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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Remarks on photographic colour equivalents and related problems,

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1. When stars are too faint for spectral photometry there are mainly two methods available for the photographic determination of their colour equivalents. The one uses the difference between the photographic and the photovisual magnitude and the other the effective wavelength.

An advantage of the first method is, that the colour equivalent is obtained as a byproduct of the determination of photographic and photovisual magnitudes, while the main difficulty lies in the necessity, that the photographic and photovisual scales should be exactly alike. It seems considerably safer to assume that the effective wavelengths are practically free of magnitude equation, when only images of approximately the same strength are used for bright and faint stars. A drawback of the effective wavelengths is that they cannot well be determined with refractors because of the disturbing influence of the secondary spectrum, while reflectors have only a small field.

The method of effective wavelengths has met with criticism due to the fact, that it does not well distinguish between stars, whose spectra lie between about *A* and *F* *). In order to judge the validity of the effective wavelengths they should however not be compared with spectra but with other colour equivalents. Roughly speaking the spectra represent temperatures and the effective wavelengths, being colour equivalents, represent reciprocal temperatures. It is therefore only natural that the effective wavelengths make a comparatively better discrimination among the yellow stars than among the white ones. This fact is plainly expressed in Table III of the Greenwich publication mentioned, where the dispersion in effective wavelength for stars of spectrum *K* is about 3 times as large as that of stars of spectrum *A* or *F*.

*) KNUT LUNDMARK and W. J. LUYTEN, *M. N.* 82, 495; 1922 and *Determinations of effective wavelengths of stars*, Royal Observatory, Greenwich, 1926.

It should in this connection be remembered that the spectral classification is arbitrary and that it is therefore not a priori to be expected, that the colour should change with the spectral class in an extremely simple way.

In order to compare the Greenwich effective wavelengths with other colour equivalents I have chosen the colour indices given by PARKHURST in the Yerkes Actinometry (*Ap. J.* 36, 169; 1912). The result of the comparison is graphically represented in Figure 1, where each point corresponds to one of the 188 stars *). No very serious deviation from a straight line connection between the two colour equivalents is apparent.

Dividing into 3 groups according to the sum of the coordinates measured in cm on the diagram I find the following mean values

number of stars	89	44	55
colour index	·161	·859	1^m·440
effective wavelength	4264	4398	4512 Å

These 3 points lie on a straight line, indicating an increase in the effective wavelength of 194 Å for 1 magnitude increase of the colour index.

The corresponding formula is:

$$\lambda_{eff} - 4367{\cdot}8 = 193{\cdot}7\ (m_{Yk,pg} - m_{Yk,pv} - {\cdot}698)$$

If the spectral class of each star according to PARKHURST (*l. c.*) is added on Figure 1, it will be seen that for stars of effective wavelengths between 4260 and 4300 Å there is a marked relation between spectrum (about *A* to *F*) and colour index, while no connection between effective wavelength and spectrum is shown

*) The colour index of one star, + 1^m·98, 4557 Å, is indicated by PARKHURST as being uncertain and on two places of the diagram — ^{m}·02, 4268 Å and — ^{m}·03, 4258 Å two stars fall on top of each other.

in this region of the diagram. The reason for this may possibly be that the effective wavelength, depending essentially on the positions of the two ends of the spectrum, is affected by the absorption at the head of the Balmer series, situated near the short-wavelength end of the spectrum, the Balmer series being the strongest in *A* and *F* stars.

Certainly, the effective wavelengths are not ideal colour equivalents, but they have different faults and advantages from other methods. They are able to compete with other methods.

2. In the Greenwich publication mentioned special tables are given of the whitest (Table VI) and the reddest stars (Table VII). For each of these two groups, omitting the very whitest stars with an effective wavelength less than 4199·5 Å, I have compared the visual Potsdam and the photographic Greenwich magnitude. The mean values used for the least square solutions are given in Table 1.

TABLE 1.

limits of $m_{Pd} + m_{Grw}$	number of stars	m_{Pd}	m_{Grw}	m_{Pd} $O-C$
m		m	m	m
14·0	9	6·654	6·244	— ·069
15·3	10	7·590	7.200	— ·043
16·7	16	8·263	7.825	+ ·036
18·2	26	9·022	8.704	— ·041
19·3	29	9·641	9.207	+ ·099
20·7	57	10·147	9.825	+ ·017
	66	10·731	10.495	— ·036
13·3	10	5·108	6.270	·000
16·0	22	6·927	8.018	— .036
17·4	25	7·920	8.920	·000
18·8	58	8·612	9.538	+ ·036
19·8	62	9·236	10.166	— ·007
	62	9·819	10.721	— ·013

Giving weights to the normal equations according to the number of stars, I find the following formulas

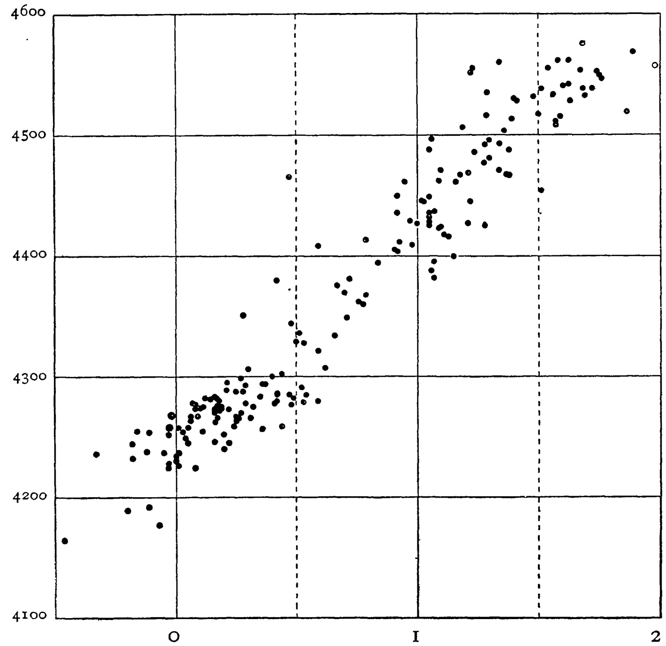
$m_{Pd} - 9{\cdot}713 = {\cdot}952\,(m_{Grw} - 9.387)$ for stars of mean
$\pm\ {\cdot}019$ effective wavelength 4239 Å

and

$m_{Pd} - 8{\cdot}713 = 1{\cdot}062\,(m_{Grw} - 9.667)$ for stars of mean
$\pm\ {\cdot}010$ effective wavelength 4534 Å

The mean errors of the coefficients are somewhat uncertain, but such evidence as there is, means that in both groups the visual and photographic scales are different. This difference is in opposite sense for white and yellow stars, in the same way as if the visual magnitudes were affected by the Purkinje phenomenon. This is not unexpected, as the comparisons, on which the Potsdam photometry are based, have not been made at the same apparent brightness. I doubt, however whether this is the right explanation. Anyhow, the effective wavelengths appear here to have revealed an interesting discrepancy, which, I think, it would have been difficult to find without their help.

FIGURE 1.



Abscissa: colour index, Yerkes Photometry.
Ordinate: effective wavelength, Greenwich.

The two lines representing the two formulas for white and yellow stars respectively intersect at $m_{Pd} = 20{\cdot}94$, $m_{Grw} = 21{\cdot}18$. The simplest assumption seems to be that the lines m_{Pd}, m_{Grw} representing other colours all pass through this same point. In that case the formula connecting m_{Pd}, m_{Grw} and λ_{eff} will, when λ_{eff} is supposed to change linearly with m_{Grw} for a given value of m_{Pd}, be

$$m_{Grw} - 21{\cdot}18 = -\,{\cdot}000369\,(m_{Pd} - 20{\cdot}94)\,(\lambda_{eff} - 7085).$$

A sliderule was made corresponding to this formula, but the effective wavelengths derived in this way from m_{Pd} and m_{Grw} did not agree well with the observed ones. The mean of 239 values of $(O-C)^2$ was $(\pm\ 57\ \text{Å})^2$.

3. I now proceeded to a comparison of the Greenwich photographic magnitudes with those of the Yerkes Actinometry *). It was soon recognized, that a linear formula would not be sufficient. Consequently

*) The following errata were noted: *Ap. J.* 36, p. 192, column *S—Pa* delete + ·31 for *BD* + 78°73 and add + ·09 for *BD* + 82°51; p. 208 the colour index of *BD* + 75°640 should be + ·94 instead of + ·96; p. 213 the colour index of *BD* + 75°822 should be + ·20 instead of + ·02.

quadratic terms were included in the least square solution. After having excluded uncertain cases the 487 stars, for which both m_{Grw}, $m_{Yk,pg}$ and $m_{Yk,pv}$ were known, were divided into 48 groups of approximately 10 stars each as indicated in Table 2.

Giving each group equal weight and putting $I' = m_{Yk,pg} - m_{Yk,pv} - {}^{\cdot}638$, $m'_{Grw} = m_{Grw} - 7^{\cdot}269$ and $m'_{Yk,pg} = m_{Yk,pg} - 7^{\cdot}410$ the following formula was derived according to least squares

$$m'_{Grw} = -{}^{\cdot}0355 - {}^{\cdot}0410\, I' + {}^{\cdot}9440\; m'_{Yk,pg} - {}^{\cdot}0428\, I'^2 + {}^{\cdot}0088\, I' m'_{Yk,pg} + {}^{\cdot}0571\; m'^2_{Yk,pg}$$
$$\pm{}^{\cdot}0107 \quad \pm{}^{\cdot}0143 \quad \pm{}^{\cdot}0096 \qquad \pm{}^{\cdot}0264 \quad \pm{}^{\cdot}0187 \qquad \pm{}^{\cdot}0090 \text{ (m. e.)}$$

The normal equations, where C is the constant term, are given in Table 3. The mean error in m_{Grw} of one equation is $\pm\,{}^{m}{}^{\cdot}041$.

TABLE 2.

number of stars	$m_{Yk,pg}$	I $m_{Yk,pg} - m_{Yk,pv}$	m_{Grw}	m_{Grw} $O-C$
	m	m	m	m
10	5·019	— ·257	5·274	— ·06
10	5·901	— ·113	5·946	— 1
10	6·452	— ·081	6·421	+ 3
10	6·812	— ·089	6·688	— 1
10	7·150	— ·139	7·079	+ 8
10	7·618	— ·128	7·462	+ 2
10	5·802	+ ·060	5·943	+ 7
10	6·776	+ ·062	6·670	— 0
10	7·197	+ ·044	7·026	— 2
10	7·859	+ ·057	7·649	— 3
10	5·609	+ ·228	5·740	— 0
10	6·268	+ ·209	6·268	+ 2
10	6·584	+ ·215	6·540	+ 4
10	6·901	+ ·234	6·757	— 2
9	7·113	+ ·263	6·946	— 2
10	7·280	+ ·249	7·097	— 2
10	7·428	+ ·258	7·270	+ 1
10	7·563	+ ·251	7·326	— 6
10	7·686	+ ·236	7·529	+ 2
10	7·870	+ ·233	7·617	— 7
10	8·258	+ ·317	8·064	— 2
10	5·779	+ ·641	5·835	— 1
9	6·721	+ ·560	6·640	+ 3
10	7·272	+ ·625	7·087	— 1
10	7·509	+ ·545	7·297	— 3
10	7·705	+ ·576	7·499	— 3
10	7·872	+ ·630	7·633	— 5
10	8·076	+ ·611	7·897	+ 1
9	8·267	+ ·606	8·187	+ 10
9	8·566	+ ·701	8·418	+ 1
10	6·827	+ 1·042	6·673	— 1
11	7·359	+ 1·041	7·167	+ 1
10	7·726	+ ·999	7·551	+ 3
10	7·904	+ 1·029	7·638	— 5
10	8·110	+ 1·033	7·949	+ 5
10	8·317	+ 1·041	8·107	— 1
10	8·454	+ 1·040	8·274	+ 1
10	8·805	+ 1·029	8·635	— 1
10	6·130	+ 1·442	6·056	+ 1
10	7·085	+ 1·564	6·870	+ 1
10	7·612	+ 1·399	7·369	0
10	7·809	+ 1·390	7·503	— 7
11	8·006	+ 1·510	7·717	— 4
11	8·223	+ 1·442	8·002	+ 2
12	8·378	+ 1·421	8·224	+ 8
12	8·524	+ 1·444	8·344	+ 5
12	8·605	+ 1·532	8·387	+ 0
12	8·915	+ 1·642	8·661	— 5

In the formula found the coefficient of $m'^2_{Yk,pg}$ is the most serious one. As it is 6 times its mean error, there cannot be much doubt about the reality. It means that the photographic scales of Greenwich and Yerkes are markedly curved against each other. In fact, the differentiation gives

$$\frac{dm_{Grw}}{dm_{Yk,pg}} = +{}^{\cdot}9440 + {}^{\cdot}0088\, I' + {}^{\cdot}1142\, m'_{Yk,pg}$$
$$\pm{}^{\cdot}0096 \quad \pm{}^{\cdot}0187 \quad \pm{}^{\cdot}0180 \text{ (m. e.)}$$

Neglecting the I' term it is seen that for $m'_{Yk,pg} = {}^{\cdot}49$ or $m_{Yk,pg} = 7^{\cdot}90$ the photographic scales of Greenwich and Yerkes are equal, while for $m'_{Yk,pg} = -1.26$ or $m_{Yk,pg} = 6^{\cdot}15$ the magnitude difference on the Greenwich scale corresponding to a difference of one magnitude on the Yerkes scale is only ·8. This is a considerable discrepancy. It shows that the establishment of a trustworthy photographic magnitude scale is more difficult than is sometimes assumed.

The coefficient of $I'\, m'_{Yk,pg}$, being less than half its mean error, shows that the proportion between the two scales does not depend sensibly on the colour of the stars.

The coefficient of I', being nearly 3 times its mean error, makes it probable that there is in the mean a small difference in the effective wavelengths of the Greenwich and the Yerkes photographic magnitudes, those of Greenwich being the larger. But the amount of this difference is rather uncertain.

4. In order to try to get some information as to whether the relative curvature of the scales of Greenwich and Yerkes is due to one or the other, I have separately compared the photographic Yerkes and Greenwich magnitudes with those of the Göttingen Actinometry for stars north of + 80°. Omitting the cases where the magnitude depends on only one plate, and arranging according to $m_{Gt} + m_{Yk,pg}$ and $m_{Gt} + m_{Grw}$ respectively, I found for groups of about 10 stars each the mean values given in Table 4. It is seen that in both cases the Göttingen magnitudes of the faintest stars are relatively too small. The explanation of this is given by SCHWARZSCHILD on p. 219, 220 of the Yerkes Actinometry. Before the least square solution I therefore excluded the last group in both cases. The rest of the material is however too limited to show relative curvature of the scales. The values of

O—C given in Table 4 are derived from the two linear formulas

$$m_{Gt} - 6{\cdot}919 = \underset{\pm\,{\cdot}012}{{\cdot}933}\,(m_{Yk,pg} - 6{\cdot}950)$$

and

$$m_{Gt} - 6{\cdot}942 = \underset{\pm\,{\cdot}021}{1{\cdot}104}\,(m_{Grw} - 6{\cdot}840)$$

From these formulas it is seen that at $6^{m}{\cdot}9$ the Göttingen scale of the polar stars is about midway between the scales of Yerkes and Greenwich. The proportion between the scales of Greenwich and Yerkes found here, $dm_{Grw} / dm_{Yk,pg} = {\cdot}845$, agrees sufficiently well with that derived from the formula given in the preceeding section for $m_{Yk,pg} = 6{\cdot}95$, viz. ·89.

5. The question whether the photographic plate shows a different gradation to light of different wave-lengths has been treated in detail by FRANK E. ROSS (*Ap. J.* **52**, 86; 1920 and **56**, 345; 1922), but in actual stellar photometry this point has received but little attention. Eventually (*A. N.* 4972, **208**, 53; 1919) I found from 183 exposures with a grating before the *UV* Zeisstriplet of Potsdam that the yellow star *BD* + 27°3911 showed a steeper gradation than the white star *BD* + 27°3868. The proportion between the two gradations was 1·030 ± ·007 (m. e.).

TABLE 3.

48 × C	− ·0200 × I'	+ ·0500 × $m'_{Yk,pg}$	+ 15·0300 × I'^2	+ 13·1400 × $I'm'_{Yk,pg}$	+ 39·1000 × $m'^2_{Yk,pg}$	= + ·0500
− ·0200 ×	+ 15·0300 ×	+ 13·1604 ×	+ 2·2955 ×	− ·2212 ×	− 2·7501 ×	= + 11·5509
+ ·0500 ×	+ 13·1604 ×	+ 39·1000 ×	− ·1917 ×	− 2·6896 ×	− 20·9303 ×	= + 35·1591
+ 15·0300 ×	+ 2·2955 ×	− ·1917 ×	+ 8·4019 ×	+ 7·4245 ×	+ 16·3240 ×	= − ·1705
+ 13·1400 ×	− ·2212 ×	− 2·6896 ×	+ 7·4245 ×	+ 16·3416 ×	+ 28·9862 ×	= − 1·5153
+ 39·1000 ×	− 2·7501 ×	− 20·9303 ×	+ 16·3240 ×	+ 28·9862 ×	+ 87·9760 ×	= − 16·4539

TABLE 4.

number of stars	m_{Gt}	$m_{Yk,pg}$	m_{Gt} O—C	number of stars	m_{Gt}	m_{Grw}	m_{Gt} O—C
	m	m	m		m	m	m
10	5·273	5·151	+ ·032	10	5·416	5·485	− ·029
10	5·974	5·965	− 26	10	6·068	6·043	+ 7
10	6·264	6·276	− 26	10	6·368	6·289	+ 35
10	6·520	6·525	− 2	10	6·546	6·534	− 57
10	6·656	6·682	− 13	10	6·742	6·642	+ 20
10	6·853	6·881	− 2	10	6·919	6·816	+ 4
10	7·042	7·023	+ 55	10	7·121	6·972	+ 34
10	7·158	7·251	− 42	10	7·244	7·076	+ 42
10	7·273	7·303	+ 25	10	7·315	7·226	− 52
10	7·348	7·404	+ 5	10	7·472	7·270	+ 56
10	7·456	7·538	− 12	10	7·571	7·375	+ 39
10	7·572	7·625	+ 23	10	7·665	7·518	− 25
10	7·642	7·771	− 43	12	7·795	7·680	− 73
13	7·838	7·904	+ 29	13	7·942	7·948	(− ·222)
13	7·946	8·180	(− ·121)				

In part *A* of the Göttingen Actinometry the observations are given in such detail that a possible difference in gradation between white and yellow stars may manifest itself.

For each star and each plate the stellar magnitude derived from the 3 individual images of exposure times 9,3 and $1 \times 3^{m}45^{s}$ respectively is given. I now looked up pairs of a white and a yellow star on the same plate and approximately of the same magnitude. For each such pair the expression

$$(m_{y,3t} - m_{y,t}) - (m_{w,3t} - m_{w,t}),$$

where y = yellow, w = white and t is the exposure time in an arbitrary unit, was calculated. E. g. the longest and next longest exposures of the yellow star no. 206 (p. 48) gave the magnitudes 6·37 and 6·34 respectively and of the white star no. 207 6·46 and 6·46. The above expression is therefore in this case + ·03 corresponding to a weaker gradation of the yellow star, the magnitude derived from the longest exposure being relatively too great for that star. The difference in colour index between the two stars is $\Delta I_H = {\cdot}70$. In this way I found from 237 comparisons the mean value of the above expression to be + ·0265 ± ·0041 (m. e.) for $\Delta I_H = 1$. This result is surprising as it has the opposite sign of what would be expected, the white stars being here found to have the steepest gradation. It must be left to further investigation to find the explanation of this effect, but it should be remembered that the plates have been measured in the Hartmann microphotometer, in which a central portion of the disc, forming the image, is measured. The Eberhard- or neighbourhood effect may here affect white and yellow stars in a different way.