Mean parallaxes of faint stars, derived from a combination of the Pulkovo and Radcliffe Catalogues of proper motions

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BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS

1943 November 20 Volume X No. 362

COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN

Mean parallaxes of faint stars, derived from a combination of the Pulkovo and Radcliffe Catalogues of proper motions, by *L. Binnendijk*.

A comparison was made between the Pulkovo and Radcliffe catalogues of proper motions. Only for a few areas a serious magnitude equation was found, the difference between Pulkovo and Radcliffe proper motions exceeding in some cases ″·003 per magnitude (Table 1, last columns). The precision found for the catalogues is in agreement with that given by the authors (Table 6*a*, column 4). The mean probable error of the proper motions in the combined areas is ± ″·0030; it was found that the proper motions of stars between $12^{m}\cdot5$ and $13^{m}\cdot5$ are the most accurate, with a mean probable error of ± ″·0026 (Table 3).

Accurate mean parallaxes have been derived for the faint stars between 10·5 and 14·9 photographic magnitude. They were derived from the residuals in the *l*- and *b*- components independently, giving a good agreement between them (Tables 6*a* and 6*b*). In particular the new values for the mean parallaxes, derived in three galactic zones, are more reliable than earlier results, because the effect of the errors in the proper motions is much diminished (Table 7 and Figures 1—3). For all latitudes the mean parallaxes can be well represented by the formula $\log \bar{p}_m = \log \bar{p}_{12} - \cdot 115\ (m_{pg} - 12)$ (c.f. Figure 4 and Table 8); the first term is tabulated against galactic latitude in Table 10. The probable error of the mean parallaxes is now about 3%. A linear relation was found to exist between the logarithms of the parallax of stars of magnitude *m* and the logarithms of the number of stars $N(m)$ down to this magnitude, the value of $N(m)$ increasing by a factor 6·8 if the mean parallax is halved. The same linear relation holds if for a given magnitude rich and poor fields at the same latitude are compared (Figures 5 and 6). Relative parallaxes were calculated from mean algebraic averages of the υ-components (Table 11). They were reduced to absolute values by means of the parallaxes found by the first method (Table 7). In all, about 17000 stars in 113 areas were used. The notation is the same as in *B.A.N.* No. 290.

Magnitude equation and true probable errors.

The object of the present investigation is to derive mean parallaxes of faint stars as accurately as possible. Two methods were used, namely from residuals and from υ-components. The first method, giving absolute values of the parallaxes, seems to be preferable to the second.

First we made a comparison between the two catalogues used, the *Pulkovo Catalogue* [1]) compiled by A. N. Deutsch and the *Radcliffe Catalogue of Proper Motions* [2]) compiled by H. Knox-Shaw and H. Scott Barrett. Both contain photographic determinations of proper motions for stars down to about $15^{m}\cdot0$ in Kapteyn's northern Selected Areas.

For each star in the 73 areas which were contained in both catalogues the differences between the two catalogues were determined. These were combined in the following five photographic magnitude groups: 10·5—11·4, 11·5—12·4, 12·5—13·4, 13·5—14·4, 14·5—14·9. The mean differences Pulkovo *minus* Radcliffe for the five groups of magnitude are given for each area in Table 1 (unit ″·001).

From these data the magnitude equations were calculated in α and δ separately by the method of least squares, weights being given proportional to the numbers of stars (n). The equation of condition was:

$$(P-R) = A + B\ (m_{pg} - 10)$$

where P = Pulkovo, R = Radcliffe. The gradient B is given in the last column of Table 1 for each area; its probable error is ± ″·0005 on the average. Except in a few areas where B is greater than ″·003, no serious magnitude equation exists between the two catalogues. In Table 2 the probable error caused by the unknown magnitude errors in the average of the Pulkovo and Radcliffe motions is given. This was calculated from the coefficients B, separately for α- and δ-components, and will be used later.

In each area and each magnitude group the residuals in $P-R$ for individual stars were formed by subtracting from the observed values their algebraic average. The average residuals so formed were multiplied by a factor $\sqrt{n/(n-1)}$ to correct for statistical error and by ·845 in order to reduce to probable error.

A comparison between the probable errors of $P-R$ so computed from the differences between the two catalogues, with those found from the published prob-

1) *Publ. Pulkovo*, Série 2, Vol. IV, 1940.
2) Oxford Un. Press, London 1934.

TABLE 1. Pulkovo minus Radcliffe (unit ″·001).

area	11m·0 α	11m·0 δ	11m·0 (n)	12m·0 α	12m·0 δ	12m·0 (n)	13m·0 α	13m·0 δ	13m·0 (n)	14m·0 α	14m·0 δ	14m·0 (n)	14m·7 α	14m·7 δ	14m·7 (n)	B α	B δ
2	12	12	(2)	1	3	(10)	0	4	(23)	−3	1	(28)	−4	−1	(15)	−2·7	−2·2
5	−12	7	(9)	−6	1	(13)	−2	−2	(26)	2	0	(31)	1	−1	(46)	+1·7	−1·2
7	2	−5	(9)	−3	−9	(11)	−1	−5	(25)	0	0	(64)	−2	1	(57)	−·4	+2·7
8	−2	2	(8)	0	−3	(25)	3	1	(42)	−3	−1	(94)	−5	−1	(97)	−2·2	−·2
9	5	−1	(4)	6	4	(9)	1	2	(28)	2	1	(36)	1	2	(19)	−1·0	−·1
12	−4	−4	(1)	−2	8	(12)	−1	−2	(28)	−1	−4	(63)	−2	−2	(27)	·0	−2·5
13	1	0	(5)	5	0	(9)	2	−2	(21)	1	−1	(27)	1	1	(12)	−·7	+·2
14	4	−3	(2)	4	0	(12)	3	−6	(22)	2	−3	(30)	6	−8	(18)	+·3	−1·7
15	3	−3	(4)	2	3	(15)	−3	3	(32)	−2	3	(58)	2	−3	(18)	−·4	−·7
17	−7	−5	(3)	−11	−15	(15)	−5	−7	(28)	−4	−1	(61)	−8	2	(95)	·0	+5·2
18	1	−2	(5)	3	2	(22)	3	0	(40)	1	1	(68)	1	0	(63)	−·9	−·2
19	−4	−9	(12)	0	−4	(26)	−1	0	(51)	2	1	(111)	0	2	(81)	+·9	+2·5
20	−3	0	(10)	−1	−2	(22)	3	−1	(44)	2	1	(63)	5	3	(39)	+1·8	+1·4
21	16	−20	(9)	5	−5	(23)	−4	2	(41)	−1	−1	(70)	−2	−1	(13)	−3·5	+3·2
22	6	1	(2)	2	−3	(12)	1	−2	(43)	0	0	(92)	−1	−1	(66)	−1·3	+·6
23	1	−9	(6)	4	−6	(9)	2	−2	(35)	−1	−2	(57)	2	−1	(49)	−·4	+1·6
24	9	−13	(2)	2	−7	(4)	3	−3	(30)	−1	0	(41)	−2	4	(31)	−2·9	+4·4
25	−3	−2	(2)	−2	−10	(6)	−1	−2	(23)	0	−1	(53)	−2	−3	(42)	−·1	+·7
28	−5	1	(8)	−6	1	(11)	−5	−3	(23)	2	−3	(38)	1	−3	(22)	+2·9	−1·2
29	−6	−7	(8)	−1	−8	(13)	5	−2	(27)	2	−1	(35)	5	1	(22)	+2·1	+2·5
30	12	4	(5)	10	4	(11)	0	−3	(25)	−3	−3	(33)	−5	1	(10)	−5·1	−2·0
31	−4	−12	(8)	0	12	(8)	−4	4	(19)	−8	0	(33)	1	2	(3)	−1·5	+1·1
32	−2	−17	(4)	−1	2	(12)	−2	−3	(10)	3	3	(27)	0	1	(24)	+·9	+2·4
33	2	4	(4)	5	4	(4)	7	4	(23)	4	0	(13)	0	−2	(20)	−1·7	−2·3
34	1	7	(3)	−2	−1	(9)	−2	3	(21)	−1	−2	(39)	−2	−4	(23)	−·1	−2·5
35	−19	11	(3)	−14	5	(7)	−1	1	(26)	2	2	(75)	−3	2	(11)	+5·0	−1·3
38	6	−7	(7)	0	−3	(23)	1	−3	(44)	−2	−4	(68)				−1·9	+·1
39	−9	3	(7)	−4	5	(28)	−1	0	(60)	−1	−2	(84)	−2	−3	(73)	+·9	−2·4
40	−5	−3	(11)	−3	1	(27)	−2	1	(46)	−1	−1	(78)	2	−2	(37)	+1·6	−·5
41	−3	−7	(7)	−1	3	(29)	1	1	(50)	1	0	(209)	7	2	(43)	+2·0	+·2
42	−3	3	(8)	−3	4	(15)	−3	2	(30)	−4	−3	(68)	−2	−7	(30)	−1·2	−3·5
43	7	−4	(4)	1	−2	(15)	0	1	(67)	−1	2	(69)				−1·5	+1·8
44	−3	9	(6)	0	2	(19)	0	2	(22)	2	1	(78)	1	3	(18)	+1·0	−·7
45	−1	4	(5)	−2	1	(13)	0	4	(17)	−1	3	(46)	1	1	(54)	+·8	−·5
46	−3	−10	(2)	1	4	(19)	1	0	(15)	1	−1	(53)	−2	−3	(27)	−·5	−1·3
47	0	6	(6)	−3	−1	(8)	0	0	(8)	−2	3	(8)	0	6	(5)	+·2	+·4
48	−18	−9	(1)	−1	−4	(6)	−2	−2	(18)	−4	4	(37)	−4	2	(36)	−·4	+2·6
49	−6	0	(6)	2	5	(11)	−3	1	(25)	−1	0	(64)	0	−3	(52)	+·8	−1·8
50	−7	−5	(4)	2	−3	(24)	1	−2	(62)	0	−4	(92)	0	−5	(84)	−·3	−1·0
51	−2	−6	(9)	−1	−7	(18)	−1	0	(26)	−1	0	(77)	−1	0	(29)	−·4	+2·1
52	5	−11	(10)	3	−3	(15)	−2	0	(22)	0	2	(56)	2	2	(11)	−1·1	+3·3
53	−9	11	(7)	3	−1	(14)	−1	−2	(25)	2	−4	(52)	2	−8	(26)	+1·6	−3·7
54	−5	6	(3)	−2	1	(13)	−2	1	(14)	4	1	(28)	3	−8	(34)	+2·4	−3·4
55	−11	7	(4)	−3	4	(8)	−2	7	(16)	−2	3	(38)	−3	2	(14)	+1·2	−1·4
56	0	11	(1)	8	−4	(8)	0	0	(24)	−1	3	(33)	−1	5	(11)	−2·5	+2·5
57				4	1	(3)	2	5	(17)	2	1	(37)	2	−2	(20)	−·3	−3·0
58				−6	−7	(13)	−2	−4	(14)	0	−2	(38)	−6	−2	(38)	−·2	+1·9
59	3	−11	(3)	3	−6	(7)	5	−4	(10)	2	1	(48)	1	3	(16)	−·9	+3·8
64	4	1	(16)	0	−1	(36)	1	0	(105)	0	−2	(220)	−1	−1	(115)	−·8	−·5
65	9	13	(5)	0	5	(14)	1	1	(51)	3	−1	(96)	1	3	(43)	−·2	−·9
66	−1	10	(8)	4	6	(27)	−1	3	(74)	−2	1	(142)	4	3	(25)	−·6	−2·1
67	−10	16	(6)	−6	3	(19)	−2	0	(36)	−2	−2	(60)	−3	−2	(53)	+1·2	−2·9
68	10	−8	(6)	7	−3	(16)	3	−2	(22)	3	−4	(47)	1	0	(26)	−2·1	+1·0
69	−24	−1	(2)	−2	−3	(13)	−2	−2	(15)	3	1	(55)	1	2	(34)	+2·9	+1·8
70				4	7	(10)	0	0	(23)	−1	2	(16)	−7	−3	(13)	−3·5	−2·6
72	−15	5	(1)	5	6	(4)	0	4	(15)	4	−3	(25)	−5	−1	(11)	−·5	−3·6
73				−10	−6	(5)	−3	0	(19)	−1	2	(52)	1	−1	(41)	+3·5	+·6
74	−3	2	(5)	0	0	(18)	−1	−1	(25)	−1	−3	(56)	−1	−3	(44)	·0	−1·3
75	0	2	(4)	0	7	(6)	−1	3	(40)	−2	0	(88)	−4	1	(52)	−1·4	−1·3
76	8	13	(1)	−4	4	(10)	−1	1	(43)	2	−1	(66)	4	−5	(21)	+2·6	−3·3
77	−5	2	(4)	−5	2	(20)	3	−2	(22)	0	−1	(47)				−·9	−2·0
78	5	−8	(3)	3	0	(11)	−1	−1	(29)	2	1	(29)	−1	−2	(33)	−·9	+·2
79	−8	5	(4)	−4	0	(7)	−5	−3	(25)	−4	2	(36)	−2	4	(7)	+1·0	+1·0
80	0	−1	(3)	−1	−2	(9)	1	0	(24)	−1	1	(23)	4	−1	(5)	+·3	+·8
81	−3	−2	(1)	1	−4	(7)	0	−3	(25)	3	−2	(34)	−2	−4	(27)	−·5	−·1
82	12	8	(2)	2	6	(10)	2	9	(18)	2	6	(33)				−1·0	−·6
83	2	7	(7)	10	−2	(14)	2	0	(18)	−2	−6	(49)	−1	−10	(23)	−2·8	−3·9
84	7	−30	(1)	−1	1	(15)	0	−2	(44)	−1	1	(81)	1	9	(33)	+·1	+3·4
87	−2	4	(7)	−2	−4	(7)	−2	−2	(37)	1	−1	(63)	−1	−1	(43)	+·7	−·2
88	6	−5	(17)	1	0	(20)	3	−4	(52)	2	−4	(127)	3	−4	(57)	−·4	−·3
89	1	−2	(5)	0	−7	(20)	−1	−1	(48)	−2	0	(71)	−5	−3	(57)	−1·8	+·7
90	−1	−6	(4)	−3	−2	(22)	0	−1	(34)	−3	−3	(62)	3	−2	(23)	+·8	·0
91	4	−2	(5)	5	1	(15)	3	−4	(19)	2	−3	(61)				−1·1	−1·9

TABLE 2.
(unit ″·0001)

b	S_α	S_δ
0°—20°	10·7	12·5
20 —45	9·3	15·8
45 —90	13·0	15·8

TABLE 3.

b	11·0 m	12·0 m	13·0 m	14·0 m	14·7 m	probable error diff.	probable error publ.
0°—20°	1·08	·95	·84	·97	1·15	± ·0029	± ·0030
20 —45	1·04	·97	·92	1·02	1·18	± 30	± 32
45 —90	1·04	·95	·84	1·01	1·21	± 31	± 30

able errors is given in Table 3 for the three galactic zones 0°—20°, 20°—45°, 45°—90°. The mean probable error is ± ″·0030. The numbers in the first part of this table show the ratios of the probable error from differences and the probable error from published errors. It may be observed that the best accuracy has been obtained for the magnitude group 12·5—13·4, where the probable error is ± ″·0026 on the average. The last two columns show the average probable errors of the combined proper motions as found from *P*—*R* and from the published probable errors respectively. In these columns all magnitudes have been combined. The agreement is most satisfactory.

The proper motions.

In forming the mean values of the proper motions of both catalogues, weights have been assigned inversely proportional to the squares of the probable errors as given in the catalogues. However, in case the ratio of weights of the Radcliffe and Pulkovo catalogues was less than 2 they were considered equal. The relative weights are given in Table 6*a*, column 5. In order to extend the material the 40 Radcliffe areas for which no Pulkovo data exist have been added.

The galactic components of the proper motion were computed for each star and the means of the residual proper motions were derived for the various intervals of magnitude in each area.

For each magnitude group in an area the residuals

TABLE 4.
Uncorrected average residuals $\overline{|\nu_i|}$ (unit ″·0001).

m_{pg} \ b	0°—20° l	0°—20° b	20°—45° l	20°—45° b	45°—90° l	45°—90° b
11·0	98	59	114	95	144	125
12·0	78	58	104	84	133	127
13·0	70	53	94	79	114	108
14·0	66	51	84	74	110	108
14·7	70	59	89	78	103	103

$\mu_i - \overline{\mu}_i$ have been formed in l and b separately. The average residuals so formed were multiplied again by a factor $\sqrt{n/(n-1)}$ to correct for statistical error. The uncorrected average residuals $\overline{|\nu_i|}$ are given in Table 4.

If μ_i denotes a component of the true residual proper motion, ν_i an observed value of this component and r the probable error of the proper motions we obtain:

$$\overline{|\nu_i|}^2 = \overline{|\mu_i|}^2 + (1{\cdot}08\ r)^2.$$

For r we have adopted the values given in Table 6*a*, column 4 (diff.), resp. Table 6*b*, column 4, multiplied in each magnitude group by the factors given in Table 3. The factor 1·08 (see *B.A.N.* No. 290, p. 79) used by OORT was checked and found to be preferable to the factor 1·18 given in Table 1 of the Introduction of the Radcliffe Catalogue and used by VAN HOOF.

In a study like the present the large proper motions always cause difficulties. All total proper motions in excess of ″/a·050 have at first been excluded, but their effect was smoothed out over all areas and was added again to the final result. This has been done for the three galactic zones separately and divided smoothly over the five magnitude groups. The effect on the residuals is found from the expression $k.\Delta n/n$, where Δn is the mean number of large proper motions in one area read from a graph through the numbers of large proper motions plotted against the magnitude, k is the mean value of these proper motions in the group considered and n is the number of stars in this group. The factor $k.\Delta n$ is given in Table 5 (unit ″·001).

TABLE 5.
The factor $k.\Delta n$ for large proper motions (unit ″·001).

m_{pg}	b = 0°—20° l, b	0°—20° v	20°—45° l, b	20°—45° v	45°—90° l, b	45°—90° v
11·0	16	12	32	24	40	37
12·0	20	15	42	30	60	45
13·0	24	18	51	36	81	55
14·0	27	21	59	45	100	68
14·7	12	10	27	22	49	34

A subdivision was made into three groups according to the value of the cosine of the angle between the component used and the major axis of the velocity ellipsoid. The direction adopted for this axis was at 335° galactic longitude, 0° latitude ($\alpha = 17^h\ 58^m$, $\delta = -22°$). In each subdivision the influence upon the residuals was calculated in a similar manner and used in the present study. These corrections were applied to the averages of the observed proper motions. For the bright stars they become quite considerable, for the faint stars they are usually small, ranging from about 5% of the true average motions for faint low-

TABLE 6*a*. Results for Pulkovo and Radcliffe Areas (unit ″·0001).

area	l	b	m_l	p. error diff.	p. error publ.	p_R/p_P	$11^{m}{\cdot}0$ l	b	(n)	$12^{m}{\cdot}0$ l	b	(n)	$13^{m}{\cdot}0$ l	b	(n)	$14^{m}{\cdot}0$ l	b	(n)	$14^{m}{\cdot}7$ l	b	(n)
2	88°	+ 13°	14·5	± 36	± 28	1·0				18	30	(10)	21	20	(23)	18	17	(26)	17	5	(15)*
5	91	+ 42	15·2	30	37	2·6	35	35	(8)	29	27	(13)	22	21	(26)	17	17	(30)	17	12	(46)
7	76	+ 20	15·0	32	35	1·3	50	38	(7)	49	51	(10)	21	28	(24)	14	20	(60)	18	16	(57)
8	92	− 2	15·0	30	33	1·3	19	17	(8)	16	17	(25)	17	15	(42)	10	10	(93)	10	7	(96)
9	106	+ 3	15·4	22	27	1·5	30	16	(4)	20	24	(9)	17	20	(28)	21	15	(36)	16	20	(19)
12	123	+ 41	14·9	33	34	0·6				34	43	(11)	27	26	(25)	19	18	(59)	20	20	(26)
13	111	+ 53	15·2	27	27	1·9	27	74	(4)	45	43	(8)	36	13	(18)	23	27	(26)	17	19	(11)
14	81	+ 57	15·2	27	33	0·8	64	56	(2)	29	37	(12)	32	23	(19)	24	30	(30)	16	16	(18)
15	62	+ 48	14·1	36	31	0·7	64	34	(3)	35	47	(12)	22	33	(30)	19	20	(57)*			
17	59	+ 19	14·2	39	39	1·9	45	12	(2)	30	46	(15)	19	25	(28)	16	17	(59)*	11	10	(94)*
18	68	+ 6	14·8	27	30	1·2	31	31	(5)	13	17	(20)	13	13	(38)	9	12	(68)	12	5	(63)
19	81	− 1	14·8	23	25	0·9	34	23	(11)	18	12	(25)	12	12	(51)	10	9	(111)	11	9	(81)
20	89	− 17	14·4	29	28	3·4	16	19	(8)	23	32	(22)	16	18	(44)	14	18	(62)	10	12	(39)*
21	99	− 17	14·4	28	31	2·1	15	17	(9)	15	18	(23)	18	17	(40)	10	15	(69)	14	22	(13)*
22	111	− 13	15·2	26	27	0·8				18	17	(12)	14	16	(43)	8	11	(91)	9	11	(66)
23	120	− 7	15·6	27	28	4·1	16	43	(5)	9	10	(9)	19	22	(34)	14	10	(56)	10	12	(49)
24	128	0	15·2	26	29	1·4				38	34	(4)	14	15	(30)	14	13	(41)	14	14	(31)
25	133	+ 9	14·8	27	26	1·6	39	35	(2)	27	23	(6)	14	14	(23)	13	12	(53)	10	7	(42)
28	143	+ 39	15·5	24	26	1·2	12	25	(8)	27	30	(10)	22	20	(22)	15	15	(37)	16	14	(22)
29	142	+ 50	15·0	25	25	2·2	30	40	(7)	48	29	(11)	32	26	(26)	21	18	(35)	17	19	(22)
30	135	+ 60	14·8	39	34	0·8	48	57	(3)	41	41	(9)	27	29	(24)	29	28	(29)	15	21	(10)
31	123	+ 68	14·6	31	34	3·1	54	31	(6)	28	42	(7)	23	29	(18)	28	27	(31)			
32	84	+ 72	15·5	27	29	2·0	58	40	(4)	50	59	(9)	29	30	(9)	22	19	(26)	17	18	(22)
33	57	+ 67	15·0	30	28	1·0	—	42	(1)	58	53	(4)	30	41	(21)	36	45	(13)	24	13	(19)
34	42	+ 59	14·5	42	41	1·6	75	—	(1)	35	56	(9)	33	33	(20)	21	24	(38)	18	23	(22)*
35	37	+ 49	14·9	36	32	1·5	93	38	(3)	60	48	(6)	28	20	(25)	19	18	(73)	26	13	(11)
38	42	+ 18	14·0	34	38	2·7	40	42	(7)	26	14	(23)	17	25	(42)	16	20	(68)*			
39	47	+ 9	15·0	31	31	1·3	25	12	(7)	21	21	(28)	18	8	(58)	13	11	(84)	10	14	(72)
40	53	0	14·5	30	30	0·7	29	21	(11)	23	25	(27)	20	16	(46)	14	14	(78)	13	18	(37)*
41	61	− 8	14·4	35	27	0·6	34	32	(7)	21	19	(29)	13	18	(50)	10	12	(209)	10	11	(43)*
42	70	− 13	15·0	21	26	0·7	51	39	(8)	17	11	(13)	20	15	(30)	9	12	(68)	10	15	(30)
43	80	− 17	14·0	25	24	0·9	32	30	(4)	24	21	(14)	12	15	(64)	12	12	(67)*			
44	85	− 32	14·6	26	25	1·9	24	28	(6)	30	28	(18)	16	27	(22)	14	17	(78)	17	14	(18)*
45	101	− 32	15·3	26	31	2·0	45	36	(4)	28	27	(12)	16	27	(17)	18	20	(45)	15	14	(53)
46	117	− 27	14·5	35	39	1·7				31	43	(16)	24	35	(15)	15	13	(52)	17	14	(27)*
47	127	− 21	15·2	23	25	2·9	40	45	(6)	57	50	(7)	39	51	(6)	30	33	(8)	28	19	(5)
48	136	− 12	15·2	31	32	1·8				20	24	(6)	13	15	(17)	12	13	(37)	13	12	(36)
49	145	− 2	15·6	28	30	2·1	24	35	(5)	12	14	(11)	19	17	(25)	10	7	(64)	9	12	(52)
50	151	+ 10	14·5	33	30	1·3	33	47	(4)	15	20	(24)	16	11	(62)	11	12	(92)	10	10	(84)*
51	156	+ 22	14·8	27	31	1·2	27	46	(9)	22	19	(18)	19	17	(26)	15	11	(77)	18	11	(29)
52	161	+ 35	15·0	30	31	1·6	40	31	(10)	24	29	(15)	19	19	(22)	20	16	(56)	26	19	(10)
53	165	+ 47	15·0	31	30	0·8	25	42	(6)	23	31	(13)	26	27	(23)	19	20	(48)	22	16	(26)
54	166	+ 60	14·5	36	33	1·8	43	53	(3)	31	35	(11)	34	39	(13)	22	22	(26)	14	17	(34)*
55	166	+ 74	15·4	23	25	1·1	88	60	(4)	77	44	(8)	39	36	(15)	20	20	(35)	18	19	(14)
56	163	+ 80	15·0	35	33	1·5				39	41	(6)	25	25	(24)	30	25	(33)	17	28	(10)
57	30	+ 84	15·0	36	27	1·2				56	66	(3)	33	34	(15)	22	24	(37)	20	16	(20)
58	12	+ 72	14·6	33	32	2·8				35	45	(12)	33	42	(14)	26	21	(37)	20	17	(37)*
59	12	+ 59	14·8	43	34	0·9	79	71	(2)	62	66	(7)	38	47	(10)	25	24	(44)	25	25	(15)
64	35	− 1	14·4	32	31	1·4	13	19	(16)	17	13	(36)	17	11	(105)	11	11	(218)	10	11	(115)*
65	43	− 12	14·6	30	28	1·0	35	31	(5)	26	20	(14)	14	18	(50)	14	9	(96)	15	12	(43)*
66	53	− 21	14·5	35	33	0·6	20	41	(8)	20	17	(27)	18	15	(72)	13	13	(141)	13	20	(25)*
67	65	− 28	15·0	27	34	1·2	38	33	(5)	21	24	(19)	23	21	(34)	14	16	(60)	15	17	(53)
68	79	− 47	14·5	29	28	1·3	44	37	(4)	30	28	(14)	23	25	(21)	22	20	(44)	17	18	(25)*
69	102	− 47	14·3	40	36	1·2	—	80	(1)	35	41	(13)	35	44	(14)	18	26	(49)	13	20	(32)*
70	121	− 42	14·8	31	31	2·7				48	36	(8)	26	36	(19)	28	32	(16)	12	16	(13)
72	146	− 24	15·2	27	28	1·4				63	29	(3)	34	27	(14)	18	28	(25)	18	26	(11)
73	156	− 11	15·1	30	30	1·5				27	26	(5)	20	11	(19)	15	12	(52)	13	8	(41)
74	163	+ 1	15·0	28	33	1·0	31	28	(5)	23	15	(18)	10	17	(24)	11	12	(56)	17	10	(44)
75	170	+ 14	14·6	25	26	1·8				30	24	(6)	17	16	(40)	11	9	(88)	15	14	(52)*
76	177	+ 27	15·2	30	32	1·4				43	31	(10)	23	21	(41)	17	13	(65)	11	14	(21)
77	184	+ 39	14·2	25	29	3·3	49	48	(4)	31	34	(19)	32	20	(22)	16	17	(46)*			
78	192	+ 54	14·9	30	32	1·9	78	81	(3)	35	45	(11)	25	21	(29)	21	21	(29)	16	15	(30)
79	208	+ 67	14·8	23	25	2·7	65	88	(2)	49	50	(6)	24	26	(24)	23	25	(35)	27	22	(6)
80	240	+ 76	15·1	19	24	3·7	63	73	(3)	46	25	(7)	31	31	(21)	24	27	(21)	40	43	(5)
81	301	+ 75	14·6	25	26	6·4				60	77	(6)	25	21	(24)	26	29	(31)	16	18	(27)*
82	334	+ 65	14·5	37	32	2·2	64	89	(2)	52	31	(7)	38	34	(18)	26	25	(32)			
83	347	+ 53	14·6	26	31	4·1	33	52	(7)	48	51	(13)	26	36	(18)	22	27	(47)	17	23	(23)*
84	357	+ 40	14·0	39	39	3·8				30	31	(13)	21	20	(42)	17	21	(79)*	23	22	(31)*
87	17	0	14·6	27	30	3·1	46	32	(7)	16	24	(7)	16	20	(37)	14	14	(62)	16	19	(43)*
88	24	− 12	14·4	39	39	1·2	17	19	(15)	24	13	(20)	14	15	(52)	13	12	(127)	18	19	(57)*
89	33	− 23	14·8	30	30	4·0	39	54	(4)	20	22	(20)	22	19	(48)	15	13	(70)	17	15	(57)
90	45	− 34	15·0	30	32	1·8	50	—	(1)	25	17	(22)	25	19	(31)	20	19	(61)	16	14	(23)
91	61	− 43	14·2	34	34	1·0	36	46	(5)	28	26	(14)	23	19	(19)	18	23	(55)*			

Table 6b.

Results for Radcliffe Areas (unit ″·0001).

area	l	b	m_l	p. error publ.	$11^{m}·0$ l	b	$(\frac{1}{2}n)$	$12^{m}·0$ l	b	$(\frac{1}{2}n)$	$13^{m}·0$ l	b	$(\frac{1}{2}n)$	$14^{m}·0$ l	b	$(\frac{1}{2}n)$	$14^{m}·7$ l	b	$(\frac{1}{2}n)$
3	102°	+ 18°	14·6	± 40	32	39	(1)	33	9	(2)	22	18	(11)	13	13	(30)	6	18	(25)*
4	107	+ 32	14·7	47	45	42	(5)	23	30	(7)	21	22	(16)	14	16	(34)	15	21	(55)*
10	118	+ 13	14·9	46				39	43	(3)	18	22	(8)	8	16	(27)	19	14	(20)
11	123	+ 27	15·0	46				29	29	(8)	22	15	(20)	17	9	(28)	11	10	(23)
16	56	+ 33	15·0	40	32	41	(6)	27	29	(7)	27	19	(13)	18	25	(29)	23	27	(30)
26	138	+ 18	15·3	42	40	23	(2)	21	22	(5)	16	10	(20)	11	14	(34)	6	12	(29)
27	141	+ 29	15·0	35	70	36	(1)	31	26	(11)	23	26	(19)	11	13	(30)	14	13	(42)
36	38	+ 39	14·7	44	53	68	(2)	26	37	(6)	28	35	(12)	21	26	(24)	15	21	(26)*
37	39	+ 28	14·5	32	29	41	(4)	30	30	(7)	23	25	(25)	14	19	(47)	17	18	(39)*
60	15	+ 47	14·6	45	73	70	(2)	43	44	(5)	24	26	(12)	24	25	(28)	40	47	(10)*
61	19	+ 34	14·9	45	47	25	(3)	46	41	(5)	21	22	(13)	22	21	(27)	18	20	(30)
62	24	+ 23	14·6	42	41	31	(2)	31	24	(16)	23	19	(27)	21	17	(45)	15	23	(68)*
63	29	+ 10	13·9	60	40	18	(4)	21	27	(17)	21	17	(31)	19	22	(116)*			
71	136	− 34	15·5	32	—	61	(1)	28	30	(6)	25	32	(8)	18	18	(20)	17	16	(23)
85	4	+ 27	14·4	28	25	41	(5)	25	23	(7)	24	29	(18)	15	18	(45)	15	16	(44)*
86	10	+ 13	14·6	38	36	34	(4)	19	26	(13)	21	20	(24)	14	22	(52)	20	25	(52)*
92	95	− 62	14·6	49	55	55	(3)	38	39	(5)	25	24	(11)	27	24	(15)	20	21	(15)*
93	123	− 58	15·0	29	45	39	(3)	52	51	(3)	28	30	(12)	19	21	(20)	16	13	(17)
94	143	− 48	15·0	27	50	53	(2)	50	34	(3)	27	27	(15)	25	20	(18)	16	15	(17)
95	157	− 37	15·0	31	—	85	(1)	41	42	(3)	37	38	(7)	21	19	(10)	20	17	(10)
96	166	− 25	14·6	33	27	32	(6)	27	34	(8)	20	20	(17)	20	16	(30)	19	13	(24)*
97	174	− 11	15·3	43	23	22	(3)	39	35	(5)	16	22	(10)	22	12	(14)	5	19	(18)
98	181	+ 1	15·0	37	23	10	(10)	13	19	(24)	8	11	(55)	8	5	(112)	12	12	(111)
99	188	+ 15	14·8	30	20	27	(2)	14	22	(8)	12	15	(28)	12	10	(55)	11	11	(82)
100	196	+ 28	14·8	31	40	35	(4)	30	34	(8)	19	22	(23)	13	15	(45)	12	19	(37)
101	208	+ 41	14·5	48	53	39	(2)	28	21	(6)	22	21	(13)	18	19	(35)	21	22	(23)*
102	221	+ 52	14·4	40	74	49	(1)	33	41	(5)	26	25	(13)	20	21	(30)	26	28	(6)*
103	245	+ 60	15·0	26	41	45	(3)	34	46	(5)	31	28	(8)	21	24	(18)	17	12	(15)
104	269	+ 63	14·4	55	85	100	(1)	47	35	(3)	31	35	(10)	22	30	(24)	24	32	(12)*
105	295	+ 60	14·6	50	33	71	(2)	41	37	(4)	19	19	(13)	20	22	(20)	20	19	(15)*
106	320	+ 51	14·6	28	84	92	(1)	40	50	(4)	40	40	(6)	21	24	(22)	17	19	(27)*
107	334	+ 40	14·5	35	59	52	(2)	18	35	(6)	20	21	(11)	22	20	(32)	14	22	(11)*
108	343	+ 28	14·6	26	55	66	(1)	25	34	(5)	21	28	(12)	21	17	(24)	16	19	(25)*
109	353	+ 13	14·5	33	42	54	(1)	33	34	(3)	18	25	(9)	16	16	(38)	16	19	(26)*
110	0	+ 1	14·0	29	43	45	(3)	15	37	(6)	30	37	(10)	25	27	(15)*			
111	6	− 11	14·6	42	—	29	(1)	14	19	(5)	12	17	(8)	15	9	(37)	24	12	(38)*
112	15	− 25	14·3	37	40	42	(3)	28	21	(8)	19	18	(30)	18	17	(43)*	18	19	(42)*
113	24	− 38	14·6	35	48	38	(3)	34	28	(4)	26	23	(12)	17	19	(31)	14	18	(10)*
114	38	− 49	14·9	32	70	34	(2)	56	46	(2)	32	22	(13)	20	20	(26)	16	17	(39)
115	59	− 58	14·6	33	43	31	(2)	42	34	(8)	29	23	(12)	21	18	(23)	19	23	(6)*

latitude groups to about 35% for 12th magnitude stars in high latitudes. The effect of excluding all total proper motions larger than ″/a·100 would be 2% and 10% in these two cases respectively. Both ways of smoothing give however concordant results for the mean parallaxes.

The linear velocity.

The average linear velocity $a/\sqrt{\gamma}$ in the direction of the major axis of the velocity ellipsoid, measured in astronomical units per year, was read from the graphs given by Oort (see *B.A.N.* No. 290, p. 85), the factor $1/\sqrt{\gamma}$ was applied in order to take account of the increase of average velocity with increasing distances from the galactic plane.

The mean velocity in the direction of the component used in each area, expressed in the mean velocity along the greatest axis of the velocity ellipsoid as unit is now found from:

$$\sqrt{l^2 + \cdot 40\, m^2 + \cdot 32\, n^2},$$

where l, m, n are the direction cosines of the component with respect to the three axes. The ratio between the axis perpendicular to the galactic plane and the major axis has been taken equal to ·57 instead of ·50, in accordance with more recent computations by van Hoof [1]), Bok [2]), van Rhijn [3]) and Veldt [4]).

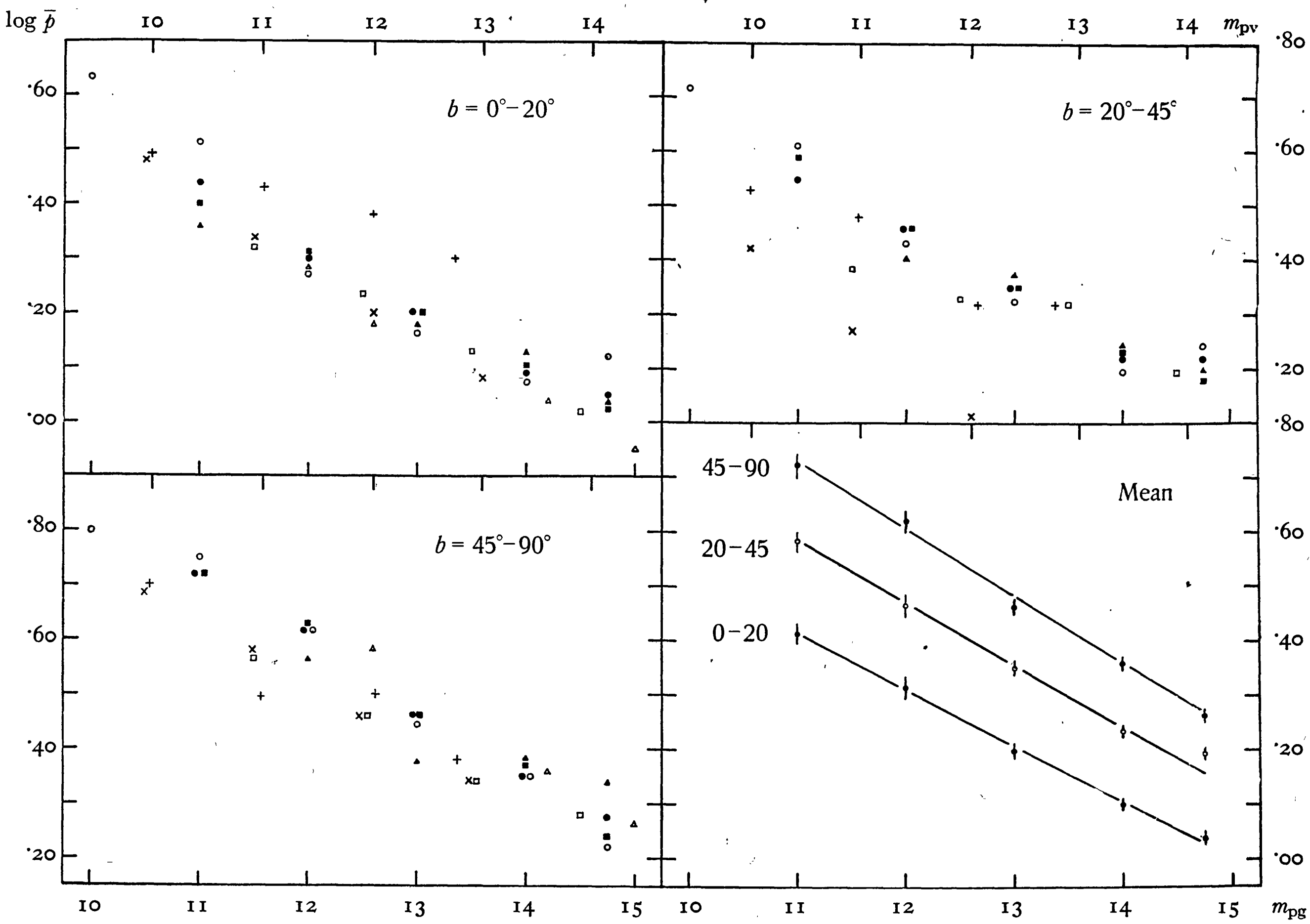
The effect of the velocity of the sun has been taken into account in the same way as was done by Oort:

$$\overline{|v_i'|}^2 = \overline{|v_i|}^2 + (2\cdot 32\, c_A)^2,$$

where $\bar{v}_i$ is the average peculiar velocity in the direction of the component used and c_A the cosine of the angle between this component and the direction of the velocity of the sun. The solar apex was assumed at $\alpha = 18^h\, 0^m$, $\delta = +30°$.

1) *B.A.N.* No. 269, 271, 1935.
2) *H. C.* No. 400, 1935.
3) *Gron. Publ.* No. 49, 1939.
4) Unpublished.

FIGURES 1—4.



Figures 1–3 give the relations between the logarithms of mean parallaxes (unit ″·001) and apparent magnitude for three galactic latitudes. The points by the author were derived from l-residuals (dots), b-residuals (squares) and v-components (triangles). The figures show also the results found by OORT (open circles), VAN HOOF (open squares), VAN MAANEN (open triangles), VAN RHIJN and BOK (crosses), VAN DE KAMP and VYSSOTSKY (plus signs). In the zone 20°—45° the cross for m_{pg} = 13·6 is omitted. Figure 4 gives for each galactic zone the mean of the values found by the author; the lines indicate probable errors.

The mean parallaxes (all spectral classes combined).

The mean parallaxes $\bar{p}$ have been obtained from:

$$\overline{|\mu_i|} = \frac{1}{f} \bar{p} \, \overline{|v_i'|},$$

where the factor f was taken from *B.A.N.* No. 290, p. 86. The resulting mean parallaxes are given in Tables 6*a* and 6*b* (unit ″·0001) both for the l- and b-components; n indicates the number of stars in the group. Columns 3 in these Tables show the limiting magnitude m_l down to which the proper motions are considered by the Radcliffe authors as being of full weight. Asterisks in the faintest magnitude groups denote that the group is incomplete and has not been used in forming the mean. For these faintest magnitudes a considerable number of Radcliffe stars have not been used, because they do not occur in the Pulkovo catalogue. The agreement between the parallaxes from the two components in each area is satisfactory.

The mean results for each of the two components are given in Table 7, columns 2, 3 and Figures 1—3, where also a comparison is made with previous results. Weights have been given equal to the number of stars in the group (n), but for the areas contained only in the Radcliffe Catalogue the weight was taken equal to $\frac{1}{2}n$ (see Table 6*b*). The probable errors of the mean parallaxes have been found from the differences between the different areas. From the spectra of SCHWASSMANN (*Bergedorfer Spektral Durchmusterung*) [1]) a mean colourindex of + ^{m}·6 has been found for these faint stars, in accordance with SEARES [2]). In each area the averages of the parallaxes found from l- and b-residuals have also been formed.

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[1]) Band 2, Table 3, 1938.
[2]) *Ap. J.* 61, 114, 1924.

TABLE 7.

Average parallaxes and their probable errors (unit ″·00001, last three columns ″·0001).

m_{pg}	m_{pv}	BINNENDIJK *l*-res. (2)	BINNENDIJK *b*-res. (3)	OORT *ψ*-res. (4)	v. HOOF *τ*-res. (5)	BINNENDIJK *υ*-comp. (6)	v. MAANEN *υ*-comp. (7)	v. RHIJN BOK *υ*-comp. (8)	v. D. KAMP VYSSOTSKY *υ*-, *τ*-comp. (9)
(1)									
b = 0°–20°									
10·0	9·5			425 ± 53					
10·5	10·0							30	31
11·0	10·4	279 ± 13	252 ± 12	330 ± 25		227 ± 15			
11·5	10·9				210 ± 26			22	27
12·0	11·4	200 ± 6	205 ± 8	195 ± 10		194 ± 13			
12·5	11·9				175 ± 13		18	16	24
13·0	12·4	159 ± 4	158 ± 5	143 ± 7		151 ± 5			22
13·5	12·9				136 ± 9			12	
14·0	13·4	123 ± 3	125 ± 4	119 ± 4		135 ± 4	11		
14·5	13·9				104 ± 14				
14·7	14·1	112 ± 4	108 ± 5	132 —		111 ± 4			
15·0	14·4						09		
b = 20°–45°									
10·0	9·5			520 ± 80					
10·5	10·0							26	34
11·0	10·4	356 ± 14	391 ± 12	410 ± 40					
11·5	10·9				240 ± 17			19	30
12·0	11·4	291 ± 9	287 ± 9	267 ± 11		251 ± 11			
12·5	11·9				215 ± 13		—	13	21
13·0	12·4	224 ± 5	222 ± 6	208 ± 7		233 ± 7			21
13·5	12·9				208 ± 12			9	
14·0	13·4	167 ± 3	172 ± 5	154 ± 7		175 ± 6	—		
14·5	13·9				159 ± 11				
14·7	14·1	165 ± 5	151 ± 6	174 —		163 ± 10			
15·0	14·4						—		
b = 45°–90°									
10·0	9·5			630 ± 60					
10·5	10·0							49	50
11·0	10·4	523 ± 25	526 ± 21	560 ± 40					
11·5	10·9				370 ± 32			38	31
12·0	11·4	416 ± 13	429 ± 12	425 ± 19		359 ± 16			
12·5	11·9				286 ± 15		38	29	32
13·0	12·4	287 ± 6	287 ± 8	278 ± 8		232 ± 10			26
13·5	12·9				218 ± 7			22	
14·0	13·4	225 ± 4	233 ± 5	222 ± 6		235 ± 8	23		
14·5	13·9				189 ± 7				
14·7	14·1	188 ± 7	174 ± 8	167 ± 9		220 ± 18			
15·0	14·4						18		

TABLE 8.

Average parallaxes from combined *l*- and *b*-results (unit ″·00001).

m_{pg}	*b* = 0°–10°	10°–20°	20°–30°	30°–45°	45°–60°	60°–90°
11·0	268 ± 14	275 ± 19	374 ± 12	379 ± 20	501 ± 27	566 ± 26
12·0	187 ± 7	226 ± 10	282 ± 15	293 ± 8	404 ± 14	444 ± 15
13·0	150 ± 6	166 ± 4	213 ± 7	237 ± 6	280 ± 7	295 ± 9
14·0	120 ± 6	129 ± 4	153 ± 5	187 ± 4	214 ± 4	246 ± 6
14·7	106 ± 5	111 ± 5	154 ± 6	170 ± 8	173 ± 6	192 ± 12
11·0	265 ± 11		377 ± 11		527 ± 18	
12·0	203 ± 6		287 ± 9		421 ± 11	
13·0	158 ± 4		222 ± 5		287 ± 6	
14·0	124 ± 3		169 ± 3		227 ± 4	
14·7	108 ± 3		161 ± 5		180 ± 6	

TABLE 9.

Ratios of observed and theoretical probable errors of the mean parallaxes.

b	$11^m\cdot0$ l	$11^m\cdot0$ b	$11^m\cdot0$ l, b	$12^m\cdot0$ l	$12^m\cdot0$ b	$12^m\cdot0$ l, b	$13^m\cdot0$ l	$13^m\cdot0$ b	$13^m\cdot0$ l, b	$14^m\cdot0$ l	$14^m\cdot0$ b	$14^m\cdot0$ l, b	$14^m\cdot7$ l	$14^m\cdot7$ b	$14^m\cdot7$ l, b
0°—20°	1·4	1·4	1·9	1·5	2·0	2·1	1·9	2·4	2·6	3·1	3·9	4·5	2·5	3·3	3·0
20 —45	1·2	1·0	1·3	1·5	1·5	2·0	1·4	1·9	2·1	2·0	2·7	2·8	1·8	2·2	2·4
45 —90	1·1	1·0	1·2	1·2	1·1	1·4	1·1	1·4	1·6	1·3	1·6	1·8	1·5	1·7	2·0

In Table 8 the mean results of these combined data are given for six and three galactic zones respectively.

If there had been no systematic differences between the areas the probable error of an average parallax $\bar{p}$ for an area or group of areas containing a total of n stars would have been equal to $\cdot50\ \bar{p}/\sqrt{n}$ for the l- and b-results respectively and equal to $\cdot50\ \bar{p}/\sqrt{2n}$ for the combined data (l, b-results). In reality it comes out larger by a factor which is tabulated in Table 9. A division in six zones gave similar results. In general the factors for the mean l, b-results are greater than those for the l- and b-data separately. It is not quite clear how this discrepancy and the considerable size of the factors in Table 9 can be explained. A correction for the difference in star density (see below) gives no better agreement.

The probable errors of the mean parallaxes are now however about 3% and therefore small enough to conclude that the relation between m and $\log \bar{p}$ is linear up to the faintest stars and small galactic latitude (Figure 4 and Table 8, lower half). The earlier results for the faintest magnitudes by OORT [1]) and VAN HOOF [2]) were considered as doubtful because it was uncertain whether the adopted probable errors held also for the faintest stars observed. In the present work this uncertainty has been removed completely because the accidental errors for all magnitudes have now been determined from the intercomparison of the two catalogues. For all practical purposes the mean parallaxes may be represented by the formula:

$$\log \bar{p}_m = \log \bar{p}_{12} - \cdot115\,(m_{pg} - 12)\,.$$

The values of $\log \bar{p}$ for various latitudes are given in Table 10.

TABLE 10.

Values of $\log \bar{p}_{12}$ (unit of $\bar{p}$ ″·001).

b	$\log \bar{p}_{12}$	b	$\log \bar{p}_{12}$
0°	·262	50°	·563
10	·311	60	·596
20	·382	70	·618
30	·452	80	·629
40	·516	90	·630

[1]) *B.A.N.* No. 290, 75, 1936, Table 10.
[2]) *B.A.N.* No. 289, 67, 1936, Table 5.

The parallaxes derived by OORT were found from the residuals in the so-called ψ-components, chosen in such a way that for each area it coincides as closely as possible with the direction of the principal axis of the velocity ellipsoid and contains as little as possible of the solar motion. VAN HOOF gives parallaxes derived from residuals in the τ-component. To reduce his results to the factor 1·08 explained on page 11 all his data have been multiplied by the factors given in Table 6 of his publication. The probable errors in this case have been calculated by the present author from these data, giving weights proportional to the numbers of stars used. The results of VAN MAANEN from υ-components have been taken from a private communication to Professor OORT. The values given by VAN RHIJN and BOK [1]) from υ-components have been deduced from all data up to the year 1931. The results are discordant for the zone of 20°—45° galactic latitude. The results of VAN DE KAMP and VYSSOTSKY [2]) were derived from υ- and τ-components of faint stars surrounding P.G.C. stars observed in the parallax programme. In the galactic zone these values deviate considerably from those given by other authors. Also the McCormick observers found that the mean parallax in the zone from 0° to 10° latitude was larger than that in the zone 10° to 20°; this increase is contradicted by the present results. The accuracy of the Groningen and McCormick mean parallaxes is less than of those derived from the Radcliffe and Pulkovo material.

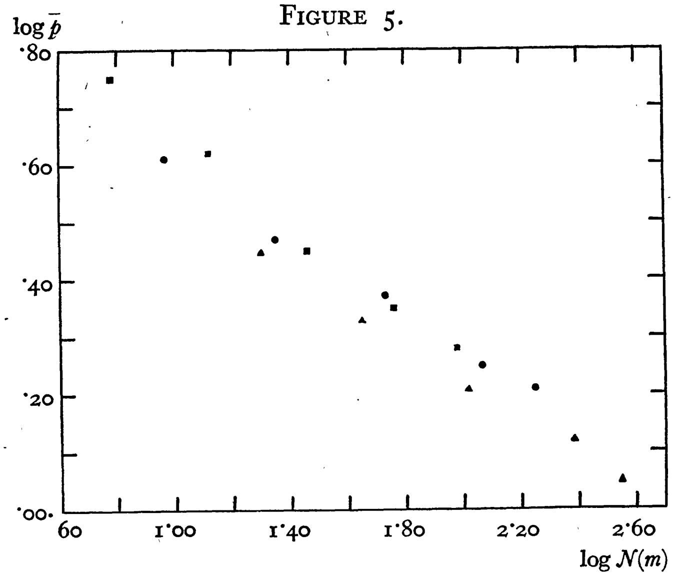
In order to investigate the relation between mean parallax and star density the results obtained in each area from the l-components, the b-components and those found by OORT and VAN HOOF were averaged. In Figure 5 we have plotted the logarithms of the total average parallaxes against the logarithms of the number of stars $N(m)$ per square degree down to the magnitude considered. The relation is practically linear, as shown in Figure 5; the galactic zone deviates slightly from the other zones. The relation can be represented by:

$$\log \bar{p} = \cdot98 - \cdot36 \log N(m),$$

where the unit of $\bar{p}$ is ″·001.

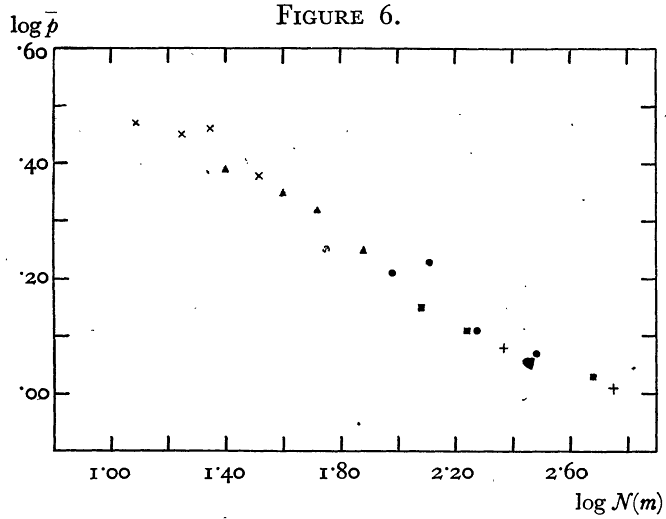
[1]) *Gron. Publ.* No. 45, 1931, Table 7.
[2]) *Publ. McCormick Obs.* **7**, 26–37, 1937; *A.J.* **46**, 9, 1937, Table 13.

FIGURE 5.



Relation between the logarithms of the mean parallax (unit ″·001) and the logarithms of the number of stars $N(m)$ per square degree down to the magnitude considered. Triangles refer to the zone of galactic latitude 0°—20°, dots to the zone 20°—45°, squares to the zone 45°—90°. From left to right the points in one galactic zone correspond with the photographic magnitudes 11·0, 12·0, 13·0, 14·0 and 14·7 respectively.

FIGURE 6.



The same relation as given in Figure 5, for the galactic zone 0°—20°, but now in each magnitude group (11^m crosses, 12^m triangles, 13^m dots, 14^m squares, $14^m·7$ plus signs) divided into four groups, according to log $N(m)$.

It is also of interest to investigate whether a similar relation holds if the mean parallaxes in one and the same group of latitude and magnitude are plotted against log $N(m)$. In each magnitude interval the areas were therefore divided into four groups according to the values of log $N(m)$. For the galactic zone the result of this division is shown in Figure 6. It will be seen that the relation between log $\bar{p}$ and log $N(m)$ within such a magnitude group is sensibly the same as the more general relation between log $\bar{p}$ and log $N(m)$ just considered.

In the areas Nos. 47, 72, 110 the parallaxes are much greater than the average parallax at this galactic latitude. A comparison with HUBBLE's [1]) counts of extra-galactic nebulae shows that these areas are within the zone of avoidance. It is probable that the deviation in these zones is caused by dark material. The first two areas are near the Pleiades, the last near the central line of the Milky Way.

For finding the average distances in parsecs ($\bar{\rho}$) we may use the formula: $\bar{\rho}.\bar{p} = 1·6$ (See *B.A.N.* No. 290, p. 90 and p. 102).

The υ-components.

Ordinary secular parallaxes were computed by means of the algebraic averages of the υ-components. The reduction to absolute values given in the Radcliffe Catalogue being too uncertain to give significant results for these faint stars, we have to content ourselves with the derivation of the differences between the mean parallaxes of the various magnitude groups.

All total proper motions in excess of ″/a·050 have primarily been excluded; their effect was again smoothed out over all areas. This has been done for three galactic zones and the five magnitude groups separately. The algebraic value of the υ-components of these stars was considered. The effect on the residuals in the five magnitude groups may be found from $k.\Delta n/n$, where n is the mean number of large proper motions in one area, read from a graph through the numbers of large proper motions plotted against the magnitude, k is the mean algebraic value of these υ-components in the group considered and n is the number of stars. The factor $k.\Delta n$ is given in Table 5 (unit ″·001). Division by n gives the influence upon the mean residual in one area. The relative corrections for the groups of relative υ-components can now be added.

For each area the relative secular parallax $\upsilon/\sin\lambda$ was calculated from the averages of the proper motions in α and δ (λ is the distance of the area to the apex of the solar motion).

The following procedure was used to determine the weights. The weight of the difference between the average α-components of two consecutive magnitude groups was taken inversely proportional to:

$$\frac{r_m^2}{n_m} + \frac{r_{m+1}^2}{n_{m+1}} + S_\alpha^2$$

where r_m is the probable error of the α-components for the magnitude m calculated from the residuals in one area and n_m is the number of stars in the group. S_α is the probable error caused by the unknown magnitude

[1]) *Mt Wilson Contr.* No. 485, 139, 1933, Figure 3; *Ap. J.* **79**, 8, 1934, Figure 3.