BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS.

1938 March 14. Volume VIII. No. 306.

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Note on the light-variation of Cepheid-variables with secondary period,

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Theoretical considerations show the possibility of the existence of a class of stars, which necessarily must set up a state of motion corresponding to radial pulsation with a definite amplitude with one or two periods. As the observational material on Cepheid-variation with variable light-curve is growing, it seems appropriate to recapitulate some features of the said motion in case of a double period and to explain some obvious consequences from a theoretical point of view.

In the state of motion referred to the star performs a free oscillation in either one or two of the fundamental modes of vibration; only the latter case will be considered in this note. Whereas in the radial velocity the two pulsations may be superposed approximately linearly, in the light-variation a large degree of intermingling must be taken into account. Hence, if the phase-arguments of the two fundamental vibrations are w_1 and w_2, the light-variation is a double-periodic function of these two arguments with periods 2π that may be developed in the series:

$$\sum_{s_1, s_2 = -\infty}^{+\infty} \gamma_{s_1 s_2} \cos (s_1 w_1 + s_2 w_2) + \sum_{s_1, s_2 = -\infty}^{+\infty} \sigma_{s_1 s_2} \sin (s_1 w_1 + s_2 w_2),$$

the combination-terms being not at all negligible. As in the theory referred to the frequencies of w_1, w_2 must be nearly commensurable in the ratio 1 : 2, it seems preferable to introduce the two arguments w_1 and $w_2 - 2w_1$; then the second argument is long-periodic and shows by its presence clearly the variability of the resulting light-curve.

The theory of the light-variation has not been developed far enough to make possible a determination of the coefficients γ, σ. Hence, in this note the consideration is restricted to the rough approximation resulting from the supposition that the effect of the second fundamental vibration may be superposed linearly as a small perturbation in the light-variation: the luminosity L is determined by the relation:

$$L = f(w_1) + \varphi(w_2),$$

f and φ being two functions periodic with period 2π; the arguments w_1, w_2 are linear functions of the time t: $w_1 = n_1 t + \varepsilon_1$, $w_2 = n_2 t + \varepsilon_2$; $\frac{n_2}{2n_1} = 1 + \lambda$, λ being a small positive or negative fraction.

Consider the epoch of maximum luminosity: then the equation

$$f'(w_1) + 2\varphi'(w_2) = 0$$

must be approximately valid; the accent denotes the derivative of the function with respect to its argument. If $\varphi' = 0$, the resulting values of w_1 are

$$w_1 = (w_1)_0 + 2\pi n,$$

n being integer.

If φ' is taken into account, the values of w_1 receive an addition δw_1 determined by the equation:

$$f''((w_1)_0)\, \delta w_1 + 2\varphi'(w_2) = 0.$$

As $w_2 = w_2 - 2w_1 + 2w_1$, the equation may be transformed into the relation:

$$f''((w_1)_0)\delta w_1 + 2\varphi'(w_2 - 2w_1 + 2(w_1)_0) = 0.$$

The argument $w_2 - 2w_1 + 2(w_1)_0$ is a linear function of t, say $\nu t + \gamma$, with period $\frac{2\pi}{n_2 - 2n_1} = \frac{\pi}{n_1} \frac{1}{\lambda}$.

As λ is small, the perturbation of the epoch of maximum is a periodic function of t with a period that is long compared with the period of the light-variation.

Some quantitative results may be derived by supposing the functions f and φ to be harmonic functions: $f(w_1) = a_1 \cos(w_1 + \alpha_1)$, $\varphi(w_2) = a_2 \cos(w_2 + \alpha_2)$, a_1, a_2, α_1, α_2 being constants; a_1, a_2 are supposed positive. Then the value of δw_1 is determined by the relation:

$$-a_1 \delta w_1 - 2a_2 \sin(\nu t + \gamma + \alpha_2) = 0.$$

In the same way the maximum of the luminosity suffers a perturbation $\varphi(w_2 - 2w_1 + 2(w_1)_0)$, hence in the case of the harmonic functions f and φ: $a_2 \cos(\nu t + \gamma + \alpha_2)$.

So, the ratio between the amplitude of the oscillation in the epoch of maximum and the amplitude of the relative oscillation in the height of the maximum is equal to

$$\frac{2\,a_2}{a_1 n_1} : \frac{a_2}{a_1} = \frac{2}{n_1}.$$

If one takes into account that ν may be positive or negative, then the relation of the phases of these oscillations is made clear by comparing the two values:

perturbation of the epoch of maximum:

$$\delta t_{\max} = -\frac{2a_2}{a_1 n_1} \sin(\nu t + \gamma + \alpha_2);$$

perturbation of the relative height of maximum:

$$\frac{\delta L_{\max}}{L_{\max}} = \frac{a_2}{a_1} \cos(\nu t + \gamma + \alpha_2).$$

In the case considered the largest maximum luminosity and the largest minimum luminosity correspond.

It is to be remembered that all the quantitative relations are only rough approximations, on account of the complex character of the light-variation already referred to.
