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The Formation of Galaxies and the Origin of the High-Velocity Hydrogen

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The observations of the high-velocity hydrogen at intermediate and high galactic latitudes are reviewed. The phenomena indicate that intergalactic gas is streaming into the Galaxy. No more than a rough estimate can be made of the total flow. The inflow provisionally adopted in the article is 2×10^{17} H atoms per cm² of the galactic plane per million years, corresponding with an increase of the Galaxy's mass of 0.9% per 10^{9} years. The cloudy structure of the high-velocity gas is probably caused by the structure of the halo.

An inflow of this order is plausible if one considers the probable origin of spiral galaxies. It is shown that these cannot generally have obtained their angular momentum by the action of external gravitational fields or by collisions in the course of their evolution. Therefore they must have been endowed with their angular momentum from the beginning, and cannot have formed by gravitational instability from large-scale fluctuations directly after the fireball stage of the universe, as commonly supposed. The universe must thus have had a high degree of turbulence on a galactic scale, as was first proposed by von Weizsäcker. In order that enough momentum can be contained in a "proto-spiral" the scale of the universe at the time of its inception should have been about 1/30 of the present scale. Initially the protogalaxies presumably expanded with the universe, though at a much reduced rate. Their collapse into actual galaxies should have come much later (at $t \geq 1 \times 10^9$ years). The age of most frequent galaxy collapses may well correspond with that of the birth of the majority of powerful radio sources, at $z \geq 1$ (Section 7.3).

Transition regions at the outer boundaries of the cells from which protogalaxies are formed, as well as natural irregularities in the rotation of these cells will make the infall of gas into the galaxies continue long after the principal collapse. The present-day inflow of intergalactic gas expected from this process of galaxy formation is of the order of that inferred from the observations.

Compelling independent evidence for the existence of a great density of intergalactic matter is given by the relative motion of the Andromeda nebula and the Galaxy, as has previously been emphasized by Kahn and Woltjer. The intergalactic gas must have a high temperature.

 $\textit{Key words:}\ \text{cosmology}\ -\ \text{galaxies}\ (\text{origin})\ -\ \text{high-velocity gas}\ -\ \text{local group}\ -\ \text{universe}\ -\ \text{galactic halo}\ -\ \text{radio sources}$

1. Introduction

Systematic observations of clouds of high velocity at high galactic latitudes made mostly with the radio telescope at Dwingeloo, and discussed in Groningen and Leiden (Blaauw and Tolbert, 1966; Hulsbosch and Raimond, 1966; Blaauw et al., 1967; Hulsbosch, 1968) haveled to the idea that these clouds have been set into motion by intergalactic gas falling into the galactic halo (Oort, 1966, 1967, 1968 and 1969). This would imply that the galaxies are still growing, and also that there exists a considerable quantity of intergalactic gas.

In the following article an attempt is made to estimate what amount of infall should be expected for galaxies formed in an expanding universe. It appears that plausible models of galaxy formation yield influxes which correspond, in order of magnitude, with that deduced from our interpretation of the high-velocity features.

Evidently this problem cannot be properly discussed without considering the process by which the galaxies have initially been formed. In the present article I use as a working hypothesis that spiral galaxies have originated from large-scale streamings in the universe at an epoch when the scale of the universe was about 1/30 of the present scale. The reasons for this supposition are explained in Sections 3 and 5.

2. Distances of High-velocity Features and Estimated Inflow into the Galactic System. Structure of Halo

Estimates of the flow of gas into the Galaxy have been based on the assumption that the clouds of "high" and "intermediate") velocity observed in

¹⁾ For the definition of these terms cf. Blaauw et al. (1967). For simplicity we shall in the following frequently use the term high-velocity gas for all the gas having velocities well outside the range of normal galactic clouds.

directions well away from the galactic plane are situated within the Galactic System. Before considering the problems mentioned it is well to review the evidence for and against this hypothesis. Some of this has been discussed in an earlier article (Oort, 1966, cf. in particular its Section 2.4), but some new data have recently become available, which will be reviewed below.

At present the only observations which can provide direct information on the distances of the highvelocity features are the optical high-dispersion spectra of stars in high galactic latitudes, most of which were obtained and discussed by Münch and Zirin (1961). They found a number of components with velocities greatly in excess of the random motions usually encountered in the galactic layer. The gas causing these absorption lines is evidently within the Galactic System, and, as the authors point out, probably at rather large distances from the galactic plane. In order to investigate in how far there is a connection between this gas and the "high"or "intermediate-velocity" clouds found in the 21-cm programmes Habing (1969) has determined accurate 21-cm profiles in the directions of all stars above 20° latitude for which measures of highdispersion spectrograms have been published. He finally restricted the discussion to stars of latitude 30° and higher, because at the lower latitudes there was still the possibility that the velocities would be due to differential rotation. In this material he found intermediate-velocity gas (with velocities larger than 30 km/s relative to the l.s.r.) in the directions of five stars, this gas being always distinctly separated from the low-velocity gas. For a sixth star, HD 119608, he found hydrogen at +20 km/s, with a high surface density, and again well separated from the lowvelocity gas. Although the following discussion will in general be limited to features with velocities higher than 30 km/s the data for HD 119608 have been added in Table 1 because of the high surface density at this velocity. Table 1 lists the relevant data for the six stars. The galactic co-ordinates are on the new system, the spectral types and estimated distances from the galactic plane are from Münch and Zirin, except for the last star. The column H I emission shows the range of velocities (relative to the l.s.r.) in which hydrogen of appreciable surface density was found, the average brightness temperature being added in parentheses. Only velocities higher than 20 km/s are given. In all cases there is also low-velocity gas.

The last column indicates the velocities relative to the l.s.r. of the Ca II components found by Münch and Zirin. Again the components with velocities below 20 km/s are omitted because they are irrelevant for our present purpose. The numbers in parentheses indicate relative intensities.

We note in the first place that in three stars Ca Π absorption lines are found in the same velocity range where HI emission is observed. In three cases the Ca II components correspond even to a maximum in the 21-cm profiles. Habing has shown that in each of these cases the hydrogen found in the direction of the star is part of a large cloud of longish shape, with separate condensations, similar to what is found in the high-velocity clouds investigated by Hulsbosch (1968). In the other three stars around which intermediate-velocity hydrogen was found no corressponding $Ca \pi$ absorption has been observed. These stars are all less than 700 pc from the galactic plane, while the former three were more than 1000 pc from this plane. It is therefore tempting to think that the absence of Ca Π absorption in these clouds is due to their lying beyond the stars. This suggests that the intermediate-velocity clouds investigated have zvalues between 600 and 1000 pc. The distance estimate is evidently very uncertain. It can be stated, however, that in all cases where high-dis-

Table 1

HD	l	ь	Sp.	z	H I emission	Ca II absorption
93 521	183°	+62°	09 Vp	1800:	-56 (1:0)	-34(1), -55(3)
97 991	262	+52	B 2	600	$-20 \text{ to } -35 \ (>3.0)$	_
100 600	239	+69	B 3 V	300	-20 to -47 (2.0)	
119 608	320	+43	B 1 Ib	2700:	+20 (5.0)	+22 (3)
215 733	85	-36	B1II	1300	$-30 \text{ to } -55^{\circ} (>2.5)$	-36(3), -50(2)
217 891	79	-50	В 5 ре	90	-20 to -50 (1.3)	

^a Maximum at -50.

persion spectra of distant stars situated in the direction of hydrogen clouds with velocities in excess of $\pm 20 \text{ km/s}$ are available, Ca II components at roughly corresponding velocities have been found. Among the stars above 30° latitude in Münch and Zirin's list there are two more with |z| > 1000 pc, and two with |z| between 800 and 1000, in which no H I with intermediate or high velocity was found. These stars show no higher-velocity Ca II components either. For the distant stars in Münch and Zirin's material there thus appears to be a close correspondence between the presence of H I and Ca II of intermediate velocity, and, if present, between the velocities at which H I and Ca II is observed. Although the material is small it seems improbable that this correspondence is accidental. There is thus a strong indication that at least the intermediate-velocity clouds are situated within the Galactic System.

There are no data as yet on clouds with velocities in excess of 60 km/s. But the properties of the intermediate-velocity features and the high-velocity ones are so similar that it appears reasonable to assume, as a provisional working hypothesis, that the latter are also within the Galaxy, and at relatively large distances from the galactic plane.

For estimating the accretion of gas by the Galaxy there is, however, no need to insist on this similarity. The intermediate-velocity hydrogen itself shows a very outspoken systematic flow towards the galactic plane. The strength of this flow can be estimated from the rough knowledge of the distances discussed above. This is all we need for our present purpose.

We have attempted to estimate the amount of intergalactic gas required to explain the motions of the high- and intermediate-velocity clouds in two ways. In the first attempt the flow was computed from the known clouds with radial velocities in excess of 100 km/s. It was assumed, somewhat arbitrarily, that the layer in which these are situated has a thickness of 1 kpc in the z-direction, that within this layer in the direction of a given cloud the highvelocity object shows all the gas that has recently been accelerated by the intergalactic wind, and that all of its velocity is due to this acceleration. We assume, further, that the average surface density of a high-velocity cloud is 3×10^{19} cm⁻² (Hulsbosch, 1968), that the average space velocity is 150 km/s, and makes an angle of 40° with the galactic plane (Oort, 1966, p. 430). The resulting flow is then 3×10^{18} cm⁻² (10⁶ y)⁻¹. As the original velocity, before deceleration, must have been about 500 km/s, the original flow into the Galactic System should have been roughly 30% of that observed. This is relative to the rotating local standard of rest. Relative to the galactic standard of rest the flow would be $\frac{380}{500}$ times as large, i.e., 0.7×10^{18} cm⁻² (10^6 y)⁻¹. As the direction of infall makes an angle of about 40° with the galactic plane the flow per cm² of the galactic plane becomes 0.45×10^{18} . It should be pointed out that this estimate rests on the assumption that the extragalactic wind is homogeneous, and falls in also in the spaces between the clouds.

I have tried to make another estimate from the survey at intermediate velocities which was made in Groningen. The data were taken from a communication by Blaauw, Fejes, Tolbert, Hulsbosch and Raimond (1967). From Fig. 3 of that paper I estimate that the average surface density of hydrogen with velocities between -30 and -80 km/s in the region $l~80^{\circ}$ to 180° , $b>+20^{\circ}$ is 7×10^{19} cm⁻². Judging from the density of positive velocities in this range the probable contribution from randomly moving hydrogen in the galactic layer will be about 2×10^{19} cm⁻². Subtracting this, and assuming an average space velocity of 60 km/s and a column length of half that assumed for the higher-velocity features, I arrive at an original flow, outside the halo, of 0.23×10^{18} per 106 years per cm² of the galactic plane. This is a minimum estimate, because also this intermediatevelocity gas is concentrated in more or less discrete features in which the surface density is higher than the smeared out density quoted above, requiring therefore more infalling gas to accelerate them.

The factor of 2 difference between this estimate and the one from the higher-velocity objects gives an indication of the uncertainty.

The above calculations are unsatisfactory because neither the way in which the infalling gas interacts with the halo nor the formation mechanism of the discrete clouds has been considered. For a proper understanding of the phenomena it is essential to have some knowledge of the manner in which the gaseous halo of the Galaxy is maintained. Interesting information on the structure of the halo has been obtained from a recent investigation by Miss Kepner (1970; cf. also Oort, 1969 and 1970). She found that the velocities of distant halo clouds cluster around discrete values which correspond with the velocities of the spiral arms in the plane. Thus, even at elevations between 1 and 2 kpc above the galactic layer, the neutral halo gas appears still to be concentrated in the regions above and below the spiral arms. Like the arms themselves it has a clumpy structure. The largest "clumps" seem to have masses of several times $10^5\,M_\odot$; there are probably numerous smaller halo clouds. In fact, it seems likely that the entire neutral halo consists of discrete clouds.

These clouds must have been expelled from the spiral arms with which they are associated. We do not know the mechanism by which this has been done. If it is by an explosion, the energy involved must evidently have been extremely large. An important independent indication that superexplosions of the required magnitude actually occur is furnished by the three remarkable formations discovered by Hindman (1967) in the Small Magellanic Cloud. The most direct interpretation of the phenomena observed in these formations is that they are expanding shells, with HI masses of the order of $10^7 M_{\odot}$, radii between 0.5 and 0.9 kpc, and expansion velocities of about 20 km/s. Mass and momentum are comparable with what would be needed to produce the largest halo complexes observed by Miss Kepner. If the probability of their occurrence per unit mass of gas would be the same in the Galaxy as in the Small Cloud the frequency of these giant explosions would be approximately of the right order to maintain the galactic halo. With an estimated mean observable life of 15 million years for a Hindman shell there would then be one superexplosion per kpc of a spiral arm in 100 million years, while about one per 50 million years would be needed to maintain a complete halo.

The hypothesis that the halo has been produced by superexplosions is not in contradiction with the observation that the halo clouds are concentrated above and below the arms, and do not occur in between. That part of the disk gas which is thrown out nearly perpendicularly to the galactic plane will in general get the highest velocity, and reach the largest distance from the plane. A rough calculation shows that at the highest elevations the dispersion in the radial components due to the spread in the angles under which the gas is expelled will be about 15 km/s. This is of the same order as the dispersions observed by Miss Kepner.

Considering the surface densities involved and the probable strength of the intergalactic pressure the pushing up of the largest clouds would be only slightly hindered by the pressure of the infalling gas. But if our hypothesis for the origin of the peculiar motions of the high-velocity objects is correct, conditions would be different for structures with surface densities of the order observed in the high-velocity clouds. After the initial expulsion their motions would have been governed only partly by gravitation and

for an important part by the pressure of the intergalactic "wind". More or less direct evidence that this is so is furnished by the fact that for several clouds the negative radial velocities are considerably higher than what could have been obtained by a free fall, even from heights as large as 3 or 4 kpc. If, like the halo clouds observed by Miss Kepner, these highvelocity clouds have come from the local arm a large part of the velocities must therefore have been produced by a pressure from outside. A possible alternative, which has been discussed in a previous article (Oort, 1966), is that they have come from an other arm, so that they could have large velocity components parallel to the galactic plane. Such an origin meets, however, with such serious quantitative difficulties that it should be considered as quite improbable.

We may imagine the following primitive working model: by superexplosions within the galactic layer part of the gas in this layer is at times locally thrown up to heights of the order of 1 kpc, and forms such structures as studied by Miss Kepner. A few thin clouds reach greater heights, up to perhaps 3 kpc. These are subsequently accelerated down, as well as sideways, by the intergalactic wind pressure in addition to the gravitational force. If they have surface densities of about 3×10^{19} cm⁻², such as we observe in the high-velocity complexes, they can reach velocities of about 200 km/s. The exact value of the wind pressure needed depends on several uncertain factors. B. van Leer has estimated that the available data can be fitted with an intergalactic density of between 2 and $3 \times 10^{-4} \,\mathrm{H}$ at cm^{-3} just outside the Galaxy, and with a flux of extragalactic hydrogen into the Galaxy between 3 and $4 \times 10^{17} \, \mathrm{H}$ atoms per cm² per 10⁶ years.

Evidently, the available data can yield no more than the order of magnitude of the inflow of intergalactic gas. But even this order of magnitude is interesting. If we reduce to the galactic standard of rest, and assume that the "wind" makes an angle of 40° with the galactic plane, the amount of intergalactic hydrogen falling into 1 cm2 of this plane becomes about 2×10^{17} atoms per 10^6 years. That this result is a factor 2 less than the flow estimated on p 383 in an apparently rather different way is probably due to the fact that in the first estimate the acceleration of the neutral clouds was ascribed entirely to the action of the infalling gas, while in van Leer's computation about half of it is due to the gravitational attraction of the Galactic System. I provisionally adopt the latter value.

As the average number of H atoms in a column of 1 cm² of the galactic layer is 7×10^{20} , the amount of gas in the disk would increase by 0.03% per 10^6 years, or by 6% per galactic revolution of 245 million years. If we assume a helium content of 30% (by weight) for the intergalactic gas the mass of the Galaxy would increase by 0.22% per revolution.

It should be noted that the high-velocity features at lower latitudes, such as Hulsbosch' anticentre complex and the gas of high negative velocity around the anticentre at latitudes between $+6^{\circ}$ and $+10^{\circ}$ discussed by Miss Kepner, may well be connected with the Perseus or the outer arm rather than with the local arm.

I am indebted to B. van Leer for discussions on the interaction of an intergalactic wind with halo filaments. The estimates given in the preceding paragraphs have partly resulted from these discussions.

3. Formation of Galaxies and Acquisition of Angular Momentum

The problem of the origin of the galaxies is closely tied up with that of the acquisition of angular momentum. If, as recently assumed by various investigators (cf. Peebles, 1967 and Field, 1970; references to earlier investigations are given by the latter), the formation of galaxies was due to gravitational instability in the period shortly after the decoupling of matter and radiation, when the scale of the universe was about 1/1000th of its present value, their present angular momentum cannot have been contained in them from the beginning, because they were too small. In a universe with critical density the radius of a sphere containing a mass equal to that of our Galaxy was then about 0.6 kpc. Such a sphere could at most contain 15% of the present angular momentum of the Galactic System. And to possess this much angular momentum it would have to rotate with circular velocity, which on the equator would be 1070 km/s, or $1/\sqrt{2}$ times the average expansion velocity of the universe at the distance of 0.6 kpc. Such velocities are incompatible with the hypothesis of gravitational instability, where the initial deviations from homogeneity are supposed to be of the order of only 1%. On this hypothesis the initial angular momentum should therefore have been entirely negligible.

It follows that if the spiral galaxies had been created around this epoch the angular momentum

must have been imparted to them in a much later period, presumably near the time when the protogalaxies reached their maximum radii.

Angular momentum can be imparted to a protogalaxy in two ways, viz., by couples exerted by neighbouring systems in a period when the protogalaxy deviated strongly from a symmetrical shape, or by a collision with another protogalaxy. The asymmetry might be due to anisotropy in the density distribution and the velocity field at the time of detachment from the surroundings, or it might have been caused by tidal forces of the perturbing system. The latter are generally too small, and the process, if it works at all, must depend on initial asymmetries.

So far as I know, the suggestion that the angular momentum may have been put in by extraneous forces after the birth of the protogalaxy was first discussed by Hoyle (1951). He estimated the perturbations exerted by a neighbouring cluster of galaxies on a non-expanding protogalaxy. On the basis of the old Hubble constant, which was 7 times larger than the value adopted at present, he found that a sufficient transfer of angular momentum would be possible. However, with a Hubble constant of 75 km/s per kpc the perturbing forces are 73 times smaller, the time in which they work is 7 times longer, so that the effect would become 50 times smaller. The perturbations considered would then no longer be sufficient.

The subject has recently been revived by Peebles (1969). His calculations give an expected transfer of angular momentum of about 1/6th of that contained in our Galaxy. In his opinion this factor can be ascribed to the uncertainties in the data and the computation, and he concludes tentatively that the observed angular momentum has originated in this way.

In view of the crucial importance of this problem for understanding the process of galaxy formation, as well as for investigating the properties of the universe, I have made an independent attempt (see Section 8) to estimate the effects of interaction, starting from what we know about our immediate surroundings. While at the epoch where gravitational instability is supposed to have started conditions in the universe can only be surmised on the basis of very uncertain general theories, the circumstances at the very much later epoch where transfer of momentum may have taken place must have been much more similar to present circumstances. We may hope therefore that a rough estimate of the conditions under which the principal transfer of angular momentum would

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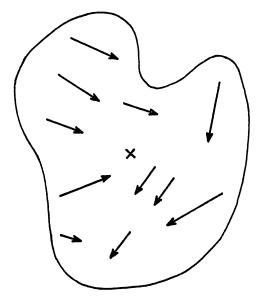


Fig. 1. Streaming which might lead to the formation of a galaxy

have taken place can be obtained from the universe as it is today.

The new computations indicate that the amount of angular momentum that could have been imparted to the Galaxy is two orders of magnitude too low. These results seem to prove that proto-spirals have not developed from gravitational instabilities in the very early stages of the universe, but that they date from a much later period, when their formation was caused by large-scale currents, which at the same time provided their angular momentum.

The considerations given in the present article are based on this latter concept as a provisional working hypothesis. This notion that large-scale "turbulence" has played an essential role in the formation of galaxies has previously been introduced by von Weizsäcker (1951) and by Gamov (1952). Von Weizsäcker estimates that the galaxies were formed at the time when the density in the universe was of the same order as the mean density in the galaxies before their collapse into a disk. He supposed that in this period the galaxies were directly adjacent to each other. In the present article it is assumed that the proto-spirals were born at the epoch when the density of the universe was such that they could just contain their present angular momentum. The protogalaxies were far from adjacent at that time. Von Weizsäcker considered the fact that random motions of galaxies are of the same order as the universal expansion over distances corresponding to those between neighbouring galaxies as an indication that turbulent velocities were sufficient to counterbalance the expansion at the time of birth of the protogalaxies. However, it now appears more probable that the galaxies acquired their random motions at a much later epoch by their collapse into groups and clusters, and that at the birth time of the protogalaxies these motions were negligible compared to the expansion (cf. Sections 8 and 9).

We must therefore accept that the theory of galaxy formation from rotating turbulent elements rests on an ad hoc hypothesis, whose only, but important, justification lies in the observed rotations. A discussion of the possible origin of the large-scale currents is beyond the scope of this article. Presumably they derived from still higher-velocity "turbulence" at earlier times. The velocities in these prior stages may have been sufficiently high to prevent the formation of protogalaxies that could stay together, and may at the same time have kept the medium at a high temperature. It is still unknown how this turbulence has developed. But Ozernoy and Chernin (1968, 1969) have made the very interesting suggestion that primordial "photon eddies" of galactic and larger mass can have survived through the fireball stage of the universe, and after the decoupling of matter and radiation have created matter turbulence of high velocity (cf. also Rees, 1970, for a discussion of these problems).

4. Schematic Models

If, in the cell which is to form a protogalaxy, the angular momentum of a galaxy is already present in the form of large-scale currents it is to be expected that these currents must in general have also radial components, and that, measured relative to a frame of reference expanding with the average velocity of expansion of the universe, these were of the same order as the transverse ones.

Relative to this frame we might have motions like those sketched in Fig. 1. There will be regions of the universe where the radial components are preponderantly outward, and others where they are mainly in an inward direction. In the former case they will locally enhance the expansion of the universe, in the latter case they will diminish it. If the velocities are high enough the latter regions will, after an initial expansion, collapse, and may then lead to the formation of a rotating galaxy.

We shall try to approach the problem of the origin of spiral galaxies in a semi-empirical manner, by starting from the observed facts that galaxies have been formed, that we know their masses, and that they are endowed with known amounts of angular momentum. We try to follow their evolution in the expanding universe back to the time when they first distinguish themselves as separate units.

We shall see that, for any given average present density in the universe, the data mentioned permit one to conclude with fair accuracy at what epoch the galaxies considered must have first detached themselves, and after what time they collapsed into a proper galaxy.

The first question to be answered is: are the expected radial motions sufficiently large to lead to formation of galaxies in a reasonable time? It turns out that with initial radial motions relative to the expanding reference frame of the same order as the transverse motions required to explain the angular momentum, the time of collapse for galaxies like our own can be of the order of 1×10^9 years, and is therefore sufficiently short to fit the observed ages of the oldest stars. The collapse time depends only to a small degree on the average density of the universe.

A second question is whether in a collapsing galactic mass, when the gas will be heated to one or two million degrees, the cooling time will be sufficiently short to get a real collapse. It seems probable that the densities would indeed become high enough for this. It will be assumed in the following that galaxies can have formed in this way, and that, though the exact way in which this happens is not yet clear, the cooling rate is also sufficient to let the galaxies act as efficient sinks for gas falling in at later times.

In a sense, the fact that galaxies have so much angular momentum thus appears to give a direct insight into the way they can have been formed in an expanding universe. This property also gives us a picture of the general characteristics of the turbulent streamings in the universe at the epoch of inception of the galaxies. Further knowledge about this turbulence can be obtained from the observed groupings and clusters of galaxies.

In the "turbulent cells" which formed the protogalaxies there must be the required amount of rotation as well as the amount of radial streaming needed to counterbalance the expansion. The actual motions as well as the density distribution in the "cells" will undoubtedly be irregular, but because we do not know how the turbulent cell was formed it seems

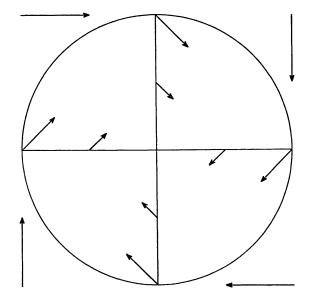


Fig. 2. Schematized picture of radial and transverse currents corresponding to suitable velocity gradients in the plane of the resultant angular momentum

difficult to make sensible predictions of these actual conditions. For the present purpose it therefore seemed best to start with the simplest possible model fulfilling the two conditions just mentioned. For a first model I assume that the cell is spherical with an abrupt outer boundary, that it is homogeneous and has a density equal to the average density in the universe. The cell is thought to subsist as an independent unit during its further evolution. The sphere is assumed to rotate with the same angular velocity throughout. The surface rotation velocity at the equator is taken equal to the circular velocity corresponding to the mass considered, the underlying idea being that if it had been less the mass would have detached itself already at an earlier epoch in the expanding universe (when the velocity of rotation, and, presumably, also the velocities of the radial streams, were higher), while it can hardly exceed the circular velocity, because then the turbulent element considered would presumably have intermingled with adjacent elements. The sphere will expand as a consequence of the expansion of the universe, but it will be assumed that there are local streamings in a radial direction which diminish the expansion. It is these which predisposes the element to become a galaxy. Regions where there are no such streamings, or where they have the opposite sign, will evidently not lead to the birth of a galaxy. It is natural to suppose that these radial streams will be largest in the plane where the transverse streams are largest i.e., in the plane of rotation. The radial streams will be supposed to have the same velocity as the transverse streams.

Conditions like this can result from proper velocity gradients such as indicated in Fig. 2, which represents the plane of rotation. In this example

$$\frac{\delta v_x}{\delta x} = \frac{\delta v_y}{\delta y} = \frac{\delta v_x}{\delta y} = -\frac{\delta v_y}{\delta x} = -C, \qquad (1)$$

where C is a positive quantity. The velocity gradients in the z co-ordinate, perpendicular to this plane, are supposed to be small, as it is plausible that the co-ordinates in which the largest gradients occur will tend to define the plane of rotation.

The assumption made above that the mass considered would be homogeneous and have the same density as the universe may well appear illogical, because a turbulent velocity field will necessarily be accompanied by density fluctuations. It is clearly unrealistic to choose initial conditions such that there are only deviations in the velocity field, and not in the density. I have nevertheless chosen a model in which the initial deviations are in the velocities rather than in the densities because the velocities are needed in any case in order to explain the rotations. The initial density fluctuations are unessential, and have been omitted in order to have a model with as few parameters as possible. In the particular case considered, where the velocity field is such that it will produce an increase of the local density, we imagine the initial epoch to be chosen just before an appreciable density increase has occurred.

It is hoped that for all its too extreme schematization the model may be usable for what it is primarily intended for, viz., the study of the possible continued growth of galaxies after their initial formation. In one respect the model as defined above is certainly too primitive for this purpose, namely in the assumed abrupt outer boundary of the velocity field. The boundary must correspond to changes in the velocity gradients, and be diffuse. We shall return to this point in Section 6.

A galaxy may be roughly specified by three parameters: mass, angular momentum and mean radius. As a first approximation I shall suppose the first two to be constant. The radius will depend on the evolution of the galaxy.

I shall use the following consistent system of units, in which the constant of gravitation is unity:

unit of distance kpc unit of time 10° years

unit of mass 2.21×10^5 solar masses

unit of velocity kpc (10° years)⁻¹, or 0.978 km s⁻¹.

The principal notations used are:

 R/R_0 ratio of the scale of the universe at time t to that at the present time

t time counted from the beginning of the universe

H Hubble constant

 ϱ average density in the universe

 $\varrho_{\rm crit}$ critical density

ratio of present average density to critical density, $\varrho_0/\varrho_{\rm crit_a}$

M mass of the galaxy considered

I angular momentum of the galaxy divided by its mass, or "angular momentum per unit mass"

turbulent the volume destined to become a galaxy, at element the time when it first detaches itself from the surrounding universe

radius of the turbulent element

x, y, z rectangular co-ordinates counted from the centre of the turbulent element; the z axis is the axis of rotation

 Π, Θ, Z velocities relative to a frame of reference expanding with the average expansional velocity of the universe, in cylindrical coordinates. Unless stated otherwise they refer to the surface of the sphere with radius r

 $\Theta_{\rm c}$ circular velocity on the equator of a spherical turbulent element

 $v_{\rm e}$ velocity of escape from the surface of the sphere; $v_{\rm e} = \Theta_{\rm c} \sqrt{2}$

radial velocity relative to the centre of the sphere

v space velocity relative to the centre of the sphere

time of revolution of a particle initially on the surface of the sphere; i.e., roughly the time between the birth of the turbulent element and its collapse into a galaxy.

Values without subscript refer to the epoch of inception of the galaxy, (i.e., the epoch of detachment from the surrounding universe), except for Ω , which refers to the present epoch. For other quantities a suffix 0 will be attached if they refer to the present time. I shall consider a universe without cosmological contant, and with $H_0=75~\rm km~s^{-1}\,Mpc^{-1}=0.0767$ units of $(10^9~\rm y)^{-1}$ (cf. Sandage, 1968). The present value of the critical density is then $1.06\times10^{-29}\rm g~cm^{-3}$,

or $157 \ M_{\odot} \ \text{kpc}^{-3}$, or 7.02×10^{-4} units, corresponding to $t_0 = \frac{2}{3} \ H_0^{-1} = 8.69$.

Following the schematical model proposed I assume

$$\Pi = -\Theta; (2)$$

in addition I introduce some contraction in the z coordinate, but considerably less than in the Π direction:

$$Z = -\beta \Theta \,, \tag{3}$$

where $\beta < 1$; Π and Θ are supposed to be independent of z.

A homogeneous sphere with radius r rotating with everywhere the same angular velocity has an angular momentum per unit mass

$$I = \frac{2}{5} r \Theta , \qquad (4)$$

where Θ is the rotation velocity on the equator of the sphere's surface.

If the initial sphere has the same mass M and the same angular momentum per unit mass as the galaxy considered, and if it rotates with a velocity equal to the circular velocity corresponding with M, so that $\Theta = \sqrt{M/r}$, its radius is given by

$$r = \frac{25}{4} M^{-1} I^2 . (5)$$

The corresponding radius of the universe is given by

$$\frac{4}{3}\pi r^3 (R_0/R)^3 \Omega \varrho_{\text{crit}_0} = M ,$$

from which

$$R/R_0 = \frac{25}{4} \left(\frac{4}{3} \pi \Omega \varrho_{\text{crit}_0}\right)^{1/3} M^{-4/3} I^2$$
$$= 0.896 \Omega^{1/3} M^{-4/3} I^2. \tag{6}$$

For Θ_c we have

$$\Theta_{\rm c} = \sqrt{M/r} = \frac{2}{5} M I^{-1} \,.$$
 (7)

On the equator

$$\dot{r} = Hr + \Pi = Hr - \Theta_c \,, \tag{8}$$

while in a direction making an angle ϕ with the equator

$$\dot{r} = Hr - \Theta_c \cos^2 \phi - \beta \Theta_c \sin^2 \phi , \qquad (9)$$

and

$$v^2 = \dot{r}^2 + \Theta_{\rm c}^2 \cos^2 \phi + (r\dot{\phi})^2$$
,

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where

$$r\dot{\phi} = -\Pi \sin \phi + Z \cos \phi = (1 - \beta) \Theta_c \sin \phi \cos \phi$$
.

Averaging over the surface of the sphere gives

$$\langle v^2 \rangle = (Hr)^2 - \frac{2}{3} (2 + \beta) Hr \Theta_c + \frac{1}{3} (4 + \beta^2) \Theta_c^2$$
 (10)

On the equator

$$v^2 = (Hr)^2 - 2 Hr \Theta_c + 2 \Theta_c^2.$$
 (11)

H is given by

$$H = \frac{R_0}{R} \sqrt{1 + \Omega\left(\frac{R_0}{R} - 1\right)} \times H_0. \tag{12}$$

For t we have well known relations (cf. Zeldovich, 1965). For a universe with $\rho < \rho_{\rm crit}$

$$t = H_0^{-1} \Omega (1 - \Omega)^{-3/2} (x \sqrt{1 + x^2} - \sinh^{-1} x),$$
 (13)

in which

$$x = \sqrt{\Omega^{-1} - 1} \times \sqrt{R/R_0}$$
.

For $\varrho > \varrho_{\rm crit}$

$$t = H_0^{-1} \Omega (\Omega - 1)^{-3/2} (\sin^{-1} x - x \sqrt{1 - x^2}), (13')$$

where

$$x = \sqrt{1 - \Omega^{-1}} \, \times \sqrt{R/R_0}$$
 .

In a universe with critical density

$$H = (R_0/R)^{3/2} H_0 , \qquad (14)$$

and

$$t = (R/R_0)^{3/2} t_0 = \frac{2}{3} H_0^{-1} (R/R_0)^{3/2}$$
. (15)

In such a universe, where $v_{\rm e}=Hr=\Theta_{\rm c}\sqrt{2}$, the expressions (10) and (11) become equal for $\beta=0.331$. In that case we can roughly simulate the evolution of the turbulent element by an isotropically expanding and contracting sphere. In the actual universe expressions (10) and (11) will generally not be equal, and the expansion and subsequent contraction will not be symmetrical and isochronous. If there is no negative velocity gradient along the z-axis the turbulent element will expand faster in this direction, and assume a cigar shape which ultimately, however, will contract again along its axis. Similarly there will be asymmetries in rotation and expansion in the equatorial plane.

In order not to complicate the rough estimates intended for this article unduly I shall start by assuming approximately isotropic expansion, and by supposing that everywhere on the sphere v^2 is given

by the expression (11). It is hoped that this further schematization will still give an approximate insight in the problems with which we are concerned. We can then compute the motion of a volume element on the surface of the original sphere as though it were under the attraction of a point mass M. Up to the moment of collapse gas pressure will be neglected. The time it will take for the sphere to expand, and to collapse again to the relatively small radius of the galaxy to be formed from it, will then be almost as long as the time of revolution in a keplerian orbit of semi-major axis a, viz.

$$T = 2 \pi a^{3/2} M^{-1/2}, \qquad (16)$$

where

$$a = M/(-2E) , \qquad (17)$$

and, therefore,

$$T = \frac{2\pi M}{(-2E)^{3/2}} \cdot \tag{18}$$

The maximum distance reached by the element considered is $r_m = 2a$. E, the energy of a unit mass in the orbit, is given by

$$E = -M/r + \frac{1}{2}v^2 = -M/r_m. (19)$$

Using the expressions (11) for v and (5) for r we can write

$$T = \frac{2\pi M}{\left\{\frac{4}{5} (Hr) M J^{-1} - (Hr)^2\right\}^{3/2}}.$$
 (20)

Hr is given by formulae (5), (6) and (12).

In a universe with critical density this reduces to

$$T = \frac{5^{3}\pi \ M^{-2}I^{3}}{2^{7/2}(\sqrt{2}-1)^{-3/2}} = 130 \ M^{-2}I^{3} \ . \eqno(21)$$

We now consider, for a moment, the case where the transverse stream velocities in the turbulent element are smaller than the circular velocities. Let Θ be equal to $\gamma\Theta_c$, where $\gamma<1$. For simplicity I assume again equality between the transverse and radial streamings, so that $\Pi=-\gamma\Theta_c$. If we limit ourselves to the case of a universe with critical density we find

$$T = \frac{5^{3}\pi M^{-2}I^{8}}{2^{7/2} \gamma^{9/2} (\sqrt{2} - \gamma)^{8/2}}$$
$$= 34.7 \gamma^{-9/2} (\sqrt{2} - \gamma)^{-3/2} M^{-2} I^{3}. \qquad (22)$$

The radius of the turbulent element and the value of R must be multiplied by γ^{-2} .

The actual times between the detachment of the turbulent element and the collapse into a galaxy will be somewhat shorter than T, because the surface

elements considered do not complete full orbital revolutions. The relatively small corrections needed will be given with the specific examples computed in the following sections.

5. Application to the Galactic System

For the Galactic System we have an approximate knowledge of the total mass and angular momentum. I shall use the model given by M. Schmidt (1965). Schmidt gives an extrapolation of the mass density and circular velocity up to 50 kpc from the centre. In reality we know nothing about the densities beyond about 15 kpc. It may well be that there is a limiting radius at much less than 50 kpc. I shall therefore estimate the mass and angular momentum for two different cut-off radii.

The mass densities projected on the galactic plane can be more or less directly derived from the parameters given by Schmidt. For determining the angular momentum for a given radius we need in addition to Schmidt's data for the circular velocity an estimate of the relative contributions of Population I, intermediate Population II and halo Population II. I have applied the following factors to the circular velocities to take account of the admixture of Population II stars: 0.9 for $\varpi \ge 10$ kpc, 0.8 for ϖ between 5 and 9, and 0.6 for $\varpi < 5$. The first factor is the most important one, because about 70% of the total angular momentum is provided by the region beyond $\varpi = 9$ kpc. It may be that I have slightly overestimated the influence of the Population II, and that the true angular momentum is some 5% higher than

Table 2. Distribution of mass and angular momentum in the Galactic System

ប	mass	⟨∅⟩	angular momentum
0- 2	0.19	120	0.02
2-4	0.23	120	0.09
4-6	0.26	159	0.21
6-8	0.23	193	0.31
8-10	0.18	221	0.33
10 - 15	0.24	213	0.62
15 - 20	0.12	184	0.38
20 - 30	0.12	159	0.45
30 - 40	0.06	134	0.27
40-50	0.036	118	0.19
0-50	1.67		2.87

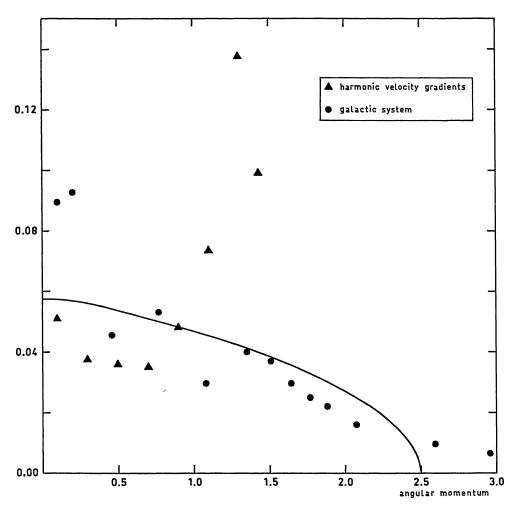


Fig. 3. Distribution of angular momentum per unit mass for a homogeneous, rotating sphere (curve) and for the Galactic System (filled circles)

Table	3
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cut-off radius (kpc)	r (kpc)	$\Theta = -\Pi$ (km/s)	Hr (km/s)	R/R_0	t (10° y)	$r_{ m max} \ m (kpc)$	$r_{\min} \ (ext{kpc})$	T (10° y)	$t_f - t $ (10° y)
20	18.1	186	262	0.030	0.045	30.7	12.7	0.79	0.70
50	25.6	168	238	0.040	0.070	43.6	18.1	1.25	1.10

the values I have given below, and which I have used in the calculations of this section.

Table 2 shows the data used to compute the mean angular momentum. The first column gives the distance from the galactic axis in kpc, the second column the total mass contained between the cylinders with the radii indicated in the first column, in units

of 10^{11} solar masses; $\langle\Theta\rangle$ in the third column gives the corresponding adopted velocity of rotation in km/s, while the last column shows the total angular momentum between the two cylinders, in units of $10^{14}~M_{\odot}\cdot {\rm km/s}\cdot {\rm kpc}$. In the first interval I have added the point mass of $0.07\times 10^{11}~M_{\odot}$ of Schmidt's model. The distribution of the angular momenta is given in

Fig. 3, which shows the fraction of the total mass within $\varpi=50$ kpc having an angular momentum per unit mass in an interval of 0.1 of the abscissae; the abscissa is the angular momentum per unit mass expressed in the total average angular momentum per unit mass. The solid curve shows the same distribution for a homogeneous sphere rotating as a solid body.

Hoyle (1963) and Mestel (1963) have both called attention to the similarity of the two distributions and have pointed to its probable significance for the theory of the origin of the Galaxy. This idea has been further worked out for other galaxies by Crampin and Hoyle (1964), who have shown that other spiral galaxies also have angular momentum distributions resembling that of a uniformly rotating uniform spheroid.

It may be noted that there is an excess of small angular momenta and of very large ones. The latter, however, depends entirely on the extrapolation of the data to 50 kpc, and may be fictitious.

A cut off at 50 kpc gives $M=1.67\times 10^{11}~M_{\odot}$, or 0.756×10^6 units, and $I=\frac{2.87\times 10^{14}}{1.67\times 10^{11}}=1720\,\mathrm{km/s\cdot kpc}$, or 1.76×10^3 units; while for a cut off at 20 kpc these quantities become $1.45\times 10^{11}~M_{\odot}$ and $1350~\mathrm{km/s\cdot kpc}$, respectively, or 0.656×10^6 and 1.38×10^3 units. I shall give calculations for the two cut-off distances, both of which are probably rather extreme.

I shall first consider a universe with critical density. The influence of the factor Ω specifying the actual density will be considered at the end of this section.

With the model described in Section 4 we then find the parameters shown in Table 3 for the turbulent element from which the Galaxy was formed. In the table Θ and Π are the velocities at the surface of the spherical element considered; Θ is equal to the circular velocity; Hr, the average expansion velocity of the universe at a distance r, is equal to the velocity of escape from the surface (because we consider a universe with critical density). The following columns give the radius of the universe and the time of the detachment of the element, computed from formulae (6) and (15); T has been computed from formula (21). The columns $r_{\rm max}$ and $r_{\rm min}$ give the maximum and minimum distance from the centre which the surface gas would reach in a keplerian motion.

We see that in this model the mass of gas from which the Galaxy originated had an outer radius of about 20 kpc. For a homogeneous sphere the radius of a cylinder which contains half the mass is 0.608 r,

or 13.3 kpc for the average of the two cut-off radii considered. In the actual Galactic System this radius is 6.8 kpc, so that in its equatorial plane the initial turbulent cell must have collapsed to about half of its original size. We hope to indicate in a subsequent article that a ratio of contraction of this order is not implausible.

The main purpose of the table is to indicate that the time of collapse is short compared to the age of the Galaxy. This shows that the birth process proposed is capable of explaining the formation of spiral galaxies.

As mentioned above, the interval between the time t of detachment of the turbulent cell and the time t_f of formation of the galaxy will be somewhat shorter than T because the outer layer considered starts at some distance from its pericentre. Considering again the simplified case of keplerian motions the factor to be applied to T is found to be 0.88. The resulting time needed for the expansion and subsequent collapse of the initial turbulent element is given under $t_f - t$ in the last column.

We do not know how the turbulent elements from which the galaxies formed have come into existence in the expanding universe. But one can perhaps understand why the cell which formed the Galaxy could not have detached itself at an earlier time. If we go back further, and still have to put into the given mass the given angular momentum, the rotation velocities become larger than the circular velocities. The cell would accordingly be likely to expand faster than the surrounding universe, and mix with adjacent turbulent elements. It is only at the time corresponding to the values of R/R_0 in Table 3 that the velocity of rotation is such that it can begin to separate itself from surrounding regions.

It is clear that the process has been vastly oversimplified. Let us consider, in the first place, what happens if the currents in the turbulent element are smaller than the circular velocity, say $\gamma\Theta_c$ in both Θ and Π . The relevant expressions are given in formula (22) and the line below it. Table 4 gives values of r

Table 4

γ	r (kpc)	T (10° y)	
0.	9 31.6	1.38	
0.	8 40.0	1.88	
0.	6 71.1	4.49	

and T for the case of a cut-off at 50 kpc. We see that for $\gamma=0.6$ the collapse time becomes much too long to be reconciled with the known ages of stars and clusters. We conclude that the velocity of the currents in the cell from which the Galaxy was formed must have been rather close to the circular velocity.

Finally, I consider the influence of the density in the universe. This has no influence on the result for r. The value of R/R_0 depends to some degree on this density, but T varies very little. The changes are shown in Table 5. They have been computed from formulae (6) and (20) for the case of the Galactic System with a cut-off radius of 50 kpc.

In the following article (Oort, 1970) evidence will be given that the overall density in the universe lies probably between 0.5 and 1.5 times the critical density, which would leave only an insignificant uncertainty in the parameters concerning the formation of our Galaxy.

6. Boundary Conditions and Inflow of Matter at the Present Epoch

The turbulent element considered in the preceding sections cannot of course end abruptly at the radius r. There must be a transition layer between such a cell and the surrounding regions with smaller, or opposite, velocity gradients.

As an extreme case we might think of a universe with harmonic variations of the velocity gradients, having wavelengths of the order of the dimensions of the turbulent elements considered above. We write

$$\Theta = \Theta_1 \sin \left(\pi \varpi / \varpi_m \right) . \tag{23}$$

For the sake of simplicity we assume again that $\Pi = -\Theta$. ϖ_m is the radius of the cylinder where Π and Θ become zero. In order to make it possible to use the approximation of spherical symmetry we add again a small negative velocity gradient along the z-axis with the same linear scale, therefore

$$Z = -Z_1 \sin \left(\pi z / \overline{\omega}_m\right). \tag{24}$$

An approximately symmetrical expansion is obtained when $Z_1=0.33~\Theta_1$. We assume that the velocity field has cylindrical symmetry around the z-axis. The maximum rotation velocity Θ_1 , at $\varpi=\frac{1}{2}\varpi_m$, will be taken equal to the circular velocity at this distance. In order to avoid transverse velocities exceeding the circular velocities in the region inside $\frac{1}{2}\varpi_m$, Θ was in this region assumed to vary proportionally with ϖ instead of with $\sin{(\pi\varpi/\varpi_m)}$.

For the outermost regions, where $-\Pi$ and Θ become small, the collapse times become longer than the age of the universe. Let ϖ_g be the radius for which T=10; $\varpi_j/\varpi_{\rm m}$ will be denoted by p. The radius ϖ_g can again be expressed in M and I. Relation (5) must then be replaced by

$$\varpi_{g} = \left[\frac{2p^{5}}{3 \left\{ 2 \int_{0}^{1/2} x^{3} \sqrt{p^{2} - x^{2}} \, dx + \int_{1/2}^{p} x^{2} \sqrt{p^{2} - x^{2}} \sin(x\pi) \, dx \right\}} \right]^{2} \cdot M^{-1} I^{2} , \qquad (25)$$

where $x = \varpi/\varpi_g$.

As an example I consider the case of a cut-off radius of 50 kpc for the Galaxy, and assume again a universe with critical density. In order to fulfil the condition that at $\varpi = \varpi_g$ T = 10, p must be 0.883. This gives

$$\varpi_{\sigma} = 15.0 \ M^{-1} \ I^2$$
.

The coefficient is 2.4 times larger than in formula (5). We thus get

$$\varpi_g = 61.2, \ \varpi_m = 69.3, \ \Theta_1 = 63.0,$$

$$R/R_0 = 0.096, \ H = 2.57.$$

An interesting feature of such a model is that it permits a rough prediction of the way in which the flow of matter into a galaxy would vary with the time. In the case considered the inflow at the present time would be about 3% of the total mass per 10° years. This is rather more than the accretion of about 0.9% estimated from the high-velocity clouds, and is implausibly high. The model would therefore need adjustment in its outer regions. There are, however, more serious objections to the entire model in connection with the collapse time and the distribution of angular momentum.

The collapse times corresponding to the above parameters are too long to be reconciled with the ages of globular clusters²) and of the oldest galactic clusters: the shortest collapse times, for ϖ between 0.6 and 0.7 ϖ_m , are 3.5×10^9 years. The only way in which these can be shortened sufficiently is by increasing the radial streamings to such an extent that in a large part of the volume contraction would

²⁾ Although, as Peebles and Dicke (1968) have suggested, globular clusters may have been formed in an early stage of the universe, and therefore some clusters in our Galaxy might be older than the Galaxy itself, the fact that so many globular clusters have a relatively high metal abundance indicates that at least a large fraction were formed after the collapse of the Galactic System.

start right at the beginning, without prior expansion. For this we would have to take Π at least equal to -1.6Θ ; ϖ_a would then become still larger, viz. about 70 kpc. While these conditions appear already rather improbable by themselves, a greater, and apparently insurmountable, difficulty with a model of this type is the very great change in the distribution of angular momentum that would be required to transform it to a system resembling our Galaxy. The initial distribution of the angular momentum has been indicated in Fig. 3 by triangles. No less than 60% of the mass has an angular momentum per unit mass between 1.00 and 1.44 in the units of Fig. 3, with a sharp cut off at 1.44; only 13% has an angular momentum less than 0.3. These figures are in sharp contrast with what we find in the Galactic System, in which, even if we suppose that the system is cutoff at 20 kpc, 23% of the mass has an angular momentum larger than 1.44 (with a cut-off at 50 kpc this would be 36%), while for 27% the angular momentum is less than 0.3. It seems extremely doubtful whether such a drastic re-arrangement of the distribution of angular momentum in our model as would be needed could be attained during the initial collapse period, or even in a time comparable with the present age of our Galaxy.

For the reasons mentioned it seems that we must, at least provisionally, abandon the hypothesis that the formation of galaxies has been caused by harmonic velocity variations in the universe. There must, however, be some kind of continuous transition from the element considered in our first model to the less systematic velocity fields outside, and any such transition will correspond with an inflow continuing up to the present time and beyond.

In order to keep the collapse times sufficiently short, and give a reasonable distribution of angular momentum, the boundary layer must be relatively thin. For a schematic model I assume again that the initial turbulent element rotates as a solid body, with velocities equal to circular velocities, up to a radius ϖ_1 , where the velocity of rotation is Θ_1 . Beyond ϖ_1 Θ will be assumed to decrease exponentially:

$$\Theta = \Theta_1 \, e^{-(\varpi - \varpi_1)/b} \,, \tag{26}$$

b being small compared to w_1 . The stream velocities in radial direction are again taken equal to the transverse velocities. Let w_{σ} denote again the radius at which in the equatorial plane the collapse time becomes 10^{10} years, so that a sphere with initial radius w_{σ} must contain a mass equal to that of our Galaxy, for which we use the model with a cut-off

radius at 50 kpc. As in the preceding calculations we consider a universe of critical density. We find then that the following choice of the parameters yields the correct value for the total average angular momentum per unit mass, I, and an inflow of gas at T=8.7 equal to 0.5%³) of the mass of the Galaxy per 10^9 years:

$$\varpi_a = 34.0 \text{ kpc}, \quad b = 2.0 \text{ kpc}.$$

From this one finds the following results for other relevant quantities $R/R_0 = 0.054$, H = 6.18 units, $\varpi_1 = 29.8$ kpc, $\Theta_1 = 130$ units, $\Theta_g = 15.9$ units. These values, except Θ_g , are comparable with those in the last line of Table 3. In the shell between ϖ_1 and $\varpi = 31.5$ the collapse times $t_f - t$ are less than 1.7×10^9 years, the minimum being 1.2×10^9 . The distribution of angular momentum is not greatly different from that in the model of Table 3.

Figure 4 illustrates the way in which the mass of the Galaxy would have grown in the course of time. The ordinates give the mass collected in fractions of the present mass.

It may be noted that for the gas of the outermost shell, which should now be falling into the Galaxy, the orbital apocentre lies at 230 kpc, while the pericentre distance computed on the basis of a keplerian orbit is only 0.2 kpc. In reality the incoming stream will not fall in so close to the centre, because in the actual universe there will have been random currents superimposed on the regular velocity field considered, and moreover the motion will be considerably influenced by the interaction with the galactic halo. Nevertheless it may well be that at the present epoch the inflow of gas is much stronger near the galactic centre than in our neighbourhood; this might possibly contribute to the peculiar hydrogen motions observed in the central region.

We must now consider the consequence of a dispersion in the values of Θ and Π on the general intensity of the incoming stream. For simplicity I use the first model, viz. that with a sudden cut-off at the boundary of the sphere rotating with constant

 $^{^{\}rm s})$ This 0.5% was based on an earlier result for the flow as estimated from the high-velocity clouds. Recent estimates give about 0.22% per revolution of 0.245 \times 10° years, or or 0.9% per 10° years (cf. Section 2). However, in view of the uncertainty of these estimates I have not made a new calculation to fit this latter value. There was the less reason to do this because in reality the motions will not be so regular as assumed in the above model, and the superposition of irregularities will tend to increase the computed inflow. As we shall see below it will roughly be doubled by introducing a reasonable dispersion in the stream velocities.

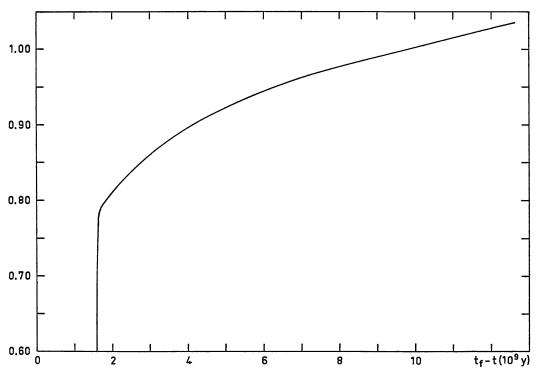


Fig. 4. Growth in mass of the Galactic System in the course of time

angular velocity, but I now suppose that at each radius Θ has a gaussian distribution with dispersion σ around the circular velocity. Using again a cut-off radius for our Galaxy at 50 kpc, and $\Omega=1$, I find that a dispersion between ± 35 and ± 80 units at the equator, i.e. between 20 and 50% of the circular velocity itself, would result in a present inflow into the Galaxy of 0.5% of the present mass per 10^9 years. If the dispersion is $\pm 20\%$ the mass condensed at T=10 would be 91% of the total mass of the sphere, if it is $\pm 40\%$ it would be 75% of the total mass. In order to get the total mass to agree with that of the Galactic System the radius of the initial sphere would have to be somewhat revised, but I did not think it necessary to carry through this calculation.

A similar dispersion in Π has a much smaller effect.

The distribution of the directions of the velocities to be expected for the approaching gas if this has come from a universe with random stream velocities has been investigated by P. C. van der Kruit. This will be published in a following article.

We conclude from this section that a thin transition layer in the original turbulent cell, and a small dispersion in Θ , result in continued inflow of matter

which may easily be of the order of that observed. A dispersion of only 20% in Θ would already produce such an inflow. The transition layer cannot have been thicker than about 30% of the radius of the uniformly rotating sphere or it would have produced too large an accretion at the present time. It is likely to have been considerably thinner. In the case considered above the boundary corresponding to matter falling in at present had only a 14% larger radius than the part rotating with circular velocity.

The models for the turbulent cell considered in the present article are too simplistic to reproduce the actual density distribution in the Galactic System. The problem of how initial conditions can be chosen so as to get better resemblance to real spiral systems is being studied by Dr. K. Robinson.

7. The Formation of other Types of Galaxies. Birth-rate of Radio Sources

7.1. Elliptical Galaxies

In all elliptical galaxies the angular momenta per unit mass are much smaller than in spirals. There are no direct observations of the rotation of the main bodies of genuine ellipticals, but we can infer from the axial ratio of the isophotes that the rotations must be relatively slow. Also the radii are considerably smaller than for spirals. The only system for which somewhat detailed measures are available is NGC 3115. This is not a pure elliptical galaxy, but an amphibious system, consisting of spheroidal components resembling an E5 galaxy and an So disk system. And even in this case we can only make order-of-magnitude estimates of I and M. Estimating that half of the mass may be contained within an equatorial radius of 2.5 kpc and that the rotation velocity of the spheroidal component, which contains the greater part of the mass, is 120 units, I may be of the order of 300. An uncertain estimate of the total mass gives 7×10^{11} solar masses, or 3×10^6 units (Minkowski, Oort and van Houten). In a universe of critical density the radius of a sphere of this mass rotating with circular velocity and having the same angular momentum per unit mass would be 0.2 kpc; R/R_0 would then be 2×10^{-4} . Other giant ellipticals may have still smaller radii and lower rotation velocities, leading to still smaller values of R/R_0 .

In reality, galaxies cannot have formed before the decoupling of matter and radiation, which took place at $R/R_0 \sim 0.001$. On the basis of the theory of the formation of spiral galaxies given above it is even unlikely that they could have formed soon after the decoupling, because of the too high degree of turbulence. Because of the low angular momentum there is no direct evidence to determine the epoch at which proto-ellipticals were generated. Lacking information on this point I assume provisionally that their detachment from the rest of the universe occurred in the same general period as for the protospirals. It may be, however, that the birth of protoellipticals extended further into the past.

7.2. Irregular Galaxies

Let us now consider the other extremes among the galaxies, those of the Magellanic Cloud type.

Table 5. Influence of deviations from the critical density

Ω	R/R_0	H	T	
0.125	0.0202	10.12	1.44	
0.25	0.0254	9.82	1.32	
0.5	0.0320	9.66	1.26	
1.0	0.0403	9.51	1.23	
2.0	0.0508	9.36	1.20	
4.0	0.0640	9.20	1.18	

Unfortunately there are as yet no suitable data for estimating the total angular momentum of such a galaxy. The systems which have been most extensively studied are the Magellanic Clouds, which, because they are satellites of a larger system, may not be representative. For the Large Cloud the inclination is moreover so uncertain that it cannot be used for our purpose. The most extensive data for the Small Cloud are those published by Hindman (1967). They indicate a linear increase of the velocity of rotation with distance from the centre up to about 2°, or 2.1 kpc. At 2.6 kpc the rotation reaches a maximum. Let us assume, in order to make a comparison with the Galactic System, that the proportion of the mass and the angular momentum beyond the distance corresponding to the maximum rotational velocity is the same in the two systems. With Hindman's estimate of 46 km/s for the rotation at 2.6 kpc I find that for the part within this distance I must be about 49 units. Adopting an axial ratio of 5 for the mass distribution the mass for the same part is found to be 2.9×10^3 units. For the Galactic System, where the maximum rotational velocity occurs around 10 kpc, I and M for the part within R = 10 are 9.0×10^2 and 4.9×10^5 units, respectively, while the corresponding values for the entire system are 15.7×10^2 and 7.1×10^5 units, and therefore larger by factors 1.74 and 1.44, respectively. Assuming the same factors for the Small Magellanic Cloud we get I=85 and M=4200 for the entire system. Formula (6) then yields $R/R_0 = 0.10$, for $\Omega = 1$. By formulae (15) and (21) the time of "detachment" would then be 0.26 and the time of collapse T = 4.5. If isolated Magellanic Cloud type systems have comparable values of I and M we see that they might have originated and collapsed into a galaxy at a considerably later epoch than spiral galaxies like our own.

7.3. Birth-rate of Radio Galaxies

As is well known the density of radio sources was much higher in the past than it is now. Counts of radio sources indicate a factor of the order of 10^3 between the birth-rate of giant radio galaxies around z=2 and the present rate (cf., for instance, Doroshkevich, Longair and Zeldovich, 1970; references to earlier articles may be found in this paper). A similarly steep increase between z=0 and z=1, with a ratio of about 100:1, has been found for the density of quasi-stellar radio sources with the aid of redshifts (Schmidt, 1968).

It is tempting to speculate whether the birth of a radio galaxy would be a consequence of the collapse of a protogalaxy, and whether the large increase in birth-rate of radio sources can be connected with the period in which most of the collapses of protogalaxies occurred. In the specific model discussed in Section 5, with a cut-off radius of 50 kpc, the collapse took place at $t=1.17\times 10^9$, which, in the universe of critical density, corresponds with z=2.8. This corresponds roughly with the period of high radio galaxy birth-rate.

The working models considered in Sections 4 and 5 contain several arbitrary assumptions, the principal one being that the inward radial motions in a "turbulent cell" are equal to the transverse components. The transverse streaming is fixed by the angular momentum - at least for the spirals -, but the radial streamings can very well have been smaller, in which case the collapse period may be considerably lengthened. A similar effect would result if the transition region between the rotating cell and its surroundings would be made wider than the rather thin layer used in the model. For these reasons it appears perfectly plausible that the collapses may mostly have taken place at a somewhat later time, say around t=2, corresponding with z=1.7, or still later. The epoch of collapse is limited only by the consideration that the known galaxies all seem to possess an old stellar population.

On the other hand some categories of protogalaxies, like the ellipticals, may well have collapsed earlier, and it would therefore be quite possible that the period of high radio source birth-rate would extend beyond z=3. We may conclude that, so far as the data go, they indicate a close resemblance between the period of galaxy formation and the period of high radio source birth-rate.

As galaxy formation from initially expanding protogalaxies is a process that probably started only in a relatively late stage of the universe it might be expected that the birth-rate of radio galaxies would decrease for $z \geq 3$. The Cambridge group has indeed found that the strong increase in density does not continue for larger values of z, a result emphasized also in the article by Doroshkevich, Longair and Zeldovich. It should, however, be pointed out that a relative scarcity of sources beyond z=3 might also be caused by the "snuffing out" of radio sources by inverse Compton effect in the black body radiation field (cf. Rees and Setti, 1968).

8. Estimate of Angular Momentum which can be Gained by Extraneous Forces

In the preceding we have assumed that the rotation of the galaxies is due to large-scale turbulence in the universe. But, as was mentioned in Section 3, there is also the possibility that in a stage when they still had irregular shapes galaxies have been set rotating through gravitational forces exerted by neighbouring concentrations of matter. This possibility has recently been investigated by Peebles (1969); he comes to the conclusion that the angular momentum of galaxies might well have originated in this way. If this would be so the process of galaxy formation could have been quite different from that envisaged above. The galaxies may then have formed by gravitational instability from density or velocity fluctuations of small amplitude but large scale in the early history of the universe, shortly after the decoupling of matter and radiation, as has often been assumed (for references, cf. Field, 1970).

In view of the importance of this problem for the birth and evolution of galaxies as well as for the nature of the turbulence in the universe, I have attempted in the following to specify what the necessary conditions would be for a transfer of the required amount of rotation. The conclusion is that these are too extreme to be plausible. Two cases are treated, in both of which our own Galaxy is taken as a representative specimen. In the first case an estimate is made of the effect of the gravitational attraction by surrounding galaxies of the general field, while in the second case the influence of the Local Group is considered. For these estimates it is assumed that the galaxies have been formed from protogalaxies born soon after the decoupling stage, and that at this stage density and expansion fluctuations had small amplitudes.

8.1. Elongated Forms of Protogalaxies

We consider a mass which detached itself at a time t_i corresponding to $R_i/R_0=0.001$, i.e. soon after the fireball stage of the universe. If angular momentum is to be imparted to this protogalaxy during its subsequent evolution its shape must be asymmetrical. This is likely to be the case for the regions of accidentally higher density from which the protogalaxies would form. Moreover, it can be shown that initial asymmetries are greatly enhanced during the evolution, so that even quite minor deviations from sphericity at the time of detachment will give rise to strongly

elongated shapes at the time the protogalaxy reaches its maximum dimensions.

The computation of the evolution in an expanding universe of a region with excess density or lower than average expansion is extremely difficult if the region is not spherically symmetrical.

Before considering this problem it is expedient to recapitulate the formulae for the spherical case, which is the only one that can at present be treated analytically. They give a useful general insight into the relations between some of the principal quantities involved.

Suppose that at $t=t_i$ there is a fractional excess density η in a sphere of radius r_i and mass M. The sphere is supposed to have no rotation. The initial expansion velocity in the sphere is assumed to be the same as in the surrounding universe, which has critical density. The velocity v of the surface is then given by

$$\label{eq:v2} \frac{1}{2} \, v^2 = \frac{M}{(1+\eta) \, r_i} \; .$$

With the aid of the energy Eq. (19) we find for the ratio of maximum to initial radius

$$\frac{r_m}{r_i} = \frac{1+\eta}{n} \,. \tag{27}$$

Alternatively, we consider a sphere of the same mass without density excess, but with a less than average expansion velocity. We denote the expansion velocity at the surface by $v=(1-\varepsilon)\,v_e$, where v_e is the escape velocity. Inserting this in formula (19), and considering that $M/r_i=\frac{1}{2}\,v_e^2$, we find

$$\frac{r_m}{r_i} = \frac{2}{2\varepsilon(1-\varepsilon/2)} . \tag{28}$$

In the theory of gravitational instability η and ε are small quantities. Approximately the same values of r_m will therefore be reached in the two cases if $\varepsilon = \eta/2$.

It may be noted that the condition that the universe has critical density is irrelevant in the present case, because for all practically possible values of Ω (between, say, 0.1 and 10) the deviation of the average density at $R/R_0 = 0.001$ from its critical value is small compared with the values of η or ε with which we shall be concerned.

From the energy Eq. (19) one can easily derive the following relation between t and y

$$t=r_m\sqrt{r_m/(2~M)}~(\sin^{-1}y-y\sqrt{1-y^2)}~,$$
 where
$$y=\sqrt{r/r_m}~. \eqno(29)$$

The relation is essentially the same as that given by formula (13').

Substituting

$$M = (1 + \eta) \frac{4}{3} \pi \varrho_{\text{crit}_0} r_i^3 (R_0/R_i)^3$$
,

using (27) to eliminate η , taking for ϱ_{crit_0} the value given on p. 389, and putting $R_0/R_i = 1000$, we get

$$t = 4.12 \times 10^{-4} \frac{r_m}{r_i} \sqrt{\frac{r_m}{r_i} - 1} \left(\sin^{-1} y - y \sqrt{1 - y^2} \right).$$
(30)

The approximate time T after which the protogalaxy recollapses may be obtained from (29) by putting $r = r_m$ and multiplying by 2. We thus find

$$T = \pi r_m^{3/2} (2M)^{-1/2} = 1.293 \times 10^{-3} (1 + \eta) \eta^{-3/2}$$
. (31)

The first expression follows also from Eqs. (17) and (18), and is Kepler's third law. From the second expression we see that in order to get a collapse in 10^9 years we must have $\eta = 0.0120$, or $\varepsilon = 0.0060$, at $R/R_0 = 0.001$.

We must now consider the influence of deviations from spherical symmetry. A preliminary investigation of these effects has been made by Mr. V. Icke. I am much indebted to him for putting his results at my disposal.

In this first attempt it was assumed that the boundary of the region where the deviation from the uniform universe occurred could be represented by a prolate spheroid. This is evidently an extreme simplification. Our hope is that the salient properties of this schematized model will give at least some indication of what would happen in the actual universe. The expansion velocities within the spheroid were assumed to be $(1 - \varepsilon) v_e$, where v_e is again the velocity of escape. In computing the evolution of the spheroid Icke made the further simplifying assumption that the attraction of the universe outside the spheroid can be neglected. This is evidently incorrect, but we believe that the assumption is usable for a first approximation.

The spheroids collapse first along their smaller axes. Icke has shown that if the initial major axis is larger than the minor axes by 10% or more the collapse perpendicular to the major axis will be complete before or at the time when the major axis reaches its maximum length. If the initial major axis was only 5% longer than the other axes collapse of the minor axes will happen at an epoch when the major axis is still close to its maximum. As a result

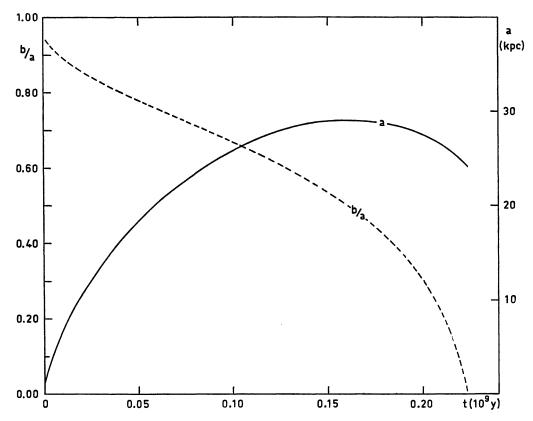


Fig. 5. Evolution of a prolate spheroidal mass. Initial values (at $R/R_0 = 0.001$): semi-major axis 0.655 kpc, axial ratio 0.95, expansion 2.2% less than average. The dotted curve gives the axial ratio (scale on the left side), the full-drawn curve shows the length of the semi-major axis (scale in kpc on the right)

Table 6. Times of collapse along the minor axis and lengths of the semi-major axis

	$b_i/a_i = 0.80$		b_i/a_i	= 0.90	$b_i/a_i = 0.9$			95	
$ \epsilon $ $ \begin{array}{c} t & a \\ \text{at } b = 0 \end{array} $			t at b =	a = 0	$\begin{array}{cc} t & a \\ \text{at } b = 0 \end{array}$		$egin{array}{ccc} t & a \ { m at} \ a_{ m max} \end{array}$		
0.0086 0.0217 0.0303	0.27 0.16 0.11	115 66 44	0.39 0.20 0.13	81 42 26	0.45 0.22 0.14	50 24 15	0.36 0.17 0.10	53 29 20	

of this collapse the gas densities must have become quite high. With a semi-major axis of 30 kpc and semi-minor axes of 1 kpc the average density would have been of the order of 10^2 at. cm⁻³. In a collapse to such densities large-scale star formation is bound to have taken place. As a consequence the ultimate stellar system will contain a sensible fraction of stars that move out to distances comparable with the length of the semi-major axis at the time when the

collapse along the minor axes took place. This length cannot therefore have been greater than the outer radius of the present galaxy.

For this reason it seems quite improbable that the semi-major axis of the protogalaxy from which the Galactic System has formed has ever been larger than about 30 kpc as an extreme estimate.

The initial shapes of the regions over which systematic density or velocity fluctuations extended were presumably quite erratic. It seems unlikely that their dimensions in different directions would generally have been equal to within less than, say, 5%. But even if the differences were no greater than 2% collapse along the smaller axes, and therefore star formation, would still happen before the contraction along the major axis could have been sensibly decelerated by gas collision. As such a close approach to sphericity is hardly probable we may, I believe, conclude that the maximum half dimensions to which the protogalaxies have expanded are less than 30 kpc.

This is an important limiting factor for the transfer of angular momentum.

Table 6 shows the relevant data for three values of the initial axial ratio and for three values of ε , the initial deviation from the mean expansion. The data, which are only approximate, are again from Icke's work. The semi-major and semi-minor axes are denoted by a and b; b_i/a_i is the initial axial ratio. In the case $b_i/a_i = 0.80$ the collapse along the minor axis occurs at a time when the major axis is still expanding. If $b_i/a_i = 0.90$ this collapse happens approximately at the epoch when the major axis has just reached its maximum extent, while with b_i/a_i = 0.95 the collapse takes place when the system is already contracting along its major axis. For this case the times at which the major axis reached its maximum length and this maximum length itself have also been given.

For estimating the maximum transfer of angular momentum I adopt the model with $b_i/a_i=0.95$ and $\varepsilon=0.0217$, for which $a_{\rm max}$ is about 30 kpc. Any other spheroidal model with the same value of $a_{\rm max}$ would have given essentially the same result. In the model chosen the relation of a and b/a with t is roughly as indicated in Fig. 5.

8.2. Perturbations by Nearest Galaxies

We consider, first, the perturbing forces due to neighbouring single protogalaxies.

At the time $t_i = 2.75 \times 10^{-4}$, corresponding with $R_i/R_0 = 0.001$, the distance between adjacent volumes which contain masses equal to those of large galaxies is about 1.5 kpc. Suppose that adjacent to the volume where the protogalaxy considered is formed there is another similar volume in which there is a fractional excess density η , which will lead to the formation of another protogalaxy. Denote this latter by M' and the original protogalaxy by M. In the theory of gravitational instability η will be no larger than a few per cent. It is only this fraction η of the mass M' that will give a perturbing force on M. The rest is counterbalanced by the attractions of the other "cells" adjacent to M. The perturbations will therefore initially be extremely small. Subsequent local contractions or expansions will not by themselves much alter this situation, as they do not considerably change the mass contained in each "cell". Appreciable perturbations can only be brought about by changes in the relative positions of M

Suppose, for instance, that, relative to the mean expanding frame of reference, M' initially had a

random velocity directed towards M, equal to a fraction ε of the normal expansion corresponding to the distance between the two galaxies. In the quiescent initial stage assumed by the theory of gravitational instability ε will be small, presumably smaller than the values which were supposed to lead to the formation of galaxies. For an example I take $\varepsilon = 0.005$. At t = 0.2, around which the protogalaxies reach their maximum lengths, the distance between M and M' will then have diminished by at most 3%compared to what it would have been in the absence of a random velocity, and the force exerted on M will accordingly have become about 6% larger. The protogalaxy M' will thus exert a perturbing force equal to that which would be produced by a mass 0.06 M' superimposed on a homogeneous universe. Let us denote this "effective" mass by fM'. If in the volume from which M' was formed there was an initial density excess of, say, 2%, the factor f would be 0.02 at $t = t_i$, and increase proportionally with tto 0.08 at t = 0.2.

The transfer of angular momentum will be largest near the time of maximum dimension of M, which according to Table 6 is around t = 0.2. At that time the distance d between M and M' will by the universal expansion have increased to 121 kpc, and is thus large compared to the dimension of M.

If $b/a \ll 1$ and $a/d \ll 1$ the couple exerted on a prolate spheroid with semi-major axis a by a mass fM' situated at a distance d from the centre of M in a direction making an angle α with the major axis of M is given by

$$C = \frac{1}{5} f M M' a^2 d^{-3} \sin 2\alpha.$$
 (32)

If b/a = 0.5 the coefficient 1/5 decreases to about 0.164; for b/a = 0.75 it is roughly 0.116, for b/a = 0.90, 0.046. As the moment of momentum of M is MI we have, for $b/a \ll 1$,

$$dI/dt = \frac{1}{5} f M' a^2 d^{-3} \sin 2\alpha.$$
 (33)

The moment of inertia around the minor axis being $\frac{1}{5} Ma^2$ the derivative of the angular velocity is

$$d\omega/dt = \int M' d^{-3} \sin 2\alpha. \qquad (34)$$

The perturbing forces are such that they make the spheroid rotate as a solid body.

After averaging over α we get

$$I = \frac{2}{3} M' \int_{t_i}^{t} f g \, a^2 \, d^{-3} \, dt \,, \tag{35}$$

where g is 0.20 for $b/a \ll 1$ and varies for larger axial ratios in the manner indicated above. The relation between a and t is given in Fig. 5, while, to a sufficient approximation, d is given by the relation for the scale value in a universe of critical density, viz., $d/d_i = (t/t_i)^{2/3}$. Let us take $d_i = 1.5$ kpc, as in the example used above, and $t_i = 2.75 \times 10^{-4}$ units, corresponding with $R_i = 0.001$ R_0 . Taking, further, for M' a mass equal to that of the Galactic System, and extending the integral to the time t at which the minor axis becomes zero we obtain I = 1.1; extrapolation to the time of complete collapse gives only about 5% increase. The result is three orders of magnitude smaller than the angular momentum per unit mass in the Galactic System.

These are the effects due to the nearest galaxy. We must next consider the tidal forces of other galaxies. If we denote by r_1 the radius of a sphere which on the average contains one galaxy, and by ΔI_1 the transfer of angular momentum by the nearest galaxy, the average distance of which is $\frac{3}{4}r_1$, the average transfer ΔI by a galaxy at distance r will at most be

$$\Delta I = \left(\frac{3}{4} r_1/r\right)^3 \Delta I_1 = 0.422 \ (r_1/r)^3 \Delta I_1 \ .$$

As the values of ΔI due to the various galaxies must be added quadratically

$$\varSigma(\varDelta I)^2 = (\varDelta I_1)^2 + \int\limits_{r_{\cdot}}^{\infty} n(r) \, (\varDelta I)^2 \, dr$$
 ,

where $n(r) dr = 3r^2 r_1^{-3} dr$ is the number of galaxies at distances between r and r + dr. Inserting the above expression for ΔI we get

$$\Sigma(\Delta I)^2 = 1.18 (\Delta I_1)^2$$
.

The influence of the other galaxies if these are randomly distributed is therefore negligible.

8.3. Perturbations by Groups of Galaxies

We must next consider the forces arising from groups and clusters of galaxies.

It seems probable that, contrary to the individual galaxies, galaxy clusters are formed from large-scale density or expansion inhomogeneities dating from the earliest stages of the universe, and that therefore their evolution since the epoch of decoupling can be schematically represented by formulae (27) to (31).

Consider at the time t_i of decoupling a spherical region of radius r_i with a density excess $\eta \varrho_i$ and a normal expansion; ϱ_i is the average density of the

universe at the time t_i . The mass contained in the region will be supposed to be considerably larger than the mass of a galaxy. Suppose that the protogalaxy M to which angular momentum is to be imparted is situated on the surface of this "protocluster" and moves with it. At the time t_i M will experience a force equivalent to that exerted by a mass $\frac{4}{3}\pi\eta\varrho_i\,r_i^3$ at a distance r_i . Due to the excess density the radius of the protocluster, to be denoted by r_c , will increase more slowly than the same distance would increase if there were no cluster. The relation of the latter distance (which I shall call r) with time is the same as that for the scale of the universe

$$r/r_i = (t/t_i)^{2/3} ,$$

while the relation for r_c is given by (30) and (27).

The resultant force on M due to the excess density in the cluster and to the fact that M has come relatively closer to its centre is

$$\frac{4}{3}\pi (1+\eta) \varrho_i r_i^3 r_c^{-2} - \frac{4}{3}\pi \varrho_i r_i^3 r^{-2} = f \frac{4}{3}\pi \varrho_i r_i^3 r_c^{-2}.$$
(36)

The "effective" mass is

$$fM' = f\frac{4}{3}\pi\varrho_i r_i^3$$
; $f = \eta + 1 - (r_c/r)^2$. (37)

The transfer of angular momentum can again be computed with the aid of formula (35) if we use (37) for fM' and replace d by r_c .

As r_c/r_i and r_c/r are independent of r_i the transfer of angular momentum is also independent of r_i , and, therefore, of the mass of the cluster, as long as the cluster's size is sufficient to make it possible to put the test galaxy on its surface, as was supposed above. With this reserve the effects sought depend only on the initial fractional density excess η .

One can get an estimate of η from the state of development of the groups and clusters of galaxies in the nearby part of the universe. Consider, first, the Local Group. As will be indicated in Section 9 the members can hardly have made more than 2 or 3 revolutions, because otherwise the mass of the group would have to be improbably high. If therefore we put the minimum time of revolution equal to 3 units we find from formula (31) a maximum value of η equal to 0.006.

The most conspicuous clustering in our general vicinity is the 10° -diameter condensed part of the Virgo cluster. Here the time of revolution can be directly estimated from the observed motions; it is found to be about 5 units (Oort, 1958). If we assume that the collapse time T was roughly equal to this

we get $\eta=0.004$. For the long appendices of this cluster, which extend to several tens of degrees on either side of it, the evolution is clearly still in an early stage: even the first collapse seems hardly to have begun. T may therefore be estimated to be around 20, and $\eta=0.0016$.

As an example I consider a case with $\eta = 0.005$. As in the case of the single neighbouring protogalaxy I extend the integration to the time at which the minor axis of the protogalaxy M becomes zero, and extrapolate the part beyond. The resulting value of I is 30. This is only 1/60 of the estimated angular momentum per unit mass of the Galactic System. Moreover, the estimate given is not representative for an average galaxy. In general, a galaxy will not lie exactly on the outer boundary of a cluster. If it lies wholly or partly inside it the perturbing forces will be smaller, in an average case by at least a factor of 2. If it lies some distance outside the boundary the effects will be similarly diminished. The discrepancy between the computed and the observed values of I is therefore at least a factor of 100.

The transfer of angular momentum can only be increased by increasing the maximum length attained by the protogalaxy during its evolution, or by decreasing the collapse time of the group or cluster of which it is supposed to be a member.

Even if we increase $a_{\rm max}$ by a factor 2, to 60 kpc, which seems impossible to reconcile with the observed present density distribution in galaxies, I would only be increased by a factor of 5.1, and would remain more than an order of magnitude too low. A similarly extreme increase of η to 0.010, corresponding with a collapse time of the cluster of 1.3×10^9 years, would give a gain in I by a factor of 2.9.

We conclude that the angular momentum possessed by spiral galaxies can have arisen neither from perturbations by neighbouring individual galaxies nor from the unevenness in the gravitational field due to neighbouring groups of galaxies, and that it therefore must have come from large-scale "turbulent" motions existing at the time when the protogalaxies became individual units.

The spiral galaxies cannot, therefore, have formed from density or velocity fluctuations soon after the decoupling of matter and radiation.

9. The Andromeda Nebula and the Galaxy

If galaxies have formed from turbulent elements it is probable that the greater part of the gas in the universe has *not* condensed into galaxies, and has

subsisted as an intergalactic medium. In this section some independent evidence is considered which points to the existence of a substratum between the galaxies.

It is well known that in several groups and clusters of galaxies the velocity dispersion is higher than what should be expected from the masses of the member galaxies. This has generally been taken to indicate that they contain a considerable intergalactic mass, probably in gaseous form. It is also possible, however, that the groups concerned are unstable, in which case we cannot conclude much about their masses from the observed motions. Galaxy clusters would then have come together only temporarily, or else, as Ambartsumian has suggested, the faster members may have been formed by ejection from a parent galaxy and have thus acquired velocities exceeding the velocity of escape from the

In the present context I therefore want to confine the discussion to the one case in which we can draw definite conclusions, namely that of the Local Group.

The Local Group contains some 20 known members. However, many of these cluster around the massive systems M31 and the Galaxy, and cannot be considered as independent. If we consider these as "satellites" of the two giant spirals, we are left with only five known "independent" members beside M31 and the Galaxy, viz., NGC 6822, IC 1613 and three Sculptor-type systems: Fornax, Leo I and Leo II. The total mass of these five is less than 1/1000 of the combined mass of M31 and the Galaxy with their satellites. As far as its visible part is concerned the Local Group might therefore be described as a binary system. In reality, however, it must be greatly different from this.

The Andromeda nebula has a radial velocity of -301 ± 3 km/s relative to the local standard of rest (cf. the comprehensive discussion by Mrs. Rubin and d'Odorico, 1969). After correction for a rotational velocity of the Galactic System of 250 km/s the velocity relative to the galactic centre is found to be -102 km/s. The principal uncertainty is in the rotation velocity of the Galaxy. Recent investigations on the distribution of RR Lyrae variables in the central region of the Galaxy by Plaut (1968, 1970) indicate that the distance to the centre is probably smaller than the value of 10 kpc which has recently been used as a standard value. In this case the rotation would also have to be decreased, which would increase the relative velocity of M31 and the Galaxy. However, for the present I adopt the above value.

As was first clearly set forth by Kahn and Woltjer (1959) the high velocity of approach of the two systems proves decisively that there must be a large extra mass in the local group beside that of M31 and the Galaxy. We can see this directly from the energy Eq. (19) and Eqs. (31) and (29). If we assume that the two galaxies have masses of, roughly, 4×10^{11} and 2×10^{11} solar masses respectively, as indicated by their rotations, then, if there is no other mass in the Local Group, the observed velocity exceeds the velocity of escape. This seems impossible for systems moving towards one another, as it would require inacceptable initial conditions. I think that enough is known about the distribution of other galaxies in the nearer part of the universe to say that no other forces but the attraction by the Local Group can have contributed appreciably to the relative motion, the influence of the Virgo cluster and of the irregularities in the distribution of the galaxies in this general region being far too small. Moreover, the Local Group contains other members as well, and cannot be considered as a chance encounter of two galaxies.

We first consider by what factor f the masses of the two galaxies should be increased if we would suppose that all the extra mass needed would be concentrated in, or immediately around, these galaxies. Suppose that the group detached itself soon after the decoupling of matter and radiation, and is at the present time t_0 in the process of recollapsing. One then finds that, in order to explain the present velocity of approach of 102 km/s at an assumed distance of 650 kpc, f must be taken equal to 7.1, corresponding with a combined mass of $43 \times 10^{11} \ M_{\odot}$. Truran and Cameron (1970) have recently made the interesting suggestion that galaxies may contain massive halos, and be heavier than hitherto supposed. However, at the very extreme, this might increase the masses by a factor of 2. A factor of 7 would appear to be out of the question. It would appear, then, that the only way in which the required extra mass can be provided is by distributing it over the Local Group as a whole. In this case the mass needed can be less, because the distances of M31 and the Galaxy to the centre of the group will presumably be smaller than that between the two galaxies. If one assumes, for instance, that the mass of the cluster is homogeneously distributed over a sphere of radius 400 kpc centred in the centre of gravity of M31 and the Galaxy one would need a total mass $13.2 \times 10^{11} M_{\odot}$ to explain the present motion of approach of the Galaxy. The effective mass of the two galaxies is $1.8 \times 10^{11}\,M_{\odot}$ in this case, so that $11.4 \times 10^{11}\,M_{\odot}$ must be due to the rest of the group. The mean density would then be $2.9 \times 10^{-28} \, \mathrm{g}$ cm⁻³, or 27 times the critical density. Although one cannot exclude the possibility that this mass would be provided by intergalactic *stars* it seems more plausible to assume, as Kahn and Woltjer have done, that it consists of intergalactic *gas* which has not collected into galaxies (cf. also the following article). As has been mentioned before the gas should have a very high temperature, of the order of 10^6 °K, and would be unobservable.

It may be pointed out that the existence of a large gaseous constituent will act as a stabilizing mechanism for galaxy clusters. During the collapse of a proto-cluster the gas loses kinetic energy through radiation. As a consequence the cluster will be able to retain a considerable fraction of its galaxies. Without gas, clusters would evidently disperse again after their first collapse. The existence of some large clusters that show characteristics of equilibrium configurations seems to indicate that some such stabilization process has taken place.

In the Local Group the collapse may not yet have been completed; the members may well be approaching its centre for the first time. For if they had previously passed through the group, and would now be in their second revolution, the mass of the group would have to be increased by another factor of 3.3 over the preceding estimate of $13.2 \times 10^{11} M_{\odot}$, as can be easily computed by means of the formulae mentioned. So high a mass is somewhat difficult to reconcile with the observed velocities of the other members. Radial velocities are known for M31, NGC 6822, IC 1613 and Fornax. If one assumes that the motions are mostly radial, the residual velocities remaining after correction for the motion of the Galaxy are -41, +44, -89 and -12, respectively. Application of the virial theorem results in a total mass 11×10^{11} M_{\odot} , i.e., four times smaller than the mass of 3.3 $imes 13.2 imes 10^{11} \ M_{\odot}$ surmised above. Though the statistics are poor, and the applicability of the virial theorem questionable, the data give at least an indication that the galaxies have not previously passed through the cluster.

If this inference is correct we apparently witness in the mutual approach of the Andromeda nebula and the Galaxy appenomenon which is quite analogous to that of the continuing collapse of intergalactic gas into the Galaxy discussed in the preceding sections.

If M31 and the Galaxy have not previously passed through the Local Group it is clear that they cannot have acquired their angular momentum by a collision with each other or with other members of

this group. Even if they would have moved through, the probability of their arriving near the centre simultaneously is negligibly small. The other members have too small masses to be of importance in this connection.

Whereas the process of galaxy formation appears for the majority of systems to have been essentially completed long ago, the formation of galaxy clusters seems at the present epoch to be only in a half-way stage. This is most clearly shown by the diagrams accompanying the catalogue of bright galaxies published by Shapley and Miss Ames (1932). The striking lack of randomness in the distribution of these relatively near-by galaxies is most probably the consequence of large-scale density or expansion fluctuations which existed already in the fireball stage of the universe, and is thus connected with a different property of the universe than the "turbulence" which at a later epoch gave birth to the individual spirals. Probably all the irregular, but quite pronounced, features seen in the abovementioned diagrams will ultimately collapse into more or less regular clusters. A remarkable characteristic of these structures is their strongly elongated shape. It will be shown elsewhere that such long arrays are the natural forms through which initial density or velocity fluctuations in the universe will evolve (cf. also Section 8).

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