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COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN

A new determination of the galactic pole and the distance of the sun from the galactic plane, by *J. J. M. van Tulder.*

Summary.

The galactic pole has been determined from all known objects of strong galactic concentration, as well as from planetary nebulae, from the directions of the motions of high velocity stars and from the zone of avoidance of extra-galactic nebulae. The individual results for the various types of stars are collected in Table 1 and the accompanying figure; within each type the stars have been divided into two or three groups, according to the distance. For those groups for which distances could be estimated, the distance z_0 of the sun to the galactic plane was also derived. The final results for the co-ordinates of the pole for 1900 and for z_0, with their external mean errors, are

$$\left.\begin{array}{l} \alpha = 12^h 44^m{\cdot}0 \pm 1^m{\cdot}1 \\ \quad \text{or } 191^\circ{\cdot}0 \pm 0^\circ{\cdot}3 \\ \delta = +27^\circ{\cdot}5 \pm 0^\circ{\cdot}2 \end{array}\right\} \text{from 23 groups}$$

$$z_0 = +13{\cdot}5 \text{ ps} \pm 1{\cdot}7 \text{ ps from 19 groups.}$$

This pole deviates 1°·0 from that used by OHLSSON in his *Lund Tables.*

A division of the material into three ranges of distance (average distances about 500, 1100 and 2300 ps) shows that both the pole and z_0 come out sensibly the same for these different distances (Table 3); the influence of the local cluster is largely confined to smaller distances than those considered in the present article.

I have also studied the distribution of the stars in the direction perpendicular to the galactic plane; care has been taken to reduce the influence of absorption as much as possible and to apply corrections for the errors in the assumed distances. The resulting distributions can be represented as the sum of two Gaussian distributions, the modulus of the first varying from 0·011 to 0·018 ps⁻¹, that of the second generally from 0·003 to 0·004 (Table 5). For all types considered the average distance from the galactic plane is near 50 ps, the extreme values are 37 and 77 ps. The corresponding average peculiar velocity in the z-direction is 3·8 km/sec.

Previous investigations.

More than 150 years ago the pole of the galaxy was determined by Sir WILLIAM HERSCHEL. Since then a large number of investigators have dealt with this subject. An extensive list of these investigations may be found in the introduction to EMANUELLI's „Tavole per la transformazione delle coordinate equatoriali in coordinate galattiche" [1]). Since 1929 the following determinations have been added:

VAN RHIJN	stars brighter than 4^m	$\alpha = 11^h\,11^m$	$\delta = +35^\circ{\cdot}2$	1)
VAN RHIJN	stars brighter than 5^m, 6^m	$\alpha = 12^h\,33^m$	$\delta = +26^\circ{\cdot}5$	1)
VAN RHIJN	stars brighter than 8^m, 10^m, 14^m, 16^m	$\alpha = 12^h\,55^m$	$\delta = +25^\circ{\cdot}5$	1)
PANNEKOEK	visual Milky Way	$\alpha = 12^h\,55^m$	$\delta = +27^\circ{\cdot}8$	2)
TRÜMPLER	open clusters	$\alpha = 12^h\,50^m{\cdot}4$	$\delta = +27^\circ{\cdot}7$	3)
WIRTZ	synoptical photographic Milky Way	$\alpha = 12^h\,47^m{\cdot}6$	$\delta = +31^\circ{\cdot}0$	4)
WIRTZ	Barnard's photographic Milky Way	$\alpha = 12^h\,56^m{\cdot}8$	$\delta = +28^\circ{\cdot}7$	4)
PARENAGO-GRIGORIEVA	long-period variables	$\alpha = 12^h\,39^m$	$\delta = +28^\circ{\cdot}0$	5)
MINEUR	δ Cephei variables	$\alpha = 12^h\,46^m$	$\delta = +27^\circ{\cdot}4$	6)

1) *Groningen Publ.* No. 43, 1929.
2) *Lembang Ann.* **2**, A72, 1929.
3) *Lick Bull.* No. 420, 180, 1930.
4) *V. J. S.* **65**, 64, 1930.
5) *A. J. S. U.* **14**, 335, 1937.
6) *Comptes Rendus*, 1941, p. 528.

The large deviations in some of the later determinations of the pole are probably due to the occurrence, in almost diametrically opposite parts of the sky, of the heavy obscuration in Ophiuchus and Taurus, respectively north and south of the galactic plane. PANNEKOEK [2]) has already remarked that this circumstance will shift the pole determined from the general distribution of galactic light towards higher right-ascensions; the same may hold for the general counts of faint stars, which form the basis of VAN RHIJN's third determination. The regions of the

1) *Publ. Specola Vaticana*, Series II, **7**, App. 1, 1929.
2) L.c. p. A73.

galactic system reached by studying the galactic light or the general star counts lie fully within the range of distances covered in the present article.

A list of various determinations of the distance $z_\circ$ of the sun from the galactic plane is given in an article by GERASIMOVIČ and LUYTEN[1]) who deduce an average value of $+ 33$ ps. In addition, the following determinations may be cited:

CHARLIER	bright B stars	$z_\circ = + 20$ [2])
SHAPLEY	B0–B5 stars	$z_\circ = + 15$ [3])
PARENAGO-GRIGORIEVA	long period variables	$z_\circ = + 25$ [4])
MINEUR	δ Cephei variables	$z_\circ = + 34$ [5])
TRÜMPLER	open clusters	$z_\circ = + 10$ [6])

The fact that the earlier studies gave a value of $z_\circ$ which is rather larger than that found at present may well be due to the neglect of the absorption and to overestimated distances.

Introduction.

A determination of the pole of the galaxy can be made most easily by determining the circle representing the maximum concentration of stars. If the sun itself would lie in this plane, a large circle would be found. But as the sun is situated at some distance, $z_\circ$, from this plane the formulae used in the present investigation contain a term which takes account of this fact.

In order to obtain the greatest accuracy the determination had to be made from objects which are strongly concentrated towards the galaxy. The following objects were selected: c stars, δ Cephei variables, O-type stars, Wolf-Rayet stars, B0 stars, B1-B2 stars, B0-B5 stars, planetary nebulae, the zone of avoidance shown by the extra-galactic nebulae and the directions of the high velocities.

A solution of rather different nature may be obtained from star-counts in high latitudes; a determination is given by J. H. OORT in a note following this article.

Nearly all types were sufficiently numerous to allow a division into several groups according to the distance, as derived from the apparent magnitude. For details about the material and the distances used see page 320 a.f. In total I made 23 solutions of the galactic pole. Four of these could not be used for the determination of $z_\circ$, viz. two derived from the planetary nebulae, whose distances were considered to be too uncertain, and those based on the zone of avoidance and on the motions of the high-velocity stars.

All computations have been made with the aid of the galactic co-ordinates referred to the pole $12^h\ 40^m$, $+ 28°$, which has the great advantage that it permits the use of the extensive tables by OHLSSON [1]).

The equation of condition.

The computations were based on the following equation of condition:

$$z = z_\circ + r \sin b \cos \Delta + r \cos b \cos l \sin \Delta \cos l_\circ + r \cos b \sin l \sin \Delta \sin l_\circ$$

in which:
r = distance of a star;
z = distance of a star from the true plane of symmetry;
$z_\circ$ = the distance of the sun from this plane, counted positive in the direction of the north galactic pole;
l, b = galactic co-ordinates for 1900, based on the pole $12^h\ 40^m$, $+ 28°$;
$l_\circ$, $90°-\Delta$ = galactic longitude and latitude of the pole which is to be determined (also based on the pole $12^h\ 40^m$, $+ 28°$).

Δ was assumed to be small, so that the equation of condition may be written:

$$z = z_\circ + r \sin b + (r \cos b \cos l) X + (r \cos b \sin l) Y \quad (1)$$

in which: $X = \sin \Delta \cos l_\circ$
$Y = \sin \Delta \sin l_\circ$

The solution is determined by the condition that Σz^2 must become a minimum. The equation in this form can be used only if accurate individual distances are known, which is only true for the group of δ Cephei variables nearer than 1000 ps. In all other cases the equation had to be written in the following form:

$$z/r = z_\circ/r + \sin b + (\cos b \cos l) X + (\cos b \sin l) Y \quad (2)$$

The corresponding condition is now that $\Sigma (z/r)^2$ becomes a minimum. In order to prevent that, with the often rather inhomogeneous distribution of stars, some intervals would obtain too much weight, the stars have been combined in intervals of 30° longitude; the same weight has been given to each 30° interval, irrespective of the number of stars contained in it. X, Y and $z_\circ/r$ were solved from the twelve equations corresponding to the averages of the twelve intervals of longitude; $z_\circ$ was obtained from $z_\circ/r$ by multiplication with $\bar{r}$ (see, however, also the last paragraph of the next section).

The solutions.

Objects showing great deviations from the adopted galactic plane have been excluded. The influence of these exclusions on the mean error has been duly investigated. The exclusion has been made in two approximations, as follows:

1) *Proc. Nat. Ac. Washington*, **13**, 387-390, 1927.
2) „Studies in stellar statistics"; *Lund Medd.* Series II, **14**, 41.
3) *Harv. Circ.* No. 239, 1922.
4) *A.J.S.U.* **14**, 335, 1937.
5) *Comptes Rendus*, 1941, 528.
6) *Lick Bull.* No. 420, 180, 1930.

1) *Lund Annals*, **3**, 1932.

1st Approximation. Exclusion of all stars with $|b| > 20°$ or $|r \sin b| > 500$ ps. This form was permissible, as the distance of the sun from the galactic plane had been found to be small. A disadvantage of the introduction of the first condition, $|b| > 20°$, is that among the nearby stars it excludes also objects which in reality are near the galactic plane. The condition can be avoided if individual distances are used; this has been done in the case of the nearer δ Cephei variables.

The mean errors ε were computed from the residuals of the twelve intervals of longitude. The results for the various groups were then combined with weights p ($p = 1/\varepsilon^2$) into the following first approximation:

$$l_\circ = 330°; \Delta = 1°\cdot1; z_\circ = 20 \text{ ps.}$$

2nd Approximation.

a) In each group I now computed $-20/\bar{r}$. The corresponding angle in degrees gives the 'dip' of each group, i.e. the angle by which the circle of maximum concentration deviates from a large circle. The corresponding limits of exclusion were now fixed at $b < -20° + \text{dip}$, and $b > +20° + \text{dip}$.

b) In the second place, the limits of exclusion were brought into agreement with the new position of the pole, which gave limits varying with l.

c) The limits of exclusion for $r \sin b$ were also fitted to the value of $z_\circ$ found, so that all stars were excluded which lay beyond -520 and beyond $+480$ ps. The number of stars finally excluded in each group is given under N_e in Table 1; N_u gives the number of stars used in the solution.

New least-squares solutions were then made; the results with their mean errors are given in Table 1. The corresponding values of Δ and $l_\circ$ are given in the last two columns of that table.

The mean errors found from the solutions will in general be too small, because the residuals on which they are based were found after the exclusion of the most deviating stars. In order to obtain an estimate of the measure in which the mean errors had been flattered by these exclusions, I have investigated the change in the residuals brought about by re-introducing the excluded stars. The sum of the squares of the residuals $\Sigma (z/r)^2$ was then found anew, but instead of dividing by $12-3$ as in the original solution with 3 unknowns, these sums were now divided by:

$$12-3+\frac{\text{number of changed sectors}}{4},$$ because no new solutions of the unknowns had been made. The mean errors of unit weight found in this way, divided by the values found previously for this same quantity, gave the correction factors to be applied to the mean errors of the original solutions. These factors are given in the column marked c.f. in Table 1. It accidentally happens in some cases that the average residuals in various longitude intervals are diminished by adding the excluded stars; this explains that the numbers in this column are sometimes smaller than unity. New weights were derived with the aid of the values of c.f. just found. The true weights will in general be between the old and new values. The weights assumed for the final solution are the geometrical means of the old and new weights. These final weights are given under p_x, p_y and $p_{z_\circ}$ in the table; the mean errors given are the old, uncorrected values.

Table 2 shows the weighted averages $\bar{X}$, $\bar{Y}$, and $\bar{z}_\circ$ of all solutions given in Table 1. The mean errors in the 3rd column are those computed from the weights in Table 1, while the external mean errors in the last column were derived by forming for each of the 23 groups the residuals $X-\bar{X}$, $Y-\bar{Y}$ and $z_\circ-\bar{z}_\circ$; they were found from the formula $\frac{\Sigma p (X-\bar{X})^2}{23 \; \Sigma p}$, and similar expressions with Y and $z_\circ$.

Solutions were also made using weights corresponding to the original, uncorrected mean errors, as well as with weights corresponding to the mean errors obtained by applying the full correcting factors. The results deviated only little from those given in Table 2.

The values given in Table 1 have been plotted in Figure 1. The radii of the circles are equal to the probable errors (two thirds of the mean errors in Table 1). The numbers refer to the groups in Table 1. The filled circle shows the weighted average of 23 groups and its probable error. The cross marks the position of the pole computed by NEWCOMB [1]).

I have investigated whether there is any dependence of the position of the pole, or of $z_\circ$, upon the distance. To this end the groups of Table 1 were divided into three ranges of distance as indicated in Table 3, which gives Δ, $l_\circ$, as well as the equatorial co-ordinates of the average poles; the mean errors were computed from the corrected weights. It will be seen that neither the pole nor $z_\circ$ is much dependent upon the distance. The slight deviation of the pole for the smallest distances may be due to an influence of the Orion group, which tends to displace the pole in the direction of 170° longitude. There is not much evidence of the so-called "local cluster" (SHAPLEY), which has a pole at about $\Delta = 15°$, $l_\circ = 160°$. This "local cluster" has been found very pronounced among the nearer B-type stars, which have not been included in the present investigation. It may be noted that neither the near-by group of late-type

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[1]) „On the position of the galactic and other principal planes", Washington, D.C. 1904.

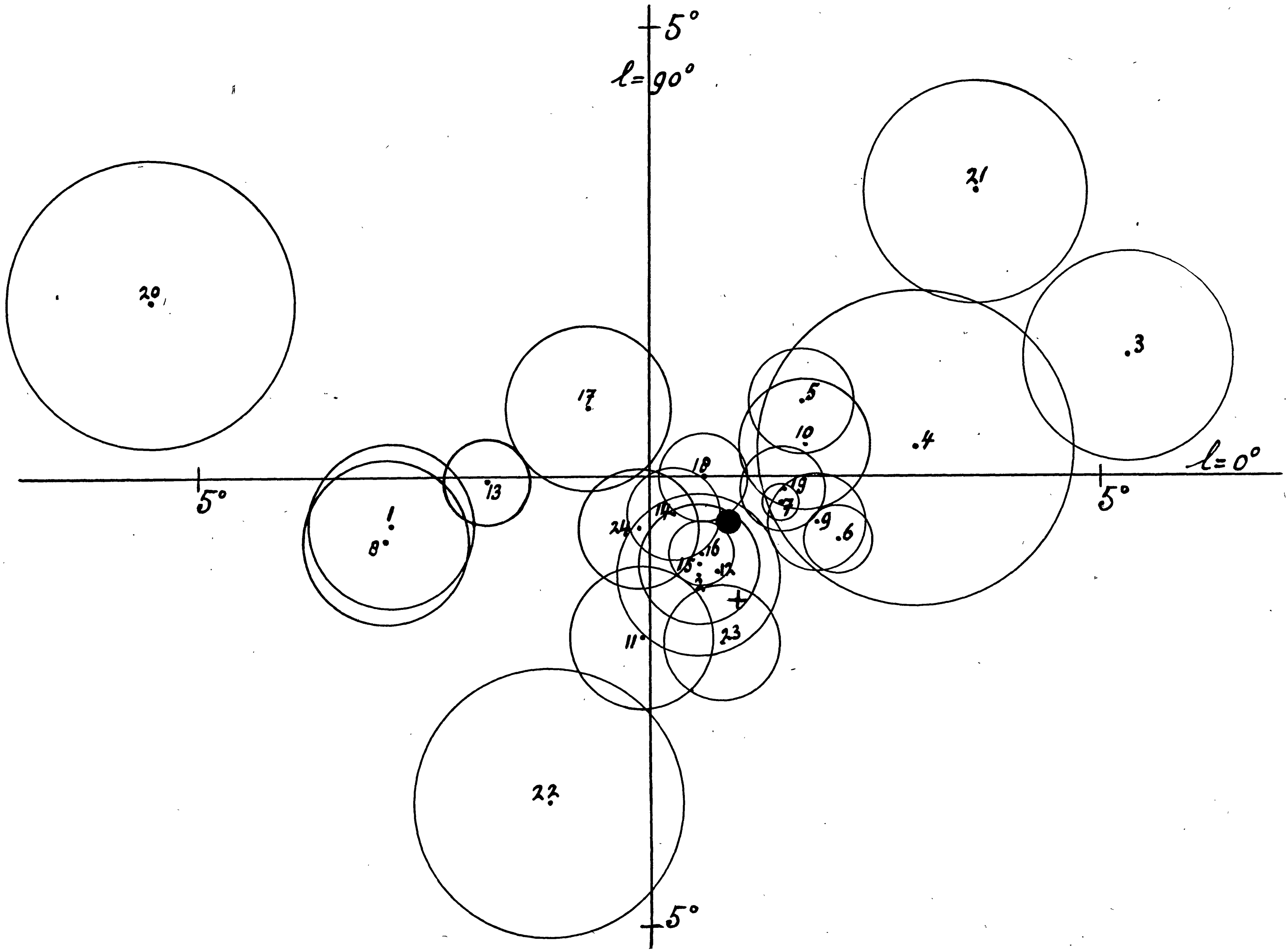

FIGURE 1

Positions of poles for the separate groups.

The centre of co-ordinates is OHLSSON's pole ($12^h\ 40^m$, $+\ 28°$). The numbers correspond to the groups in Table 1; the radii of the circles represent probable errors. The black dot indicates the pole finally adopted. NEWCOMB's pole is shown by a cross ($12^h\ 44^m{\cdot}4$, $+\ 26°{\cdot}8$).

TABLE I.

No.	type	r (ps)	$\bar{r}$ (ps)	N_u	N_e	c.f.	X (radians)	ε_X (radians)	$p_X \times 10^{-2}$	Y (radians)	ε_Y (radians)	$p_Y \times 10^{-2}$	z_0 (ps)	ε_{z_0} (ps)	$p_{z_0} \times 10^4$	Δ	l_0
1	cB0—cA6	0—1200	697	80	6	2·24	− ·050	± ·024	8	− ·010	± ·022	9	+ 31·6	± 10·9	38	2°·9	191°
2	cB0—cA6	> 1200	1931	65	6	1·75	+ ·007	± ·025	9	− ·020	± ·024	10	+ 11·0	± 33·4	5	1·2	298
3	cA7—cK	0— 350	187	73	11	1·55	+ ·093	± ·027	9	+ ·023	± ·029	8	+ 17·7	± 3·7	471	5·5	14
4	cA7—cK	> 350	764	72	14	1·48	+ ·053	± ·045	3	+ ·005	± ·042	4	+ 18·1	± 23·4	12	3·0	6
5	δ Cephei	0—1000	607	78	0	1·00	+ ·030	± ·015	46	+ ·015	± ·013	61	+ 25·9	± 6·4	244	1·9	26
6	δ Cephei	1000—1900	1430	96	0	1·00	+ ·037	± ·010	111	− ·011	± ·010	111	+ 14·3	± 9·6	109	2·2	343
7	δ Cephei	> 1900	2696	89	33	3·08	+ ·026	± ·006	85	− ·005	± ·006	82	+ 48·0	± 11·9	24	1·5	349
8	B1—B2	0— 700	460	81	12	1·46	− ·051	± ·022	14	− ·011	± ·023	13	+ 14·1	± 7·3	129	3·0	193
9	B1—B2	700—1100	902	79	5	2·09	+ ·032	± ·013	28	− ·008	± ·012	31	+ 9·4	± 7·9	77	1·9	346
10	B1—B2	> 1100	1446	73	4	1·62	+ ·032	± ·018	19	+ ·007	± ·018	20	+ 10·0	± 18·4	18	1·8	12
11	B0	0—1250	853	108	2	0·90	− ·001	± ·021	26	− ·030	± ·020	28	+ 12·7	± 12·4	73	1·7	269
12	B0	> 1250	1682	110	5	1·61	+ ·013	± ·008	95	− ·018	± ·008	105	+ 17·3	± 9·4	71	1·3	305
13	B0—B5	....	345	564	0	....	− ·032	± ·012	68	− ·001	± ·012	68	+ 15·3	± 3·0	1111	1·8	181
14	B0—B5	....	500	719	0	....	+ ·005	± ·011	86	− ·006	± ·011	86	+ 10·9	± 3·9	657	0·5	305
15	O	....	1261	78	4	0·93	+ ·010	± ·017	48	− ·017	± ·017	53	+ 23·5	± 13·1	62	1·1	301
16	WOLF-RAYET	....	1430	91	0	1·00	+ ·010	± ·011	87	− ·015	± ·008	164	+ 31·7	± 9·4	56	1·0	304
17	open clusters	0—1250	778	104	8	1·39	− ·012	± ·023	17	+ ·014	± ·022	15	+ 9·6	± 12·5	46	1·0	131
18	open clusters	1250—2250	1705	104	1	0·88	+ ·010	± ·011	93	+ ·000	± ·011	98	+ 22·0	± 13·1	66	0·6	0
19	open clusters	> 2250	3310	108	9	1·48	+ ·027	± ·010	64	− ·002	± ·010	66	+ 21·2	± 23·8	12	1·5	357
20	plan. nebulae	....	....	65	2	0·93	− ·095	± ·037	8	+ ·033	± ·042	6	....	....	....	5·8	161
21	plan. nebulae	....	....	53	12	2·10	+ ·062	± ·030	5	+ ·054	± ·030	5	....	....	....	4·8	41
22	extra-gal. neb.	....	....	....	....	....	− ·019	± ·035	8	− ·064	± ·058	3	....	....	....	3·8	253
23	high velocities	....	....	456	79	1·14	+ ·012	± ·014	45	− ·033	± ·016	34	....	....	....	2·0	290
24	stars at large distances from the galactic plane						− ·002	± ·017	....	− ·010	± ·017	....	....			0·6	281

c stars, nor the nearest δ Cephei variables show any signs of it. That the B0—B5 stars between $6^{m}{\cdot}26$ and $7^{m}{\cdot}25$ (group 13 of Table 1) show so little evidence of it, is in part due to the treatment of the clusters, by which the weight of the Orion stars was much reduced.

As there is some danger that for objects showing only moderate concentration towards the galactic circle the position of the pole is influenced by the irregular distribution of absorbing material or of the stars themselves, I have also made a solution in which the groups 3, 8, 13, 14, 17, 20, 21, 22, for which there is an appreciable spread in latitude (c.f. Table 5), were omitted. This solution gave

$$\alpha = 191^{\circ}{\cdot}2 \pm 0^{\circ}{\cdot}2;\ \delta = +27^{\circ}{\cdot}4 \pm 0^{\circ}{\cdot}2.$$

Comparing this with the pole resulting from the solution in which all groups were used we may conclude that a pole at

$$\alpha = 191^{\circ}{\cdot}0 \pm 0^{\circ}{\cdot}2 \text{ (m.e.)};\ \delta = +27^{\circ}{\cdot}5 \pm 0^{\circ}{\cdot}2 \text{ (m.e.)}$$

will probably be somewhat better than that used in OHLSSON's tables.

The values of z_0 in Table 1 were found by multiplying $\overline{(z_0/r)}$, obtained from equation (2), by the values of $\bar{r}$ given in the third column. Strictly the factor should have been the inverse mean parallax, which is a little smaller than the mean distance. The ratio $\bar{r} : (1/\overline{\pi})$ was empirically determined for groups 3 and 5; a ratio of 1·35 was found. The accidental errors in the absolute magnitudes and absorptions tend to make this result too high, and a ratio 1·2 ± ·1 (m.e.) was finally adopted. The distance of the sun to the galactic plane was thus found to be + 13·5 ps ± 1·9 (m.e.).

TABLE 2.

Comparison of internal and external mean errors of the pole (in radians) and of z_0 (in parsecs)

	mean value	mean error from Table 1	external mean error
$\overline{X}$	+ ·0126	± ·0032	± ·0049
$\overline{Y}$	− ·0078	± ·0030	± ·0026
z_0	+ 16·2	± 1·8	± 1·6
z_0 corr.	+ 13·5	± 1·9	± 1·7

TABLE 3.

Results for various distance groups

	range of $\bar{r}$	mean distance	Δ	l_0	α	ε_α	δ	ε_δ	z_0	ε_{z_0}	z_0 corr.	ε_{z_0} corr.
			°	°	°	°	°	°	ps	ps		
I	0— 750	466	0·2	164	189·8	± 0·4	+ 28·1	± 0·4	15·8	± 1·9	—	—
II	750—1500	1108	1·2	327	191·1	± 0·3	+ 27·3	± 0·3	16·1	± 4·7	—	—
III	>1500	2265	1·1	332	191·1	± 0·3	+ 27·5	± 0·3	23·3	± 7·5	—	—
All		1210	0·8	328	190·8	± 0·2	+ 27·6	± 0·2	16·2	± 1·8	13·5	± 1·9

Discussion of the material and the distances.

The corrections for absorption needed in the computation of the distances were either determined with the aid of the published colour-excesses, or, when these were not available, a general absorption of $1^{m}{\cdot}0$ per 1000 ps was assumed unless otherwise stated; it was further assumed that the absorbing layer did not extend beyond $z = \pm 150$ ps.

(a) Stars with c-characteristic.

The stars were collected successively from catalogues by MERRILL [1]); by ADAMS, JOY, HUMASON and BRAYTON (*Mt Wilson Catalogue of spectroscopic absolute magnitudes*) [2]); by STEBBINS, HUFFER and WHITFORD [3]); by R. E. WILSON [4]); and by Miss PAYNE [5]). From the latter catalogue only those stars with spectral lines marked "narrow" and "very narrow" were used (c.f. R. E. WILSON, l.c. p. 215). The estimated distances were obtained in three different ways, as follows:

For the early-type stars contained in the catalogue of STEBBINS, HUFFER and WHITFORD the distance was directly computed from the modulus $m_0 - M$ given in that catalogue; they used $M = -5{\cdot}5$. For the stars taken from the *Mt Wilson Catalogue of spectroscopic absolute magnitudes* the distance was computed from the values of M and m given in this publication, while for the remaining stars the absolute magnitudes were taken from Table 9 of R.E. WILSON's study (the adopted luminosities were roughly: B0—B5 −5·3; B7—A5 − 4·8; A6—F7 − 4·4; F8—G6 − 3·6; K0—K5 −2·3). Corrections for absorption were applied. The distances in this group are thus not quite homogeneous, but this is no great disadvantage, as the entire material of c stars is inhomogeneous in the way in which they have been discovered.

In order to prevent a disproportionately large number of exclusions the limits of exclusion were fixed at $b = \pm 30°$ for the c stars; the limits for $r \sin b$

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1) *Ap. J.* **81**, 351, 1935; Mt W. Contr. No. 512.
2) *Ap. J.* **81**, 187, 1935; Mt W. Contr. No. 511.
3) *Ap. J.* **91**, 20, 1940; Mt W. Contr. No. 621.
4) *Ap. J.* **93**, 212, 1941; Mt W. Contr. No. 643.
5) „Stars of high luminosity", p. 287, 1930.

were correspondingly enlarged to $500 \frac{\text{tg } 30°}{\text{tg } 20°} =$ 790 ps.

(b) δ Cephei variables.

These were taken from the *Katalog und Ephemeriden* for 1941, excluding variables belonging to the Magellanic Clouds. Only stars with periods in excess of $2^{d}\cdot5$ were taken, as the galactic distribution of variables with periods between $1^{d}\cdot0$ and $2^{d}\cdot5$ seemed to show more resemblance to that of RR Lyrae variables than to that of galactic δ Cephei variables. The absolute magnitudes were taken from SHAPLEY [1]), the median photographic magnitudes from a list by JOY [2]), or from the *Katalog und Ephemeriden*, while the absorption was computed in the way indicated in the beginning of this section. For the group with $r < 1000$ ps, where the absorption effects are moderate, individual distances were used for the solution.

(c) B1—B2, B0 and O stars.

These were in the first place compiled from the list of colours by STEBBINS, HUFFER and WHITFORD [3]), which is practically complete for all stars of these types contained in the *Draper Catalogue* and having declinations above —40°. For the part of the sky south of this limit the stars of these types were collected from the *Henry Draper Catalogue*. For the stars taken from the list of colours cited the distances were calculated from the moduli given in that list. These moduli had been determined with the aid of the following values of the absolute magnitudes: O —4·5; B0 —3·9; B1 —3·6; B2 —3·0. For the southern stars, for which no colour-excesses were available, the average absorption was estimated from comparison with a northern region lying on the opposite side of the galactic centre and at the same distance in longitude from it. In the latter region the average absorption estimated from the colour-excesses was $1^{m}\cdot4$ per 1000 ps; this same value was adopted for the southern regions; the distances were then found with the aid of the above absolute magnitudes. I have checked that there was no large difference between the mean distances in this southern region and those in the remainder of the sky, so that it was not necessary to apply corrections for the differences in depth between the northern and the southern parts of the *Draper Catalogue*.

Before starting the computation of the mean longitudes and latitudes in the various intervals of longitude, I have tried to diminish as far as possible the effects of clustering, which are pronounced among these types. In order to prevent arbitrariness, I have simply assumed that within each group of magnitude and sub-type all stars within a projected surface extending over 50 ps in longitude and 50 ps in latitude belonged together if the estimated distances were not more than 50 % larger or not more than 30 % smaller than the average distance corresponding to the "cluster"; the limits in radial distance were taken so large because of the uncertainty of the individual distances. Each cluster was given the weight of a single star.

(d) B0—B5 stars.

The data for this determination were borrowed from an article by SHAPLEY [1]). SHAPLEY gives median latitudes for 30° intervals of longitude in four magnitude groups. I have used only the two faintest groups, from $6^{m}\cdot26$ to $7^{m}\cdot25$ and from $7^{m}\cdot26$ to $8^{m}\cdot25$. The two brightest groups were omitted because of the overwhelming influence of the local cluster. In the fainter groups the Orion cluster has still a considerable influence on the corresponding interval of longitude. Because we had already for the O—B2 stars adopted the principle that clusterings of stars should be combined, it was decided to combine in this case the stars from 166° to 177° longitude and —14° to —20° latitude into a single average for each of the two magnitude intervals, and to recompute the median latitudes in the corresponding interval, counting the Orion cluster as one star.

The mean distances given in Table 1 were computed by adopting a mean absolute magnitude of $-1^{M}\cdot2$ and a general absorption of $1^{m}\cdot0$ per 1000 ps in the galactic layer, using the rough equation $M = m + 5 - a - 5 \log \bar{r}$.

(e) Wolf-Rayet stars.

This determination is based on the catalogue given by Miss PAYNE [2]), omitting of course the stars belonging to the Magellanic Clouds. The mean distance was again computed with the aid of the above formula, assuming $M = -3\cdot3$ in agreement with Miss PAYNE (l.c. p. 78). On account of the great uncertainty of the absolute magnitude the final weight of z_0 was halved. These stars are so unevenly distributed that it was not practicable to divide them into the twelve usual intervals of longitude. The solution was therefore made from the individual stars, instead of from average latitudes.

(f) Open clusters.

The open clusters were taken from TRÜMPLER's catalogue [3]). The final distances given in that catalogue were adopted.

(g) Planetary nebulae.

The material was taken from a catalogue by H. D. CURTIS [4]), supplemented by some recent

1) „Star Clusters", p. 135, 1930.
2) *Ap. J.* **89**, 356, 1939; *Mt W. Contr.* No. 607.
3) *Ap. J.* **91**, 20, 1940; *Mt W. Contr.* No. 621.

1) *Harv. Circ.* No. 239, 1922.
2) „Stars of high luminosity", p. 64.
3) *Lick Bull.* **14**, 170, 1928-30.
4) *Lick Publ.* **13**, 55, 1918.

discoveries. Because no trustworthy distances could be assigned a division was made according to diameters, and no solution of z_0 was given. The first group in Table 1 contains the nebulae with diameters in excess of 10″, while in the second the nebulae with diameters of 10″ and smaller are included.

(h) Zone of avoidance of extra-galactic nebulae. In this solution the pole is determined from the plane of concentration of the absorbing material. With the aid of HUBBLE's counts of nebulae [1]), the average number of nebulae for latitudes at intervals of 10° was formed in each 10° interval of longitude. These average numbers were plotted against b, and the points of symmetry were determined, corresponding to the minimum density of nebulae. These points of symmetry were then averaged for 30° intervals of longitude; the solution of the pole was based on these averages. As HUBBLE's counts do not extend south of −30° declination the solution had to be confined to the region between 340° and 210° longitude. As the distance of the absorbing material was unknown no solution of z_0 could be made.

(i) Stars of high velocity.

The material was taken from an article by MICZAIKA [2]). It has long been known that the directions of the motions of high velocity stars show a marked concentration to the galactic plane [3]), and it seemed of interest to make a solution of the galactic pole based on these motions, because the character of such a solution is so entirely different from all other solutions given. In this solution z_0 was put equal to zero, and because of the moderate galactic concentration the limit of exclusion for the apices was set at ± 30° latitude. On account of the asymmetry in the motions of the high velocity stars only eight 30° intervals (from 350° to 110°) could be used.

The spread of the stars around the galactic plane.

I have tried to represent the distribution in the z-direction, of the various types of stars considered in the present article, by the sum of two Gaussian functions. We may distinguish two categories:

1. Objects with well-determined distances, among which alone the δ Cephei variables and the open clusters were counted.

2. The remaining objects, for which there is a considerable dispersion in absolute magnitude, so that the true distances show a certain spread around the assumed values.

In this second group the observed distributions in the z-direction should first be corrected for the effect of the accidental errors. In practice it appeared to be more convenient to follow the inverse way, viz. to superimpose the effect of the errors upon Gaussian curves, and to compare these with the observed distributions. By a method of trial and error it was then fairly easy to find the moduli of the Gaussian curves which would fit the observed curves. I have assumed throughout that the true absolute magnitudes were, for each type, distributed over a normal error curve with a dispersion of ± $1^{M}\!\cdot\!0$ around the assumed mean absolute magnitude.

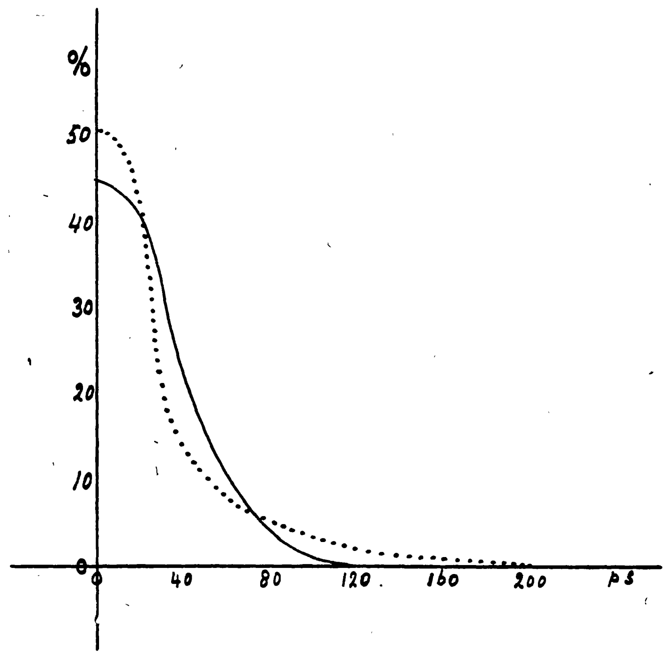

FIGURE 2
Gaussian curve for $h = 0{\cdot}020$ (full-drawn)
Corrected curve for $h = 0{\cdot}020$ (dotted)

As an illustration of the effect of the accidental errors, Figure 2 shows an error curve with modulus $h = {\cdot}020$ ps^{-1} (full-drawn curve) and also the same after the effects of accidental errors have been superimposed (dotted curve).

The material used has been confined to as small distances as possible. The large distances become increasingly uncertain on account of the absorption, while also the number of stars near the galactic plane becomes more and more incomplete compared to that farther away from the plane, thus falsifying the observed distribution. The investigation has therefore been restricted to stars situated in a cylinder whose axis is perpendicular to the galactic plane and passes through the sun. In two cases the radius was taken 500 ps, in the other cases it had to be taken as large as 1000 ps in order that a sufficient number of stars might be reached. However, in Table 4 a

1) *Ap. J.* **79**, 8, 1934; *Mt W. Contr.* No. 485.
2) *A. N.* **270**, 249, 1940.
3) OORT, *Groningen Publ.* No. 40, 1926.

comparison has been given of the z-distribution in cylinders of 500 and 1000 ps radius, from which it is apparent that there are no great differences.

In order to strengthen the determination of the densities at higher values of z the stars with latitudes larger than 21°·7, where the absorption effects have become negligible, were all counted, also when they were situated outside the cylinder. The numbers were afterwards reduced to the corresponding parts of the cylinder. For the interval from $z=200$ to $z=500$ the reduction factor to the cylindrical space of 500 ps radius is ·308, to that of 1000 ps radius ·917. For the interval from $z=500$ to $z=1000$ the factors are ·069 and ·274. The angle 21°·7 was chosen because with a radius of 500 ps the corresponding value of z is just 200 ps.

The columns O in Table 5 show the counted numbers of stars between different limits of z, reduced in the manner described; z has been counted from the median value of z in each type. The columns C give the computed distributions. For the first two types these are simply sums of two Gaussian components; for the other five types the effect of the dispersion of the observed distances has been superimposed in the calculation of the columns C. The moduli h of the Gaussian components, as well as the numbers N belonging to each of the components are given in the lower part of the table.

It should be noted that the determination of the percentage of stars in the Gaussian component with the smallest modulus is always rather uncertain, in particular because it is so difficult to decide how many of the large values of z are still due to errors in the distances. In several cases a rather different picture of the second Gaussian component would be obtained if a somewhat different dispersion were assumed for M.

TABLE 4.

Comparison of the distribution in the z-direction: I in a cylinder of 500 parsecs radius (for O type 750 ps); II between two cylinders of 500 and 1000 parsecs radius (for O type 750 and 1000 ps)

$\|z\|$	δ Cephei		open clusters		cB–cA7		O		Bo	
	I	II	I	II	I	II	I	II	I	II
0— 20	6	14	8	9	7	24	4	6	9	13
20— 40	11	12	7	11	4	9	4	6	3	11
40— 80	7	16	6	19	7	6	8	2	7	9
80— 120	3	6	6	6	2	4	1	1		11
120— 200		4	3	6	0	3	2	1		8
200— 500				5	3	1		1		
500—1000					1	0				
0—1000	27	52	30	56	24	47	19	17	19	52

TABLE 5.

Distribution of stars in the direction perpendicular to the galactic plane

$\|z\|$	δ Cephei		open clusters		cB—cA6		O		Bo		cA7—cK		B1—B2	
	O	C	O	C	O	C	O	C	O	C	O	C	O	C
0— 20	20	20·9	17	17·5	31	28·2	10	15·8	22	27·1	50	50·9	11	21·2
20— 40	23	18·7	18	16·1	13	12·6	10	6·8	14	12·7	23	21·7	14	10·2
40— 80	23	25·4	25	24·3	13	13·2	10	6·8	16	14·2	21	20·8	17	11·8
80— 120	9	10·5	12	13·3	6	6·6	2	3·2	11	7·5	12	9·5	10	6·5
120— 200	4	3·3	9	9·7	3	5·5	3	2·4	8	6·5	4	7·0	5	5·9
200— 500	0	0·2	6·4	6·4	3·6	3·4	1	1·0	0	3·0	2·8	2·7	1·5	3·0
500—1000	0	0·2		0·1	0·8	1·1	0·3	0·3	0·3	0·3	0·6	0·6	0·5	0·3
>1000	0·5	0·4			0·3	0·3						0·1		
N	0·8 + 78·7		25·0 + 62·4		8·7 + 62·0		2·0 + 34·3		1·6 + 69·7		6·4 + 106·9		1·5 + 57·5	
h	·0005 + ·012		·004 + ·011		·003 + ·016		·003 + ·017		·003 + ·013		·004 + ·018		·003 + ·012	
$\overline{\|z\|}$	58		77		53		42		46		37		51	
$\overline{\|Z\|}$	4·2		5·5		3·8		3·0		3·3		2·7		3·7	
radius	1000		1000		1000		1000		1000		500		500	

The two bottom lines of Table 5 show the average distances z corresponding to the average true distribution as given by the Gaussian components, and the corresponding values of the average velocities Z in the z-direction. The latter were computed from the values of z with the aid of a table given in an article by OORT [1]). It is remarkable to note how little the average distances and velocities in the z-direction differ for the various types of stars considered. The total average velocities are always quite small; the average value of z corresponding to the second modulus of ·0035 alone would be 161 ps, corresponding to an average velocity of 10·0 km/sec.

In conclusion I should like to express my gratitude to Professor J. H. OORT, on whose initiative this investigation was undertaken, and who has helped me with constant advice and many valuable suggestions.

1) *B.A.N.* No. 238, p. 282, 1932.