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COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN.

Remark on the colours of faint stars, by *Ejnar Hertzsprung*.

In *Astroph. Journal* **52**, 23, 1920 KAPTEYN and VAN RHIJN give new constants for the formulas of SCHWARZSCHILD (*A. N.* 4557, **190** 361, 1912) for the frequency $\varphi(M)$ of stars of different absolute magnitude M (luminosity curve)

$$\log \varphi(M) = a_1 + b_1 M + c_1 M^2,$$

where a_1, b_1 and c_1 are constants (l. c. p. 31), and for the density Δ of stars in space at different galactic latitudes and distances ρ from the sun (l. c. p. 35)

$$\log \Delta = a_2 + b_2 \log \rho + c_2 (\log \rho)^2,$$

where a_2, b_2 and c_2 are constants different for each galactic latitude.

From these formulas it is easy to calculate the frequency of stars of different luminosity for each galactic latitude and given *apparent* magnitude. In fact, apart from an additive constant, we find:

$$\log \psi(M) = a_1 + b_1 M + c_1 M^2 + a_2 + b_2 \log \rho + c_2 (\log \rho)^2 + 3 \log \rho,$$

where $5 \log \rho = m - M$. Hence

$$\log \psi(M) = a_1 + a_2 + \frac{b_2 + 3}{5} m + \frac{c_2}{25} m^2 + \left(b_1 - \frac{b_2 + 3}{5} - \frac{2c_2}{25} m\right) M + \left(c_1 + \frac{c_2}{25}\right) M^2.$$

The function $\psi(M)$ is therefore, like $\varphi(M)$, of the Gaussian form, and, as the coefficient of M^2 does not contain m, the mean deviation from the mean absolute magnitude is for each galactic latitude independent of the apparent magnitude.

Further it is seen, that the coefficient of M is a linear function of m. This means that to each increase of 1^m in the apparent magnitude corresponds a constant increase in absolute magnitude. This last increase depends only on the galactic latitude.

Using the constants of the formulas given by KAPTEYN and VAN RHIJN, I find for all apparent magnitudes and different galactic latitudes b:

	b =	0°	30°	60°	90°
mean deviation in M =		± 1.93	± 1.89	± 1.68	± 1.50
for $\Delta m = 1$	ΔM =	.41	.43	.55	.64
	$\Delta 5 \log \rho$ =	.59	.57	.45	.36

For stars contained in a given volume of space the mean deviation from the mean absolute magnitude according to KAPTEYN and VAN RHIJN is $\pm 2^m.51$. If the star density were the same at all distances, the mean deviation in M would be the same for stars of any given apparent magnitude. When the star density diminishes with increasing distance the mean deviation in M for a certain apparent magnitude m will be less. It is in accordance with this consideration, that the above table shows a decrease of the mean deviation in M with increasing galactic latitude. For stars of given m near the pole of the galaxy the mean deviation in M is only $\pm 1^m.5$. This is about the same as the mean error assigned to the absolute magnitude calculated from the apparent magnitude and the proper motion of a star, viz: ± 1.41 according to KAPTEYN (*Groningen Publ.* **8**, 23, 1901) and ± 1.63 according to SCHWARZSCHILD (*l. c.*, 373) The smaller value ± 1.27 given by the writer (*A. N.* 4975, **208**, 95, 1919) refers to stars with an annual proper motion of more than 0″.1.

For all stars taken together it is seen from the above table that according to the formulas of KAPTEYN and VAN RHIJN one magnitude change in the apparent magnitude is accounted for to about equal parts by change in the absolute magnitude and by change in the distance *). At high galactic latitudes the change in distance for a given change in the apparent magnitude is proportionally small, as was to be expected.

Further investigations are needed to see, to which extent these consequences of the formulas of KAPTEYN and VAN RHIJN hold. A more rapid falling of the density with increasing distance would imply a decrease

*) This simple way of putting the matter is due to EDDINGTON (*Month. Notices* **76**, 530, 1916).

of the mean deviation in M and an increase of $\Delta M/\Delta m$ with increasing apparent magnitude (compare *A. N.* 4422, **185**, 91, Tab. 3, 1910).

For different galactic latitudes b the following relations between the apparent magnitude m and the mean absolute magnitude M are derived from the formulas given by KAPTEYN and VAN RHIJN:

$$b = 0^\circ \quad M_0 = -7.814 + .407\, m$$
$$b = 30^\circ \quad M_0 = -7.334 + .432\, m$$
$$b = 60^\circ \quad M_0 = -7.957 + .551\, m$$
$$b = 90^\circ \quad M_0 = -8.530 + .641\, m$$

From these formulas I derived the following table, which gives the mean absolute magnitudes M for different galactic latitudes and apparent magnitudes.

m	0° m	30° m	60° m	90° m
0	−7.81	−7.33	−7.96	−8.53
5	−5.78	−5.18	−5.20	−5.33
10	−3.74	−3.02	−2.44	−2.12
15	−1.70	− .86	+ .31	+1.08
20	+ .33	+1.30	+3.07	+4.28
	±1.93	±1.89	±1.68	±1.50

At the bottom of each column is added the mean deviation in M.

As is also stated by KAPTEYN and VAN RHIJN the formulae adopted do not hold for the very nearest stars. We therefore need not pay much attention to the figures of the table relating to the brightest stars. Much greater interest attaches to the mean absolute magnitudes of the apparently faint stars. As was to be anticipated the mean absolute magnitude increases both with the apparent magnitude and the galactic latitude.

The important question now arises, whether the calculated values of these mean magnitudes are in accordance with what we know about the mean colours of faint stars and the relation between colour and absolute magnitude.

Perhaps the field where the colours of faint stars are best known is that of the North Pole. Taking the galactic latitude of this region to be 27° the calulated mean absolute magnitude of stars of the apparent magnitude 15 is $-0^m.94$.

From a paper containing mean colour equivalents of 734 stars brighter than 5^m visually (*Annalen van de Sterrewacht te Leiden*, Deel XIV, eerste stuk) I find in connection with the most reliable determinations of stellar parallax available, that the mean value of c_2/T for stars of the absolute magnitude $-0^m.94$ is 2.5 corresponding to a colour index of $+{}^m.53$ in the scale of KING (*Harv. Ann.* **76**, 117, 1916).

The visual magnitude 15 in the Harvard scale used by KAPTEYN and VAN RHIJN corresponds to the photovisual magnitude 15.3 in the scale of SEARES (*Mount Wilson Contr.* No. 97, *Ap. J.* 41, **206**, 1915). For stars of this magnitude in the North Polar region the mean colour index according to SEARES is $+{}^m.90$. Using my measures of effective wavelengths as a connecting link of comparison, this is found to correspond to a colour index of $+{}^m.67$ in the scale l_H of the Göttingen Actinometry. Hence there is, for stars of the magnitude 15 near the North Pole, no serious discordance between the colour index to be expected from the above consideration and the one found by SEARES.

It is however of interest to carry the comparison between calculated and observed colours down to still fainter stars.

Now SEARES has determined the colour index on his own scale for 6 absolutely faint stars (*Journ. of the Astr. Soc. of the Pacific* **28**, 280, 1916 and **30**, 191, 1918 and *Proceedings of the National Academy of Sciences* **5**, 234, 1919). Of these six stars I omit the companion to Lal 5490 on account of its uncertain parallax and VAN MAANEN's star of great proper motion near Lal 1299 because it is known to be of exceptional colour, the colour index being + .57 and the absolute magnitude $+9^m.3$. The other 4 stars are:

	absolute magnitude	colour index
61 Cygni *A*	$+3^m.0$	$+1^m.09$
61 Cygni *B*	+3 .7	+1 .27
55 Cancri *B*	+7 .4	+1 .55
BARNARD'S star	+8 .3	+1 .76
mean	+5 .6	+1 .42

The mean colour index $+1^m.4$ is in the region of the North Pole found for stars of the photovisual magnitude 17.0 corresponding to $16^m.75$ on the visual Harvard scale. For stars of this apparent magnitude and galactic latitude 27° the mean absolute magnitude is from the formulas of KAPTEYN and VAN RHIJN calculated to be $-0^m.19$. This absolute magnitude is $5^m.6-(-{}^m.19)=5^m.79$ les than the value to be expected from the data of SEARES. The discordance is considerable. It means that the stars between the magnitude 15 and 17 in the region of the North Pole show a more rapid increase of the colour index with increasing apparent magnitude than is to be expected from the formulas of KAPTEYN and VAN RHIJN in connection with what is known about the relation between absolute magnitude and colour of the stars.

It seems to be impossible to explain this discordance without extraordinary assumptions. In fact, according to SEARES the mean colour index of the stars in the region of the North Pole increases from