	7
N895	Sed	46.0	1.5071	4.91	208.0	11.75	.26	.31	67.4	8.15	
N925	Sd	15.5	1.2417	12.10	137.8	10.53	.52	.39	30.7	7.77	7
N1055	Sb	25.9	1.0708	9.06	221.7	11.38	.28	.48	39.8	9.06	15
N1058	Sc	15.5	-	6.00	-	11.74	.63	.23	-	-	7
N1068	Sb	25.9	1.6330	9.35	248.2	9.63	.28	.21	51.5	3.00	15
N1084	Sc	25.9	1.1769	3.57	223.6	11.04	.26	.44	27.9	4.91	15
N1087	Sc	25.9	1.3057	5.21	156.7	11.45	.28	.36	20.0	5.43	15
N1140	Im	29.5	1.2401	2.52	148.8	12.40	.26	.51	21.2	9.43	
N1156	IBm	9.28	-	5.90	-	11.85	.45	.30	-	-	
N1309	Sbc:	41.2	2.3378	3.06	191.7	12.14	.27	.19	27.9	6.67	
N1337	Sed	23.5	1.0507	5.30	141.8	12.26	.29	.75	17.3	8.32	
N2268	Sbc	48.4	1.2349	3.96	261.8	12.18	.48	.29	79.2	10.7	
N2273B	SB:	44.3	1.3173	3.37	179.2	13.26	.57	.31	28.9	11.4	
N2276	Sc	52.4	-	4.00	-	11.91	.47	.23	-	-	
N2336	Sbc	47.9	1.2097	8.86	301.2	11.03	.47	.30	232.2	11.1	
N2366	IBm	4.24	1.1554	8.27	67.0	11.41	.46	.60	2.03	13.5	2
N2460	Sa	30.9	1.3787	5.13	264.7	12.33	.42	.25	38.3	16.0	
N2500	SBd	12.0	3.0994	3.84	-	12.13	.42	.24	-	-	6
N2537	IBmp	12.0	2.1319	2.85	108.7	12.08	.40	.34	5.21	10.7	6
N2541	Scd	12.0	1.1228	6.94	119.0	11.88	.40	.51	8.14	11.9	6
N2552	Sm?	12.0	1.4153	4.95	100.5	12.05	.39	.42	7.74	14.5	6
N2654	Sa?	29.4	1.0108	3.90	200.1	12.77	.36	.68	15.8	7.76	41
N2742	Sc:	31.6	1.2074	4.03	242.7	12.07	.34	.41	45.2	13.2	41
N2805	Sd	31.6	1.4813	7.34	99.2	11.68	.34	.31	19.7	4.40	41
N2841	Sb:	12.0	1.1347	9.20	342.7	10.10	.32	.36	44.7	15.7	6
N3003	Sbc?	29.4	1.0911	5.86	163.7	12.04	.29	.43	27.8	9.36	42
N3031	Sab	4.24	1.0750	26.81	236.5	7.85	.34	.46	21.9	6.95	2
N3162	Sbc	24.6	1.6006	4.63	153.7	12.03	.27	.22	16.2	9.54	47
N3184	Scd	13.1	-	9.5	-	10.28	.27	.23	-	-	12
N3198	Sbc	13.1	1.0752	9.12	172.0	10.82	.27	.63	21.3	9.95	12
N3227	Sap	24.6	1.3678	7.69	160.0	11.48	.27	.25	16.7	5.77	47
N3239	SBm	15.5	1.4127	6.81	135.6	11.77	.27	.43	25.0	24.1	LEO
N3254	Sbc	25.8	1.0515	5.33	235.5	12.14	.26	.55	46.0	20.3	54
N3319	SBcd	13.1	1.1230	7.10	133.6	11.67	.26	.51	11.5	13.2	12
N3338	Sc	15.5	1.2322	8.09	219.3	11.25	.26	.39	36.4	22.7	LEO
N3351	SBb	15.5	1.3504	8.31	185.0	10.48	.26	.25	15.2	5.31	LEO
N3368	Sab	15.5	1.4166	9.80	249.3	10.05	.26	.24	32.6	7.74	LEO
N3430	Sc	32.0	1.2247	5.28	208.2	12.05	.25	.40	44.2	13.5	43
N3432	SBm	13.1	1.0247	5.67	139.4	11.59	.25	1.20	18.6	10.6	12
N3512	Sc	28.0	2.3814	2.64	321.5	12.78	.24	.25	46.1	41.8	48
N3596	Sc	15.5	3.3114	5.95	-	11.73	.24	.24	-	-	LEO
N3623	Sa	15.5	1.0584	8.89	269.9	10.18	.24	.52	34.6	7.28	LEO
N3627	Sb	15.5	1.1107	11.01	214.4	9.65	.24	.39	27.1	3.95	LEO
N3628	Sbp	15.5	1.0090	11.71	247.2	10.23	.24	.68	38.3	7.29	LEO
N3672	Sc	29.8	1.1787	5.25	244.0	11.45	.30	.43	56.2	10.6	23
N3675	Sb	14.3	1.1844	7.49	261.8	10.85	.24	.32	25.3	13.9	10
N3684	Sbc	22.0	1.3564	4.56	198.0	12.35	.24	.25	23.7	23.4	49
N3686	SBbc	22.0	1.5671	4.27	206.9	11.97	.24	.22	24.2	17.3	49
N3718	SBap	22.7	1.2030	6.88	286.3	11.24	.25	.31	44.2	13.8	34
N3726	Sc	18.3	1.5307	7.58	222.0	10.84	.24	.30	41.2	14.0	24
N3810	Sc	15.5	1.2978	5.37	176.5	11.30	.24	.36	15.6	10.7	LEO

TABLE 7—Continued

NGC	TYPE	d	csc i	$D_H(0)$	V_m^c	m_{PG}	A_b	A_i	M_T	$\frac{M_T}{L_{PG}}$	GROUP
		Mpc		arc min	km/s	mag	mag	mag	$10^{10} M_\odot$	solar units	
N3813	Sb:	22.2	1.2310	2.86	194.5	12.64	.23	.29	8.29	10.2	17
N3893	Sc:	17.7	1.2348	5.39	190.2	10.76	.24	.39	20.8	6.46	32
N3898	Sab	22.7	1.2425	4.51	306.9	11.33	.26	.29	33.3	11.4	34
N3938	Sc	17.7	4.0698	6.74	—	10.79	.24	.24	—	—	32
N3992	SBbc	22.7	1.3314	8.54	322.2	10.62	.25	.26	121.4	22.5	34
N4051	Sbc	14.3	1.4226	7.26	190.6	10.81	.23	.24	22.7	13.1	10
N4062	Sc	15.4	1.1308	5.27	168.5	11.83	.22	.49	13.9	14.2	
N4096	Sc	17.7	1.0773	6.84	186.4	10.88	.24	.62	25.4	7.13	32
N4145	Sd	22.2	1.3790	7.53	160.0	11.33	.23	.33	36.9	13.1	17
N4151	Sab:	22.2	1.6896	8.58	131.8	11.22	.23	.21	11.4	4.09	17
N4236	SBdm	4.24	1.0397	18.72	93.6	10.05	.30	.84	5.99	10.5	2
N4303	Sbc	24.5	1.3565	9.58	116.7	10.01	.24	.25	19.3	1.78	26
N4369	Sa	22.2	2.3765	3.61	194.9	12.59	.23	.19	10.5	13.6	17
N4395	Sm:	6.94	1.4412	13.67	96.6	10.66	.22	.42	11.4	20.8	3
N4490	SBdp	14.3	1.1538	7.35	147.7	10.09	.23	.46	19.8	4.81	10
N4536	Sbc	24.5	1.1272	7.20	193.9	10.94	.24	.37	40.0	7.79	26
N4631	SBd	14.3	1.0072	12.25	161.2	9.71	.22	.92	39.2	4.43	10
N4656	SBmp	14.3	1.0215	9.93	99.1	10.74	.22	1.20	18.0	4.06	10
N4670	SB0/ap:	18.7	2.0270	2.85	214.9	13.08	.22	.13	8.49	25.9	13
N4712	Sbc	87.5	1.0867	2.88	236.9	13.46	.22	.44	85.3	12.7	
N4725	Sabp	18.7	1.7403	11.43	363.7	10.07	.22	.21	97.5	17.3	13
N4747	Sbc?	23.9	1.2004	3.82	114.0	12.90	.22	.42	7.15	8.66	
N4939	Sbc	59.5	1.2275	7.13	287.2	11.41	.28	.29	211.0	11.2	
N5033	Sc	18.6	1.1111	9.82	250.0	10.61	.22	.53	68.8	15.1	
N5055	Sbc	12.1	1.2633	13.94	256.4	9.26	.23	.28	66.9	12.5	M51
N5147	SBdm	20.4	2.0140	2.74	157.1	12.41	.25	.26	11.9	14.2	
N5194	Sbcsp	12.1	1.3183	12.59	133.1	8.88	.24	.26	16.3	2.16	M51
N5204	Sm	7.90	1.1512	6.59	79.4	11.62	.26	.61	4.23	11.7	M101
N5457	Scd	7.90	—	28	—	8.20	.25	.23	—	—	M101
N5474	Scdp	7.90	2.9811	7.08	98.4	11.22	.25	.24	3.74	10.1	M101
N5585	Sd	7.90	1.2969	7.66	106.3	11.25	.26	.36	5.89	14.6	M101
N5660	Sc	48.8	1.9581	4.42	150.8	12.10	.25	.27	29.6	4.60	37
N5676	Sbc	48.8	1.1223	4.60	276.1	11.60	.25	.37	103.2	9.22	37
N5701	SB0/a	33.9	3.2794	6.04	—	11.85	.26	.12	—	—	29
N5713	Sbcsp	33.9	1.4672	4.44	187.8	11.72	.28	.23	32.0	7.32	29
N5850	SBb	35.7	—	6.0	—	11.56	.29	.17	—	—	50
N5879	Sbc:?	19.0	1.0839	5.15	156.1	12.05	.28	.44	14.4	11.7	30
N5907	Sc:	19.0	1.0000	8.46	246	11.04	.28	.92	58.7	12.1	30
N5970	Sbc	40.9	1.4219	3.86	238.9	12.06	.30	.32	54.3	10.6	
N5985	Sb	54.4	1.2003	6.61	321.7	11.84	.30	.31	128.4	11.6	
N6015	Scd	20.9	1.1037	4.99	171.1	11.69	.32	.54	21.1	8.95	
N6217	SBbc	36.6	2.3871	5.08	279.3	11.90	.40	.19	87.5	18.8	51
N6412	Sc	36.6	2.1939	3.68	153.6	12.19	.42	.26	19.2	4.97	51
N6643	Sc	36.6	1.1549	4.21	197.5	11.61	.47	.46	36.2	4.36	51
I4182	Sm	6.94	1.9911	8.08	69.7	12.92	.22	.35	3.51	54.8	3
A814	Im	4.24	1.6672	10.32	63.4	11.14	.41	.38	2.27	15.0	2
A936	(Im)	4.24	3.6006	5.24	—	13.27	.35	.31	—	—	2

NOTES TO TABLE 7

N = NGC, I = IC, A = anonymous designation in RC1. d = distance, from average line-of-sight velocity of group members. i = inclination. $D_H(0)$ = face-on Holmberg angular diameter. $V_m^c = V_m \csc i$ = corrected rotational velocity. m_{PG} = Holmberg photographic magnitude. A_b = Galactic extinction correction. A_i = internal extinction correction. M_T = integrated mass. M_T/L_{PG} = ratio of mass to luminosity.

NOTE.—Group designations are by de Vaucouleurs 1975 except for the M51, M101, and Leo groups, which are by Sandage and Tammann 1975.

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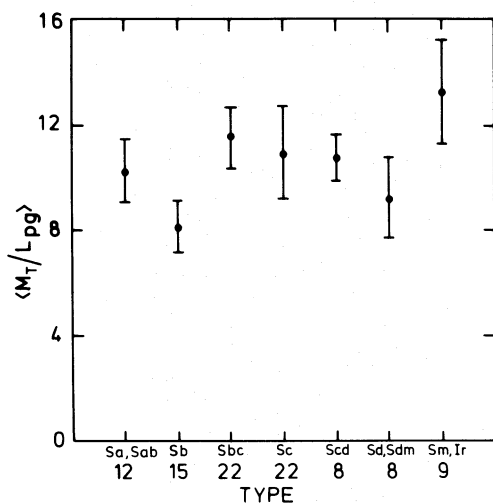


FIG. 2.—Average ratios of integrated mass to luminosity in solar units as a function of morphological type for the galaxies with data in Table 7. The error bars represent mean errors of the averages. The number of sample galaxies in each morphological type is listed below the type.

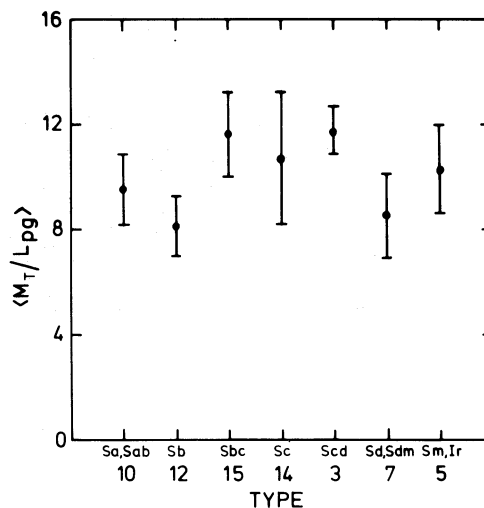


FIG. 3.—Average ratios of integrated mass to luminosity in solar units as a function of morphological type for the five brightest galaxies in groups with data listed in Table 7. The error bars represent mean errors of the averages. The number of sample galaxies in each morphological type is listed below the type.

REFERENCES

- Balkowsky, C. 1973, *Astr. Ap.*, **29**, 43.
 Brandt, J. C. 1960, *Ap. J.*, **131**, 293.
 Brosche, P. 1973, *Astr. Ap.*, **23**, 259.
 Brosche, P., and Reinhardt, M. 1977, *Astr. Ap.*, **54**, 531.
 Dean, J. F., and Davies, R. D. 1975, *M.N.R.A.S.*, **170**, 503.
 de Vaucouleurs, G. 1975, in *Stars and Stellar Systems*, Vol. 9, ed. A. Sandage, M. Sandage, and J. Kristian (Chicago: University of Chicago Press), p. 557.
 de Vaucouleurs, G., and de Vaucouleurs, A. 1964, *Reference Catalogue of Bright Galaxies* (Austin: University of Texas Press) (RC1).
 de Vaucouleurs, G., de Vaucouleurs, A., and Corwin, H. G. 1976, *Second Reference Catalogue of Bright Galaxies* (Austin: University of Texas Press) (RC2).
 Dickel, J. R., and Rood, H. J. 1975, *A.J.*, **80**, 584.
 Fisher, J. R., and Tully, R. B. 1975, *Astr. Ap.*, **44**, 151.
 Heidmann, J., Heidmann, N., and de Vaucouleurs, G. 1972, *Mem. R.A.S.*, **76**, 105.
 Holmberg, E. 1958, *Medd. Lund*, Ser. 2, No. 136.
 Holmberg, E. B. 1974, *Astr. Ap.*, **35**, 121.
 Huchra, J. 1976, *A.J.*, **81**, 952.
 Huchtmeier, W. K. 1975, *Astr. Ap.*, **45**, 259.
 Nilson, P. 1973, *Uppsala General Catalogue of Galaxies* (Uppsala: Uppsala Offset Center AB) (UGC).
 Roberts, M. S. 1969, *A.J.*, **74**, 859.
 ———. 1975, in *IAU Symposium No. 69*, Dynamics of Stellar Systems, ed. Hayli (Dordrecht: Reidel) p. 331.
 Rood, H. J., and Dickel, J. R. 1976, *Ap. J.*, **205**, 346.
 Rubin, V. C., Ford, W. K., Thonnard, N., and Roberts, M. S. 1976, *A.J.*, **81**, 687.
 Sandage, A., and Tammann, G. A. 1975, *Ap. J.*, **196**, 313.
 Shostak, G. S. 1975, *Ap. J.*, **198**, 527.
 Tully, R. B. 1972, *M.N.R.A.S.*, **159**, 35 p.
 Wakamatsu, K. 1976, *Pub. Astr. Soc. Japan*, **28**, 397.
 Zwicky, F., Herzog, E., Karpowicz, M., Kowal, C. T., and Wild, P. 1960–1968, *Catalogue of Galaxies and of Clusters of Galaxies*, Vols. 1–6 (Pasadena: California Institute of Technology).

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