Preliminary determination of the mass of Jupiter's fourth satellite from a discussion of old eclipses of satellite III

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Preliminary determination of the mass of Jupiter's fourth satellite from a discussion of old eclipses of satellite III, by *W. de Sitter*.

The determination of the mass of the fourth satellite rests on the term in the equation of the centre of the third, whose argument is $l_3 - \varpi_4$. Delambre put this coefficient at 116.^{s}76, from which by the formulas of Laplace follows

$$m_4 = \cdot 0000424751 ,$$

which is the value that, with small fluctuations, has been generally adopted to the present day.

Damoiseau in his tables uses the value 65.^{s}073 for the coefficient. Damoiseau's tables do not, like Delambre's, represent a consistent theory. The principal inequalities were derived by him anew from the observations, but for the smaller terms he adopted values based on Delambre's masses. The tables, which were published in 1836 to replace those of Delambre (which expired in 1840), thus represent the first step in a series of successive approximations. The second step, consisting in the derivation of new masses, was not completed by Damoiseau.

Souillart, in his new mathematical theory, uses Delambre's masses.

Adams substituted Damoiseau's coefficients in Laplace's equations, and thus found

$$m_4 = \cdot 0000214880,$$

but this value has not been used in any theory or tables.

If the equation of the centre in longitude is put in the form

$$\delta v_3 = k_3 \sin l_3 - h_3 \cos l_3$$

we have

$$h_3 = \Sigma_i \, \mathbf{a}_{3i} \sin \varpi_i \quad (i = 1 \ldots . 4)$$
$$k_3 = \text{same with cosines.}$$

The suffix 3 will be dropped, when no ambiguity can arise.

From observations at one epoch we can only determine h and k, but not the separate coefficients $\mathbf{a}_i$. For the determination of $\mathbf{a}_3$ and $\mathbf{a}_4$ separately we must thus combine two epochs separated by such an interval of time, that the difference of phase $\varpi_3 - \varpi_4$ differs by about 180° in the two cases. It was pointed out by me in my inaugural dissertation *) that for combination with modern observations we require observations near the epoch 1790.

All complete eclipses of satellite III (i. e. eclipses of which the disappearance and reappearance have been observed by the same observer with the same instrument), of which records were found in the litterature from the years 1770—1800, have been compared at the Leiden observatory with my theory of 1908 **). The observed times were converted from true to mean time, corrected for aberration and reduced to Greenwich mean time. For these times jovicentric coordinates of the satellite were computed. Also the jovicentric longitudes of the sun were computed from Hill's tables, and the differences

$$\alpha = v - \Lambda + 0.°0025$$

were formed ***).

The detailed discussion of these observations will be undertaken by Mr. P. FEENSTRA KUIPER as his inaugural dissertation for the doctor's degree. Preliminary results will be derived here.

In this preliminary discussion I neglected the effect of the change of the latitude and the radius-vector during the eclipse, thus assuming that the mean

$$\alpha_0 = \tfrac{1}{2} (\alpha_{in} + \alpha_{ex})$$

*) Discussion of Heliometer observations of Jupiter's Satellites, Groningen, 1902, p. 82. See also *History of the Cape Observatory*, page c.

**) On the masses and elements of Jupiter's Satellites, and the mass of the system, *Proc. Acad. Sci. Amsterdam*, Vol. X, pp. 653—673, 710—729, Feb. 1908. This paper will be quoted as »Masses and Elements".

***) Cf.: On the formulae for the comparison of observed phenomena of Jupiter's Satellites with theory, *M. N.* vol lxxi, pp. 85—101, Nov. 1910.

of the values of α for disappearance and reappearance gives directly the correction to the longitude of the satellite at the time $t_0 = \frac{1}{2}(t_{in} + t_{ex})$ of mid-eclipse.

I further combined all observations of the same eclipse by different observers to one mean, and treated all eclipses as of equal weight, irrespective of the number of observers. I then combined the observed values of α_0 to normal places according to the value of the mean longitude l of the satellite. These normal places are given in Table I. From the deviations of the separate eclipses within one normal place from their mean, the probable error of one value α_0 was found to be

$$\pm 0.^\circ 015$$

TABLE I.

Nr.	l	α_0	Res.
25	44°.9	+.°004	+.°001
15	76.4	+.033	+ 6
6	115.8	+.055	− 3
7	162.0	+.076	− 6
3	197.0	+.091	+ 31
5	243.4	+.021	− 5
13	359.7	−.022	+ 2

These observed values can be represented by the formula:

$$(1) \qquad \alpha_0 = +{}^\circ 022 \underset{\pm 25}{} \; + {}^\circ 019 \underset{\pm 3}{} \sin l \; - {}^\circ 046 \underset{\pm 3}{} \cos l$$

The probable errors have been derived from the residuals, which are given in the last Column of Table I. It will be seen that the representation is very satisfactory. It should be remarked, that the individual eclipses are not so well represented. Evidently there are several sources of error affecting the single eclipses, which are to a great extent eliminated in the normal places. This is what could have been expected a priori.

The first term of (1) is a correction to the mean longitude, which will not further be considered at present. The other two terms give a correction to the equation of the centre, amounting to

$$\Delta h = +{}^\circ 046 \pm {}^\circ 003$$
$$\Delta k = +.019 \pm .003$$

The values of h and k used in the computation of the tabular coordinates were

$$h = -.^\circ 0015 \sin \varpi_2 + .^\circ 1736 \sin \varpi_3 + .^\circ 0706 \sin \varpi_4$$
$$k = \quad \text{same with cosines}$$
$$\varpi_2 = 62^\circ.7 + .^\circ 03896\, t$$
$$\varpi_3 = 338.3 + .007032\, t$$
$$\varpi_4 = 283.15 + .001896\, t$$

where t is the time counted in days from 1900.0.

The mean epoch of the observations is 1783.34.

For comparison we have the observations by Bessel in 1836, and the Cape Heliometer observations by Gill and Finlay in 1891 and Cookson in 1901 and 1902. These have been discussed in *Masses and Elements* pp. 670—673. The observed values and the resulting corrections to the theory with their probable errors are given in Table II. The results for 1891, 1901 and 1902 have been reduced from Jupiter's vernal equinox, which was used in *Masses and Elements*, to the first point of Aries as zero of longitudes.

TABLE II.

Epoch	$h_{obs.}$	Δh	Res.	$k_{obs.}$	Δk	Res.
1783.34	+.°129	+.°046 ± .°003	.°000	+.°090	+.°019 ± .°003	−.°001
1836.0	−.033	+ 8 ± 10	− 11	−.188	+ 20 ± 14	+ 22
1891.75	−.188	− 2 ± 8	− 7	+.130	− 5 ± 7	− 1
1901.61	−.148	− 26 ± 17	− 29	+.190	+ 7 ± 17	+ 6
1902.60	−.085	+ 30 ± 12	+ 28	+.175	− 11 ± 12	− 13

In order to derive corrections to the elements of the theory I tried to represent these quantities by equations of condition of the form:

$$(2) \qquad \begin{aligned} \Delta h &= x \sin \varpi_3 + y \cos \varpi_3 + z \sin \varpi_4 \\ \Delta k &= x \cos \varpi_3 - y \sin \varpi_3 + z \cos \varpi_4 \end{aligned}$$

The meaning of the unknowns is thus

$$x = \Delta \mathbf{a}_3$$
$$y = \mathbf{a}_3 \sin \Delta\varpi_3$$
$$z = \Delta \mathbf{a}_4$$

The values of y are different for the different epochs. They are however not independent of each other. We must have

$$\Delta\varpi_3 = \beta + \gamma t$$

where γ is the correction to the motion of ϖ_3. Both γ and z depend on m_4.

As a first approximation I neglected Bessel, and combined the three modern series to one normal place, with the relative weights 2, 1, 1. Then, if y_1

and y_2 are the values of y at the old and the modern epoch respectively, we have four equations with the four unknowns x, y_1, y_2, z. Now treating y_1, y_2 and z as three independent unknowns, I find:

$$x = +.0163$$
$$y_1 = +.0318$$
$$y_2 = -.0208$$
$$z = -.0285$$

The two values of y give

$$1783.34 \quad \Delta\varpi_3 = +10.^\circ6$$
$$1896.93 \quad \Delta\varpi_3 = -\ \ 6.\ 7$$

or

$$\Delta\varpi_3 = -7.^\circ17 - .^\circ000417\ t.$$

Now the motion of ϖ_3 used in *Masses and Elements* is $+ .^\circ007032 + .^\circ00066\ \lambda_4$, where λ_4 is defined by $m_4 = \cdot000042475\ (1 + \lambda_4)$.

This motion depends on Souillart's theory. In my new theory *) I find a correction to Souillart's values, using the same masses in both cases, of $-.^\circ000097$. We have thus

$$-.000417 = -.000097 + .00066\ \lambda_4,$$

from which

$$\lambda_4 = -.485$$

From the value of z we have, by the differential formulas of *Masses and Elements*,

$$z = .0619\ \lambda_4,$$

from which

$$\lambda_4 = -.460$$

The agreement between the two determinations is excellent. I adopt

$$\lambda_4 = -.475$$

We then have

$$z = -.^\circ0296$$
$$\Delta\varpi_3 = -7.^\circ2 - .^\circ000410\ t$$

From this value of $\Delta\varpi_3$ we compute the values of y for the different epochs, and substituting in the equations (2), we find the residuals which are contained in Table II.

*) Theory of Jupiter's Satellites, II, the Variations. *Proc. Acad. Sci. Amsterdam*, Vol. XXII, p. 236 (1919).

Although the observations by Bessel are known to be affected by systematic errors, the residuals are not larger than could be expected with regard to the probable errors. The values of Δh in 1901 and 1902 of course cannot be reconciled by any system of elements. With the exception of these two, however, the residuals are very satisfactory and a second approximation does not appear necessary.

I have however derived the weights of corrections to the adopted values of x, λ_4, β in the supposition that only these three independent unknowns were introduced, and solved from all equations by the method of least squares.

As the result of this investigation, we have thus

$$(3) \qquad \begin{aligned} m_4 &= .0000\,2230 \pm 0000\,0060 \\ 2e_3 &= .^\circ1899 \pm .^\circ0046 \\ \varpi_3 &= 331.^\circ13 \pm 0.^\circ26 + .^\circ006622\ t. \end{aligned}$$

The probable errors have been derived from the residuals of Table II in the manner just explained.

It is noticeable that the resulting value of m_4 is very near the value found by Adams. The probable error is very satisfactory. Also the probable errors of e_3 and ϖ_3 are much smaller than those of the previous determinations of these elements.

Nevertheless the values (3) cannot be accepted as final. Firstly they rest on a preliminary discussion of the old eclipses. It is not probable, however, that a final discussion will alter the adopted values of Δh and Δk appreciably. More serious however is the neglect of possible corrections to all masses except m_4, to the coefficient Jb^2, and to the values and motions of ϖ_2 and ϖ_4. In a final discussion the data resulting from the present investigation must be combined with other data, and all masses must be determined together.

In this determination the mass of the fourth satellite, which up to now could be said to be practically unknown, will have a very large weight. Provisionally we can adopt the value found here, which will certainly not be very far from the truth.

ERRATA.

B. A. N. **1**, p. 3, 2nd	column,	line 17	from	top:	for 0	read	10
B. A. N. **2**, „ 5, 2nd	„	„ 3	„	bottom:	„ 1912	„	1919
„ „ 7, 1st	„	„ 31	„	top:	„ 1914	„	1919
„ „ 8, 2nd	„	„ 25	„	„	„ 1908	„	1918