

# BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS

1940 October 15

Volume IX

No. 332

## COMMUNICATION FROM THE OBSERVATORY AT LEIDEN

### Photographic photometry of the eclipsing variable WW Draconis = $\Sigma$ 2092 A, by *L. Plant.*

From observations on plates taken with the 1-inch Cooke and 16-inch Metcalf cameras of the Harvard College Observatory Miss HARWOOD found that the southern component A of the visual binary  $\Sigma$  2092 (B.D. + 60° 1691, 8<sup>m</sup>.2 = H.D. 150708, G5) is an eclipsing variable and gave a period of 3<sup>d</sup>.501 (*Harv. Circ.* No. 194, 1916; *A.N.* 207, 217, 1918). The magnitude of the combined light of both visual components was found to vary from 8<sup>m</sup>.5 to 9<sup>m</sup>.1. To this star the name WW Draconis has been given in the 21st "Benennungsliste" (*A.N.* 224, 129, 1925).

The observations of the position angle and distance of the visual binary  $\Sigma$  2092 have been collected in Table I. The fourth and fifth columns give the estimated visual magnitudes of the components, with the exception of the eighth line of these columns, where the photographic magnitudes of the Astrographic Catalogue are given. No relative motion of the two components has been observed. The proper motion of component A is given as follows in B. Boss' General Catalogue:

$$\mu_{\alpha} = +^{\circ}0017, \mu_{\delta} = -^{\circ}065$$

$$\pm 9, \quad \pm 4 \text{ (m.e.)}$$

It is probable that the two stars form a physical pair.

Since the announcement of the variability of WW Draconis by Miss HARWOOD the following has been published about this star:

GRAFF (*V.J.S.* 63, 164, 1928) mentions that he has measured the magnitudes of a sequence of comparison stars.

KUKARKIN (*N.N.V.S.* 1, No. 12, 1929), PARENAGO (*N.N.V.S.* 3, No. 25-6, 1930), HOFFMEISTER (*Sonneberg Mitt.* No. 20, 18, 1931), KANAMORI (*Kyoto Bull.* No. 247, 1933) and BEYER (*G.u.L.* II, 2, 90, 1934) give notes on small numbers of unpublished visual estimates.

ZVEREV (*N.N.V.S.* 4, No. 43, 1933 and 5, No. 52, 1937) found that the period 3<sup>d</sup>.501 is erroneous. He derives the new elements: Min. I = J.D. 2427344.447 + 4<sup>d</sup>.62963 E, visual maximum brightness 8<sup>m</sup>.84, range of the primary minimum = <sup>m</sup>.74, of the

TABLE I.  
Double star measurements of  $\Sigma$  2092 = A.D.S. 10152.

epoch	$p$	$d$	$m_A$	$m_B$	$n$	authority
			<sup>m</sup>	<sup>m</sup>		
1831.10	5.9	8.04	7.7	8.8	3n	STRUVE, <i>Mensurae micrometricae</i> , p. 158 (1837).
1845.60	5.7	8.44	—	—	1n	MÄDLER, <i>Untersuchungen über die Fixsternsysteme</i> , I, 56 (1847).
1866.20	5.7	7.96	8.0	9.0	3n	DEMBOWSKI, <i>Misure micrometriche</i> , 2, 379 (1884).
1872.9	6.0	8.64	8.7	9.3	2	<i>A. G. Katalog, Helsingfors</i> , Nr. 8924-5 (1890).
1878.42	5.2	8.18	—	—	1n	BURNHAM, <i>Mem. R.A.S.</i> 44, 210 (1879).
1879.29	4.0	8.06	8.3	8.5	1n	BURNHAM, <i>Mem. R.A.S.</i> 47, 290 (1882).
1883.22	5.7	8.03	—	—	2	SEAGRAVE, <i>Sidereal Messenger</i> , 2, 275 (1883).
1902.52	7.7	8.15	8.6	9.5	2n	<i>Cat. Astrografico Sez. Vaticana</i> , App. III, 60° 26322-3, 61° 25985-6 (1926).
1903.57	2.8	8.49	7.8	8.4	3n	ESPIN, <i>M.N.</i> 64, 677 (1904).
1905.15	5.0	8.25	—	—	2n	BURNHAM, <i>Double Star Catalogue</i> II, 724 (1906).
1915.39	3.8	8.26	—	—	3n	FRANKS, <i>M.N.</i> 76, 32 (1915).
1923.83	4.6	8.07	—	—	4n	PEEK, <i>M.N.</i> 81, 171 (1926).
1937.63	5.11	8.235	—	—	2pl	HERTZSPRUNG, <i>B.A.N.</i> No. 330 (1940).

secondary =  $m.14?$ , duration of the primary minimum  $P.08$ , and of totality  $P.02$ .

As mentioned in the Annual Report of the Mt. Wilson Observatory 1937-8 and in the Draft Report of the I.A.U. 1938, p. 240, observations of the radial velocity are or will be made at the Mt. Wilson Observatory.

Miss HARWOOD and MESSRS ZVEREV, JOY, HOFFMEISTER, BEYER and KANAMORI have been so kind to send me their unpublished observations. The dates of eleven minima out of 312 measurements by Miss HARWOOD on Harvard plates, and of two minima out of the visual estimates of HOFFMEISTER and BEYER are given in Table 11.

ZVEREV of the Sternberg Observatory at Moscow made 148 visual estimates, 39 of which were made with a magnification great enough to separate the two components. The magnitudes of the six comparison stars were determined by a comparison with the North Polar Sequence by the aid of a Graff photometer. The amplitude of the primary minimum is  $m.65$ , while that of the secondary could not be determined accurately. The minimum epochs deduced from this material together with an additional one found by ZVEREV on an older Moscow plate are given in Table 11.

Photographic observations of WW Draconis have been made with the Leiden photographic refractor ( $a = 32$  cm,  $f = 524$  cm) during three oppositions, viz. from 1934 April 2 to 1936 September 14. In total 5882 photographic exposures (14 by Prof. ZAGAR, 729 by Dr. WESSELINK, 5139 by the writer) have been made on 268 plates in 117 nights. Furthermore 18 photovisual exposures on 6 plates in 5 nights should determine the difference in colour index between the visual components. These exposures were taken on Eisenberger Ultrarapid and Voigtländer Illustraplates combined with a yellow screen OG1 of Schott und Gen. at Jena.

The photographic observations have been made on Guilleminot La Superguil plates, size  $9 \times 12$  cm. These plates are extremely fast but have a large grain and often an irregular plate fog. Unfortunately the plates received before and after July 1935 have a different colour sensitivity. The sensitivity curves for the two different emulsions, called GLS 1934 and GLS 1936, are given by WESSELINK in *B.A.N.* No. 294, 1937. The later plates proved to be sensitized at longer wavelengths.

It was the intention to make two or three series of 10 exposures on each plate. In many cases clouds or moonshine did not allow this. Exposures which have been disturbed by clouds have not been measured.

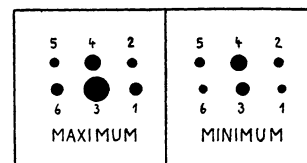
Mostly the exposure time was 75 or 90 seconds. In

the case of poor transparency and during minimum longer exposures up to 225 seconds were taken. The total time of exposure is 531285 seconds or a little more than six days. The plates have been taken two millimetres inside the photographic focus. A coarse grating with  $d = 1 = 3.8$  mm made of stainless steel has been put in front of the objective with the bars perpendicular to the declination circles. A schematic image of exposures at maximum and at minimum is given in Figure 1. The plates have been developed for 8 minutes in Agfa Rodinal 1:20 at a temperature of  $18^\circ$  C.

From the distance of the two first order images the effective wavelength of the exposures has been determined to be 4270 Å.

The plates have been measured with the second Schilt photometer of the Leiden Observatory. A diaphragm of 5 mm or  $.14$  mm as projected on the plate has been used. The B component has been used as the only comparison star. The variable component is photographically brighter than this star in maximum and fainter in minimum. The use of a comparison star which is situated close to the variable has many advantages (compare WESSELINK, *Leiden Ann.* XVII, 3), but in the case of WW Draconis the distance of both stars is only  $8''$  or  $.2$  mm on the plate. Systematic errors may be caused by this fact. The images have been measured in the order indicated in Figure 1. The galvanometer readings of the central images and the means of those of the first order images

FIGURE 1.



have been converted into provisional magnitudes by the aid of a table like that described by WESSELINK (*B.A.N.* No. 318, 1939, see also *B.A.N.* No. 190, 1930). To these provisional magnitudes the "parabola" method of HERTZSPRUNG (*A.N.* 190, 121, 1911) has been applied in order to obtain the difference  $m_A - m_B$ . Preliminarily a value of 1 $m$  has been used for the difference in magnitude between the central and the first order images. The use of this method for this kind of observations is justified by the investigations of WESSELINK (l.c.). Means of about ten values  $m_A - m_B$  have been derived from exposures taken after each other on one plate. The use of these means is allowed on account of the practically linear variation in brightness during the time of taking ten exposures. All data concerning such a mean  $m_A - m_B$  were written on a card. An example of such a card is given here:

3536b 10 P 75  
 8235.6862 .9107 .9261 .4910  
 2.53 1.19 15.8 8 14.3  
 —.703 ±.044 —.678  
 full moon

N A O T  
 JD P<sub>1</sub> P<sub>2</sub> P<sub>3</sub>  
 D G St S F  
 Δm<sub>1</sub> m.e. Δm<sub>2</sub>  
 remarks

N = number of the plate and letter of the mean,  
 A = number of exposures contained in this mean,  
 O = name of the observer,  
 T = exposure time in seconds,  
 JD = heliocentric Julian Day mean astronomical time Greenwich — 2420000,  
 P<sub>1</sub> = preliminary phase =  $d^{-1} \cdot 2160003$  (J.D. — 2420000),  
 P<sub>2</sub> = definitive phase =  $d^{-1} \cdot 21600217$  (J.D. — 2420000),  
 P<sub>3</sub> = definitive phase counted from primary minimum,  
 D = photographic density, defined as the provisional magnitude of the central image of the comparison star,  
 G = gradation, defined as the mean from both stars of the difference in provisional magnitudes between the central and the first order images (G = 1 if the gradation is the same as that of the table of provisional magnitudes),  
 St = local sidereal time,  
 S = sharpness of the images, defined as the sum of three estimates in a scale 1, 2, . . . , 14 of

very diffuse to very sharp images obtained by variation of focus,  
 F = plate fog, defined as the galvanometer reading in cm in measuring the plate fog, with a galvanometer deflection of 25 cm without a plate in the photometer,  
 Δm<sub>1</sub> = mean of  $m_A - m_B$  computed with 1<sup>m</sup> preliminary difference in magnitude between the central and the first order images,  
 m.e. = internal mean error of one exposure as derived from all exposures of a plate,  
 Δm<sub>2</sub> = definitive difference in magnitude  $m_A - m_B$ .

It is easy to examine the accidental and systematic errors by filing these cards successively according to the different quantities. In total there are 581 cards. The observations are distributed in phase as follows: maximum (P<sub>1</sub> = P.46—P.85, P.96—P.35): 326 Δm<sub>1</sub>'s  
 primary minimum (P<sub>1</sub> = P.35—P.46) : 170 ,,  
 secondary minimum (P<sub>1</sub> = P.85—P.96) : 85 ,,

In the further discussion only the means Δm<sub>1</sub> mentioned above will be used and they will be referred to as observations. From the differences in Δm<sub>1</sub> of observations following each other in phase P<sub>1</sub> the following quantities have been derived:

The square of the mean error of one observation

1°. from all differences (practically all are derived from observations in different nights)

for the maximum	$m^2 \cdot 000653 = (\pm m \cdot 026)^2$	from 326 differences,
for the primary minimum	$\cdot 000716 = (\pm \cdot 027)^2$	,, 170 ,,
for the secondary minimum	$\cdot 000617 = (\pm \cdot 025)^2$	,, 85 ,,
2°. from differences between observations during one night		
for the maximum	$\cdot 000170 = (\pm \cdot 013)^2$	,, 247 ,,
3°. from differences between observations on one plate		
for the maximum	$\cdot 000171 = (\pm \cdot 013)^2$	,, 183 ,,

The error common to all observations of one night or one plate may be called "night error" respectively "plate error". These errors and the internal error have been derived from the above quantities:

the square of the mean night error	$m^2 \cdot 000483 = (\pm m \cdot 022)^2$ ,
,, ,, ,, ,, ,, plate ,,	$\cdot 000000 = (\pm \cdot 000)^2$ ,
,, ,, ,, ,, ,, internal ,,	$\cdot 000170 = (\pm \cdot 013)^2$ .

With the aim of determining systematic errors the 326 cards containing observations at maximum have been filed successively according to the quantities JD, D, G, St, S and F. For each separate arrangement of the cards means have been computed of about twenty observations which follow each other respectively in JD, D, etc. The means are given in Tables 2 to 7. The columns of these tables give successively: the means of JD, D, etc.; the mean of Δm<sub>1</sub>,

called  $\overline{\Delta m_1}$ ; the number  $n$  of Δm<sub>1</sub>'s; the mean "internal mean error" derived from the dispersion of the individual exposures (within one mean Δm<sub>1</sub>); the mean error of  $\overline{\Delta m_1}$  derived from the dispersion of the Δm<sub>1</sub> within one  $\overline{\Delta m_1}$ ; the mean gradation G; the mean of the definitive magnitudes Δm<sub>2</sub> derived as described below, and the mean error of Δm<sub>2</sub>.

In order to obtain some more data about the systematic errors ten series of exposures (at maximum)

TABLE 2.  
Dependence of  $\Delta m_1$  on the Julian Day.

JD	$\overline{\Delta m_1}$	n	m.e.	m.e. ( $\Delta m_1$ )	G	$\overline{\Delta m_2}$	m.e. ( $\Delta m_2$ )
7532	<sup>m</sup> -·870	14	<sup>m</sup> ±·043	<sup>m</sup> ±·006	1·10	<sup>m</sup> -·831	<sup>m</sup> ±·005
7579	·873	16	45	8	1·12	·836	7
7627	·870	17	39	3	1·21	·840	7
7688	·847	22	42	4	1·26	·823	4
7825	·854	13	41	6	1·06	·812	5
7871	·857	28	41	3	1·25	·831	4
7918	·838	30	40	4	1·43	·828	3
8067	·827	8	48	14	1·37	·812	15
8210	·860	36	33	3	1·26	·835	2
8245	·824	32	38	3	1·52	·822	3
8270	·839	31	38	4	1·38	·824	3
8308	·817	31	44	6	1·48	·812	7
8341	·808	27	47	7	1·54	·807	4
8403	·838	21	50	4	1·53	·836	4

TABLE 3.  
Dependence of  $\Delta m_1$  on the photographic density.

D	$\overline{\Delta m_1}$	n	m.e.	m.e. ( $\Delta m_1$ )	G	$\overline{\Delta m_2}$	m.e. ( $\Delta m_2$ )
1·40	<sup>m</sup> -·861	20	<sup>m</sup> ±·033	<sup>m</sup> ±·004	1·26	<sup>m</sup> -·836	<sup>m</sup> ±·004
1·60	·870	20	37	5	1·28	·846	3
1·74	·836	20	41	9	1·36	·820	6
1·85	·850	20	36	5	1·34	·830	5
1·91	·856	20	39	7	1·26	·831	5
1·97	·838	20	39	6	1·38	·824	5
2·04	·849	20	37	8	1·32	·831	5
2·10	·842	20	39	5	1·38	·826	6
2·17	·835	20	43	8	1·34	·817	6
2·25	·835	20	41	5	1·42	·823	4
2·33	·815	20	42	4	1·48	·811	5
2·42	·826	20	43	6	1·38	·812	6
2·49	·840	20	49	9	1·36	·815	7
2·57	·841	20	48	7	1·45	·832	6
2·70	·837	20	44	5	1·33	·819	5
2·94	·849	26	45	3	1·30	·828	3

TABLE 4.  
Dependence of  $\Delta m_1$  on the gradation.

G	$\overline{\Delta m_1}$	n	m.e.	m.e. ( $\Delta m_1$ )	$\overline{\Delta m_2}$	m.e. ( $\Delta m_2$ )
1·00	<sup>m</sup> -·868	20	<sup>m</sup> ±·043	<sup>m</sup> ±·005	<sup>m</sup> -·822	<sup>m</sup> ±·005
1·09	·863	20	43	6	·826	6
1·15	·864	20	45	5	·829	5
1·20	·859	20	39	6	·828	6
1·23	·847	20	40	6	·820	6
1·27	·859	20	41	4	·835	4
1·29	·855	20	34	5	·833	4
1·32	·840	20	36	7	·820	7
1·36	·835	20	36	7	·824	8
1·40	·841	20	44	6	·830	6
1·44	·827	20	43	5	·817	5
1·48	·825	20	49	7	·819	7
1·52	·827	20	46	7	·824	7
1·55	·828	20	41	4	·827	4
1·59	·834	20	39	4	·837	4
1·65	·811	26	40	4	·819	4

TABLE 5.  
Dependence of  $\Delta m_1$  on the sidereal time.

St	$\overline{\Delta m_1}$	n	m.e.	m.e. ( $\Delta m_1$ )	G	$\overline{\Delta m_2}$	m.e. ( $\Delta m_2$ )
h	<sup>m</sup>					<sup>m</sup>	
11·10	-·845	20	<sup>m</sup> ±·041	<sup>m</sup> ±·005	1·26	<sup>m</sup> -·820	<sup>m</sup> ±·005
12·06	·847	20	38	6	1·35	·831	4
12·72	·853	20	41	5	1·28	·829	3
13·29	·847	20	44	5	1·30	·828	4
13·75	·835	20	37	6	1·38	·821	6
14·13	·843	20	38	6	1·33	·824	6
14·44	·849	20	38	5	1·34	·832	4
14·80	·848	20	39	6	1·34	·831	4
15·26	·852	20	36	6	1·34	·833	5
15·91	·846	20	37	7	1·39	·828	4
16·64	·833	20	45	9	1·37	·819	7
17·44	·816	20	46	10	1·46	·809	8
18·35	·828	20	44	9	1·39	·810	6
19·24	·838	20	42	6	1·41	·828	6
20·06	·844	20	44	7	1·36	·828	5
21·06	·848	26	47	4	1·32	·828	5

TABLE 6.  
Dependence of  $\Delta m_1$  on the sharpness of the images.

S	$\overline{\Delta m_1}$	n	m.e.	m.e. ( $\Delta m_1$ )	G	$\overline{\Delta m_2}$	m.e. ( $\Delta m_2$ )
6·8	<sup>m</sup> -·820	12	<sup>m</sup> ±·044	<sup>m</sup> ±·009	1·38	<sup>m</sup> -·806	<sup>m</sup> ±·009
9·0	·827	19	34	5	1·42	·816	3
10·0	·829	13	41	7	1·45	·820	6
11·0	·826	21	41	5	1·52	·824	5
12·0	·828	39	39	4	1·40	·816	4
13·0	·846	26	42	5	1·33	·828	4
14·0	·822	33	46	6	1·43	·812	4
15·0	·837	13	42	6	1·45	·828	5
16·0	·853	22	37	4	1·30	·832	4
17·0	·863	20	39	6	1·34	·844	5
18·0	·863	17	42	6	1·40	·849	5
19·5	·846	20	41	6	1·34	·828	5
21·6	·864	16	42	7	1·25	·838	6
23·3	·840	16	42	8	1·22	·812	8
25·9	·866	15	40	4	1·22	·838	8
29·2	·863	11	50	8	1·28	·840	7
33·8	·859	13	46	9	1·04	·815	7

TABLE 7.  
Dependence of  $\Delta m_1$  on the plate fog.

F	$\overline{\Delta m_1}$	n	m.e.	m.e. ( $\Delta m_1$ )	G	$\overline{\Delta m_2}$	m.e. ( $\Delta m_2$ )
7·48	<sup>m</sup> -·856	20	<sup>m</sup> ±·044	<sup>m</sup> ±·004	1·08	<sup>m</sup> -·816	<sup>m</sup> ±·003
9·50	·855	20	48	8	1·24	·827	7
10·36	·839	20	41	8	1·40	·826	5
11·40	·858	20	44	7	1·32	·838	5
12·11	·845	20	48	4	1·41	·833	3
12·62	·834	20	47	8	1·36	·818	7
13·20	·844	20	43	8	1·34	·826	7
13·83	·842	20	40	7	1·28	·819	7
14·22	·859	20	37	5	1·24	·831	5
14·50	·832	20	38	6	1·38	·822	5
14·75	·839	20	43	7	1·36	·819	6
14·93	·820	20	41	7	1·50	·815	5
15·31	·843	20	35	5	1·40	·829	3
15·80	·832	20	33	4	1·46	·824	4
16·30	·839	20	35	5	1·41	·827	5
17·40	·840	26	41	6	1·43	·830	5

TABLE 8.  
Further measurements of ten series of observations.

$(\Delta m_1)_S$	$(\Delta m_1)_H$	$(\Delta m_1)_H$ $-(\Delta m_1)_S$	$G_S$	$G_H$	G WW Dra	G Comp. B	G a	G b	G c	S	1-6	2-5	1-2 6-5	3-4
<sup>m</sup> -939	<sup>m</sup> -849	<sup>m</sup> +090	'91	10'15	'92	'89	'89	'90	'94	36	590	591	207	209
'903	'869	+ 34	1'11	11'25	1'11	1'12	1'12	1'10	1'18	31	592	586	207	206
'837	'825	+ 12	1'16	9'60	1'22	1'10	1'14	1'11	1'16	16	591	589	208	212
'847	'870	- 23	1'28	9'30	1'29	1'27	1'21	1'23	1'20	13	589	586	205	202
'801	'781	+ 20	1'32	11'25	1'35	1'29	1'24	1'19	1'26	20	590	588	196	200
'772	'810	- 38	1'37	9'00	1'45	1'29	1'26	1'15	1'41	6	583	594	200	207
'811	'859	- 48	1'45	9'85	1'50	1'40	1'40	1'29	1'42	11	588	582	197	201
'777	'810	- 33	1'52	9'65	1'64	1'40	1'37	1'28	1'61	7	589	589	196	200
'819	'890	- 71	1'60	9'80	1'77	1'43	1'46	1'42	1'70	13	587	592	197	201
'760	'822	- 62	1'72	10'60	1'80	1'64	1'50	1'49	1'69	12	589	598	200	205

covering the whole range of gradation have been measured with the Hartmann photometer. The results are given in Table 8. Columns 1 and 2 give  $\Delta m_1$  from the Schilt and the Hartmann photometer, column 3 the difference of both measurements, columns 4 and 5 the mean gradation for WW Draconis and component B (the gradation for Hartmann measurements is defined as the difference in wedge reading between the central and the first order images), columns 6 to 10 the Schilt gradation from measurements of different stars, viz. WW Draconis (spectrum G<sub>5</sub>), component B (colour index =  $+0.68 \pm 0.02$ , as found by the aid of photovisual plates), star a = BD + 61°1600, 9<sup>m</sup>.1 (colour index =  $+0.35 \pm 0.03$ ), star b = BD + 61°1603, 9<sup>m</sup>.3 (colour index =  $1.06 \pm 0.03$ ) and star c = TX Draconis = BD + 60°1688, 7<sup>m</sup>.0 (spectrum Mb); column 11 gives the sharpness S of the images. The results of measurements with the Toepfer measuring machine of the distances between the different images of the same exposure are given in columns 12 to 15 for the same series of exposures. The columns contain the distances in  $\mu$  between the images 1-6 (see Figure 1), 2-5, mean of 1-2 and 6-5, and 3-4. The distances 1-6 and 2-5 are proportional to the effective wavelength.

The following features may be derived from Tables 2 to 8:

1. There is a dependence of  $\Delta m_1$  on the sharpness S of the images. The (absolute) difference in mag-

nitude between both stars is constant for  $S > 20$  or sharp images and decreases for smaller S or more diffuse images. In the case of diffuse images especially the central image of the variable (number 3 in Figure 1) becomes so large that it may influence the measurement of neighbouring images, in this case of image 4. Then this image will be measured too bright and the (absolute) difference in magnitude becomes smaller. The influence of the first order images on each other may be supposed to be smaller. If this explanation is right there should be no dependence of  $\Delta m$  on the sharpness of the images if two stars are considered which have a larger distance on the plate.

2.  $\Delta m_1$  depends linearly on the gradation G. A least squares solution yields:

$$\Delta m_1 = -0.8424 + 0.085 (G - 1.35) \pm 0.17 \pm 0.09 \text{ (m.e.)}$$

The absolute difference in magnitude between both stars increases with decreasing gradation. This effect was also found by WESSELINK (*Leiden Ann.* XVII, 3) from similar observations of SZ Camelopardalis =  $\Sigma 485$  A. A discussion of observations of SX Aurigae made by OOSTERHOFF and measured by KOOREMAN (*B.A.N.* No. 250, 1933) reveals the same effect. The following values have been found for the coefficient K in the formula

$$\Delta m_2 = \Delta m_1 + K \Delta m_1 (G - \text{const})$$

	K	m.e.	spectra of both stars	mean gradation and dispersion	mean $ \Delta m_1 $
SX Aurigae	-0.31	$\pm 0.10$	A3 Bo	$0.92 \pm 0.11$	0.78
SZ Camelopardalis	-0.080	$\pm 0.028$	Bo Bo	$1.05 \pm 0.13$	0.12
WW Draconis	-0.101	$\pm 0.011$	G5 Go:	$1.35 \pm 0.18$	0.84

3. Relations between the gradation and the other quantities.

a. The gradation of Guilleminot La Superguil plates increased steadily from 1.10 at J.D. 2427500 to 1.55 at J.D. 2428400.

b. Further, least squares solutions yield:  
gradation =  $1.357 - 0.0130$  (sharpness-16)  
 $\pm 0.16 \pm 0.22$  (m.e.)

WESSELINK (*Dissertation*, Leiden 1938) gave an explanation of this effect and of the effect described

above under 2. If the images are sharp many silver grains fall upon each other, while in the case of diffuse images they are spread out so that the density of these images as measured with the Schilt photometer is larger than that of sharp images caused by the same amount of light. This applies only to the central images, because the first order images in any case are more diffuse. Thus the gradation increases for diffuse images and the measured difference in magnitude between two stars decreases. Later WESSELINK did not maintain this explanation (*Leiden Ann. XVII*, 3).

$$c. \text{ Gradation} = 1.340 + .00062 \text{ (density—2.00)} \\ \pm 15 \quad \pm 37 \quad \text{(m.e.)}$$

The independence of the gradation on the density (as defined here) shows that the table of provisional magnitudes gives a good approximation of a mean opacity curve.

$$d. \text{ Gradation} = 1.403 - .019 \times \text{hour angle} \\ \pm 22 \quad \pm 6 \quad \text{(m.e.)}$$

If real this relation means that the images are sharper at larger distances from the zenith in contradiction to what may be expected.

$$e. \text{ Gradation} = 1.328 + .027 \text{ (plate fog—12.5)} \\ \pm 19 \quad \pm 8 \quad \text{(m.e.)}$$

Similar relations have been found by ROSS (*The Physics of the Developed Photographic Image*, p. 118-9, 1924) and by EBERHARD (*Publ. Potsdam*, Nr. 84, 43, 1926).

4. The dependence of  $\Delta m_1$  on the other quantities may be accounted for by the above dependences of  $\Delta m_1$  on  $G$  and of  $G$  on those other quantities.

5. The value of  $\Delta m_1$  as a result of Hartmann measurements does not depend on  $G_{\text{Schilt}}$  or on  $G_{\text{Hartmann}}$ , whereas  $G_{\text{Hartmann}}$  is larger for sharp than for diffuse images. This latter fact has been mentioned by ROSS (l.c., p. 133).

6. There is no dependence of the gradation on the colour of the star.

7. The effective wavelength is the same for variable and comparison star and does not depend on the sharpness of the images.

8. The distances between the images 1 and 2, 3 and 4, 6 and 5 (see Figure 1) are measured larger in the case of very diffuse images. It cannot be decided whether this is a photographic or a physiological effect.

Preliminarily the difference in magnitude,  $\Delta m_1$ , has been corrected for the dependence on the sharpness  $S$  of the images mentioned above under 1. The corrections are given in Table 9. They reduce the observations to the case of sharp images as it ought to be if the explanation of the effect given under 1 is

TABLE 9.

Preliminary correction to  $\Delta m_1$ .

S	correction	S	correction
6	— <sup>m</sup> .047	14	— <sup>m</sup> .018
7	43	15	15
8	40	16	12
9	36	17	08
10	33	18	04
11	29	19	01
12	26	20-36	00
13	22		

right. A least squares solution of the corrected magnitudes  $\Delta m_1$  and the gradation gives:

$$\Delta m_1 = -^m.8580 + ^m.060 \text{ (gradation—1.35)} \\ \pm 18 \quad \pm 10 \quad \text{(m.e.)}$$

The fact that there remains a dependence on the gradation after the corrections of Table 9 have been applied speaks for WESSELINK's explanation of the gradation effect (see No. 3 above). Further facts supporting this explanation are 1st) the gradation effect found from other observations, where the two stars considered are at a larger distance on the plate, 2nd) the variation of the gradation in the same way for other, single, stars on the plates of WW Draconis, 3rd) the existence of the gradation effect also for very sharp images, and 4th) the independence, as shown below, of the  $\Delta m_2$ , corrected for the gradation effect, on any one of the other quantities. Therefore the preliminary correction for different sharpness has not been applied in the final reduction.

With the assumption of this explanation  $\Delta m_1$  must be reduced to that gradation for which the difference in magnitude between the central and the first order images has been determined. The fact that this reduction differs from that for the effect of the sharpness of the images is important, because the amplitudes of the light variation as obtained after correction in both manners are different.

The difference in magnitude between the central and the first order images has been determined by HERTZSPRUNG's method (*A.N.* 186, 177, 1910), which uses exposures taken with and without grating. The results from the different plates are given in Table 10. The material is not extensive enough to show a variation of  $m_1 - m_c$  with the gradation. The resulting value is:

$$m_1 - m_c = ^m.980 \pm ^m.005 \text{ (m.e.) at } G = 1.35.$$

The magnitudes  $\Delta m_1$  therefore have been corrected by the aid of the formula:

$$\Delta m_2 = \Delta m_1 [.980 + .099 \text{ (gradation—1.35)}] \\ \pm 5 \pm 11 \quad \text{(m.e.)}$$

The mean values of  $\Delta m_2$  have been computed for

TABLE 10.  
Difference in magnitude between the central and first order images.

object	n	N	G	S	$m_1 - m_c$	m.e.	weight	exposure time
<i>Photographic plates</i>								
Praesepe	1	52	1'48	15	<sup>m</sup> .977	<sup>m</sup> ±.011	8	300 <sup>s</sup>
Region of WW Dra	10	42	1'13	13	.973	11	8	75
” ” ”	4	20	1'09	21	.986	30	1	45
” ” ”	4	15	1'48	15	.991	30	1	45
” ” ”	4	28	1'19	33	.968	22	2	45
Messier 39	4	159	1'41	16	.987	8	16	20, 60
” ”	4	101	1'25	24	.989	10	10	20
” ”	4	59	1'48	18	.967	10	10	20
			1'35		<sup>m</sup> .980	<sup>m</sup> .005		
<i>Photomusical plate</i>								
Praesepe	1	36	1'74	36	.968	11	—	600

n = number of exposures with and without grating

N = number of stars measured

all the means of observations (all these are maximum observations) given in Tables 2 to 7. Herefrom the following least squares solutions have been derived:

$$\Delta m_2 = -^m.8268 + ^m.0113 \text{ (density—2.00)}$$

$$\pm 22 \quad \pm 53 \quad \text{(m.e.)}$$

$$\Delta m_2 = -^m.8206 - ^m.0016 \times \text{hour angle}$$

$$\pm 40 \quad \pm 14 \quad \text{(m.e.)}$$

$$\Delta m_2 = -^m.8253 - ^m.00076 \text{ (sharpness—16)}$$

$$\pm 31 \quad \pm 42 \quad \text{(m.e.)}$$

$$\Delta m_2 = -^m.8249 - ^m.00017 \text{ (plate fog—12.5)}$$

$$\pm 17 \quad \pm 64 \quad \text{(m.e.)}$$

The square of the mean error of one observation  
1°. from all differences

for the maximum  $^m^2.000335 = (\pm ^m.018)^2$  from 326 differences,

for the primary minimum  $.000679 = (\pm .026)^2$  ,, 170 ,,

for the secondary minimum  $.000412 = (\pm .020)^2$  ,, 85 ,,

and if the definitive phases  $P_2$  (see below, p. 129) are used:

for the primary minimum  $.000589 = (\pm .024)^2$  ,, 168 ,,

2°. from differences between observations during one night

for the maximum  $.000143 = (\pm .012)^2$  ,, 247 ,,

3°. from the differences between observations on one plate

for the maximum  $.000136 = (\pm .012)^2$  ,, 183 ,,

The square of the mean night error  $^m^2.000192 = (\pm ^m.014)^2$ ,

the square of the mean plate error  $.000007 = (\pm .003)^2$ ,

the square of the mean internal error  $.000136 = (\pm .012)^2$ .

After correcting the magnitudes for the gradation effect the night error decreased. The fact that this error, however, remains fairly large may be due to the small distance of the two stars on the plate. The real nature of the systematic errors and of the night error may possibly be found from observations of the difference in magnitude between the components of

Further, the mean  $\Delta m_2$  is  $-^m.829 \pm ^m.002$  for  $JD < 8000$  and  $-^m.822 \pm ^m.002$  for  $JD > 8000$ . The difference of these values has the opposite sign than would be expected from the higher sensitivity for longer wavelengths of the more recent plates.

There is no dependence of  $\Delta m_2$  on any of the other quantities. The  $\Delta m_2$  has been taken as definitive magnitude.

With the definitive magnitudes  $\Delta m_2$  the different kinds of mean errors have been derived once more as follows:

double stars of various kinds, made under various conditions. Such kind of observations are now being carried out at the Leiden Observatory.

The ten series of exposures of Table 8 have been measured twice with both photometers. From these measurements the following quantities have been derived:

Schilt photometer      Hartmann photometer

$$^m^2.000225 = (\pm ^m.015)^2 \quad ^m^2.003600 = (\pm ^m.060)^2$$

$$.001521 = (\pm .039)^2 \quad .003250 = (\pm .057)^2$$

$$.001746 = (\pm .042)^2 \quad .006850 = (\pm .083)^2$$

the square of the mean measuring error  
the square of the mean error inherent to the image  
the square of the mean internal error

The number of differences used is 107. These errors refer to individual exposures.

From the internal agreement of the exposures con-

for the maximum	$m^2 \cdot 001656 = (\pm 0.041)^2$	from 3260 differences,
for the primary minimum	$\cdot 001892 = (\pm 0.043)^2$	„ 1700 „
for the secondary minimum	$\cdot 001440 = (\pm 0.038)^2$	„ 850 „
mean	$\cdot 001693 = (\pm 0.041)^2$	

The dependence of the internal mean error and of the dispersion of  $\Delta m_1$  or  $\Delta m_2$  on the various quantities JD, D, etc. can be seen from Tables 2 to 7.

With the assumption that the means  $\Delta m_2$  are independent from each other the total weight of the observations would be  $1450000m^{-2}$ . If the systematic errors are taken into account this value is reduced to  $510000m^{-2}$ .

The improvement of the period has been made in the usual manner. The phase of the primary minimum has been derived by the method described in *B.A.N.* Nos. 147 and 166. Normal points between the phases  $P_1 = P.35$  and  $P.46$  have been used representing each 5 values of  $\Delta m_2$  which follow each other in the preliminary phase  $P_1$ . It may be remembered that each value  $\Delta m_2$  is the mean of about ten exposures.

The normal points outside the primary minimum have not been used, because they show small fluctuations in brightness, which may be caused by systematic errors. A second determination of the minimum

tained in one mean the square of the mean internal error of one exposure is found:

phase has been made by aid of the observations on the two branches of the minimum, viz. those with  $P_1 = P.3690-P.3890$  and  $P.4160-P.4360$ . The light variation within these phases was assumed to be linear. From least squares solutions the phases corresponding to  $\Delta m_2 = -m.200$  on both branches have been derived. The mean of these two phases gives the minimum phase. The result is the same for both methods:

$$P_1 (\text{min I}) = P.4023 \pm P.0002 (\text{m.e.}).$$

For the observations of the primary minimum phases  $P'_1$  reflected with respect to the middle of this minimum ( $P'_1 = |P_1 - P.4023|$ ) have been computed. A light curve has been drawn through normal points representing 10  $\Delta m_2$ 's which follow each other in  $P'_1$ . By the aid of this curve the times of observation have been reduced to the middle of the minimum. Means have been computed of the minimum epochs derived from observations during one night. The weights of these means are given by

$$p_1 \sim \frac{1}{(\text{night error})^2 + \frac{1}{n} (\text{internal error})^2} = \frac{1}{m^2 \cdot 000192 + \frac{1}{n} m^2 \cdot 000143}$$

Here  $n$  is the number of epochs from one night. The plate error has been neglected. These weights (see column 3 of Table 11) have been used although

they do not differ very much from each other.

Least squares solutions of the minimum epochs yield the following results:

Min I = J.D. 2427918.5171	+ 4 <sup>d</sup> .629576	E	for the descending branch,
	± 7	± 13	(m.e.)
Min I = J.D. 2427983.3334	+ 4.629579	E	for the ascending branch,
	± 7	± 16	(m.e.)
Min I = J.D. 2428020.3693	+ 4.629583	E	for both branches together.
	± 5	± 12	(m.e.)

Table 11 gives the epochs of minimum including those obtained by other observers. The minimum epochs from ZVEREV's observations have been derived in the same manner as the Leiden epochs. The O—C's of the above elements derived from both branches are given in column 6. The older observations show systematically negative residuals.

Therefore another determination of the period has been made with the use of all epochs. The weights  $p_2$  (column 4) for this new least squares solution were put inversely proportional to the square of the mean

error of one epoch. The following values have been used for these mean errors:

Harvard plates	± <sup>d</sup> .0600
visual estimates by ZVEREV	± 0.110
Leiden observations, per unit of weight $p_1$	± 0.144

The resulting period is  $4^d.629619 \pm d.000007$  (m.e.). The new O—C's (column 7) are more satisfactory.

BURNHAM, while measuring the relative position of the components, once estimated the difference in brightness to be only  $m.2$  (Table 1, line 6). The time of this observation is J.D. 2407456.71. For this time



TABLE I.  
Determination of the period.

observation	J.D.	$p_1$	$p_2$	epoch	O—C <sub>1</sub>	O—C <sub>2</sub>
Miss HARWOOD, measurement, 1 plate	d 2415205 <sup>d</sup> 599	0	I	0	—'0850	+ '0160
" " " "	5501 <sup>d</sup> 809	0	I	64	—'1680	—'0700
" " " "	5774 <sup>d</sup> 848	0	I	123	—'2740	—'1780
" " " "	5955 <sup>d</sup> 882	0	I	162	—'0940	'0000
" " " "	6390 <sup>d</sup> 638	0	I	256	—'2190	—'1280
" " " "	6418 <sup>d</sup> 486	0	I	262	—'1480	—'0580
" " " "	6603 <sup>d</sup> 702	0	I	302	—'1160	—'0260
" " " "	6969 <sup>d</sup> 44	0	I	381	—'1150	—'0280
ZVEREV, estimate, 1 plate	7131 <sup>d</sup> 500	0	I	416	—'0900	—'0050
Miss HARWOOD, measurement, 1 plate	8867 <sup>d</sup> 661	0	I	791	—'0230	+ '0490
" " " "	8955 <sup>d</sup> 495	0	I	810	—'1510	—'0800
" " " "	9191 <sup>d</sup> 586	0	I	861	—'1690	—'1000
" " " "	9890 <sup>d</sup> 777	0	I	1012	—'0450	+ '0190
HOFFMEISTER, 2 visual estimates in the minimum	2422881 <sup>d</sup> 486	0	I	1658	—'0460	—'0060
" 4 " " " "	2895 <sup>d</sup> 458	0	4	1661	+ '0370	+ '0770
BEYER, 2 " " " "	5247 <sup>d</sup> 247	0	I	2169	—'0020	+ '0190
ZVEREV, 3 visual estimates on the ascending branch	7284 <sup>d</sup> 275	0	36	2609	+ '0090	+ '0150
" 6 " " " "	7307 <sup>d</sup> 396	0	36	2614	—'0180	—'0120
KORDYLEWSKI, visual estimates (S.A.C. 1934)	7321 <sup>d</sup> 296	0	36	2617	—'0060	—'0010
ZVEREV, 2 visual estimates on the ascending branch	7335 <sup>d</sup> 197	0	36	2620	+ '0060	+ '0110
" 7 " " " " " "	7534 <sup>d</sup> 261	0	36	2663	—'0020	+ '0010
" 4 " " " " " descending "	7543 <sup>d</sup> 530	0	36	2665	+ '0080	+ '0110
" 8 " " " " " " "	7557 <sup>d</sup> 409	0	36	2668	—'0020	+ '0010
" 6 " " " " " ascending "	7557 <sup>d</sup> 418	0	36	2668	+ '0070	+ '0100
PLAUT, observations on the ascending branch	7645 <sup>d</sup> 3748	42	730	2687	+ '0017	+ '0041
" " " " " descending "	7654 <sup>d</sup> 6307	48	831	2689	—'0016	+ '0008
" " " " " " "	7691 <sup>d</sup> 6670	38	663	2697	—'0019	+ '0001
" " " " " " ascending "	7710 <sup>d</sup> 1918	42	730	2701	+ '0045	+ '0064
" " " " " " " "	7881 <sup>d</sup> 4789	42	730	2738	—'0029	—'0024
" " " " " " " descending "	7904 <sup>d</sup> 6277	38	663	2743	—'0020	—'0017
" " " " " " " ascending "	7918 <sup>d</sup> 5176	38	663	2746	—'0009	—'0006
" " " " " " " descending "	7918 <sup>d</sup> 5197	46	800	2746	+ '0012	+ '0015
" " " " " " " ascending "	7932 <sup>d</sup> 4068	45	785	2749	—'0004	—'0003
" " " " " " " descending "	7955 <sup>d</sup> 5536	47	816	2754	—'0016	—'0016
" " " " " " " ascending "	7983 <sup>d</sup> 3329	44	764	2760	+ '0002	'0000
" " " " " " " " "	8020 <sup>d</sup> 3700	42	730	2768	+ '0007	+ '0001
" " " " " " " " "	8057 <sup>d</sup> 4074	30	524	2776	+ '0014	+ '0006
" " " " " " " " "	8205 <sup>d</sup> 5510	45	785	2808	—'0016	—'0036
" " " " " " " " "	8219 <sup>d</sup> 4445	45	785	2811	+ '0031	+ '0010
" " " " " " " " "	8307 <sup>d</sup> 4054	45	785	2830	+ '0019	—'0009
" " " " " " " " descending "	8404 <sup>d</sup> 6215	30	524	2851	—'0032	—'0068

O—C<sub>1</sub> = + 0<sup>d</sup>.95 and O—C<sub>2</sub> = + 1<sup>d</sup>.09. The observation thus occurs during maximum.

There remains a possibility of a small variation of the period. Therefore the definitive phases P<sub>2</sub> have been computed with the period derived from the Leiden observations only (the third of the solutions of p. 128), viz.

$$P_2 = {}^{d-1}21600217 \text{ (J.D. — 2420000)}.$$

Table 12 gives the means  $\Delta m_2$  of about ten individual observations with the various quantities belonging to them. A dot after the Julian date indicates a new plate. The internal mean error of one exposure is given in units of m.001.

Now with the new phases P<sub>2</sub> the phases of both minima have been computed by the method of B.A.N. Nos. 147 and 166:

$$P_2 \text{ (min I)} = P.4171 \pm P.0002 \text{ (m.e.)},$$

$$P_2 \text{ (min II)} = .9099 \pm .0026 \text{ (m.e.)}.$$

TABLE 12.

J.D. 242....	P <sub>2</sub>	D	G	St	S	F	$\Delta m_1$	m.e.	$\Delta m_2$	J.D. 242....	P <sub>2</sub>	D	G	St	S	F	$\Delta m_1$	m.e.	$\Delta m_2$
7530 <sup>d</sup> 5495	P .6150	2'66	'92	14'2	17	7'6	—'877	41	—'822	7531 <sup>d</sup> 5593	P .8332	1'86	1'12	14'5	19	8'6	—'858	43	—'821
'5631	.6180	2'48	'94	14'5	17	7'6	—'882	41	—'828	'5747	.8365	1'96	1'12	14'9	22	8'6	—'866	43	—'829
'5772	.6210	2'44	'90	14'8	16	7'6	—'874	41	—'818	'5918	.8402	1'97	1'16	15'3	16	8'6	—'837	43	—'804
'5925	.6243	2'23	'96	15'2	16	7'6	—'870	41	—'819	'6230	.8469	1'67	1'20	16'0	15	7'8	—'853	44	—'824
7531 <sup>d</sup> 5454	.8301	1'95	1'22	14'1	18	8'6	—'839	43	—'811	'6520	.8532	1'58	1'16	16'7	16	7'8	—'875	44	—'841











FIGURE 2.  
Normal points of 10 means  $\Delta m_2$ .

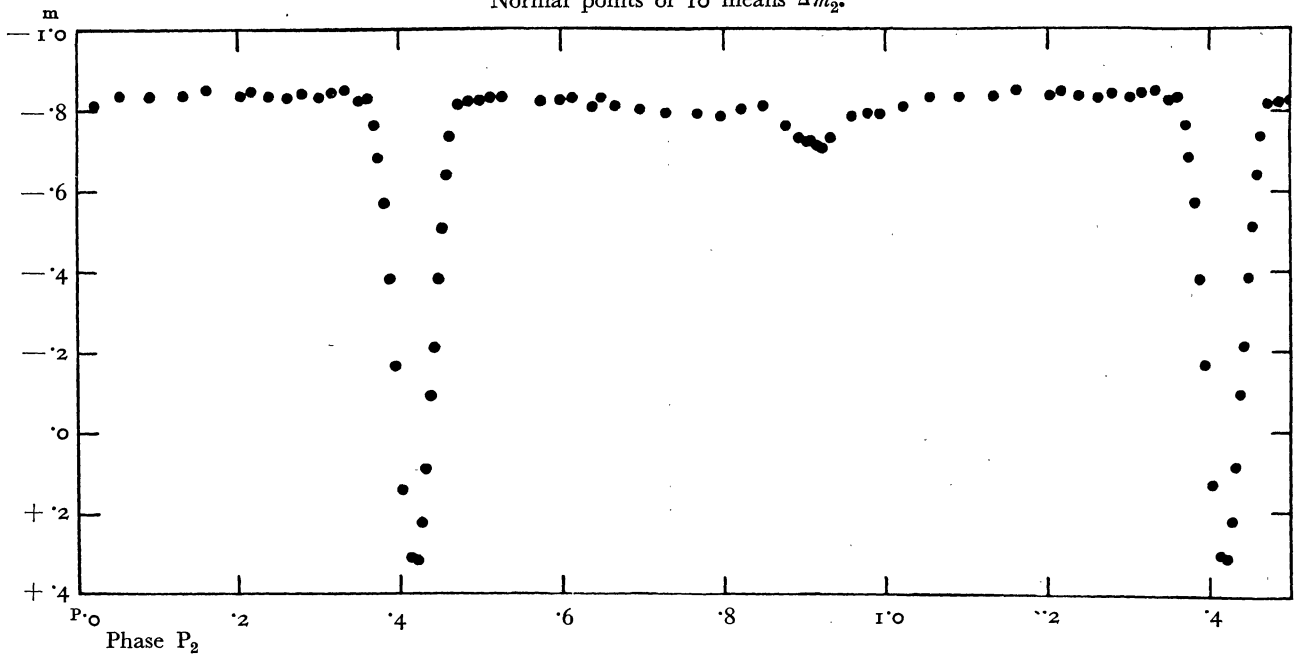
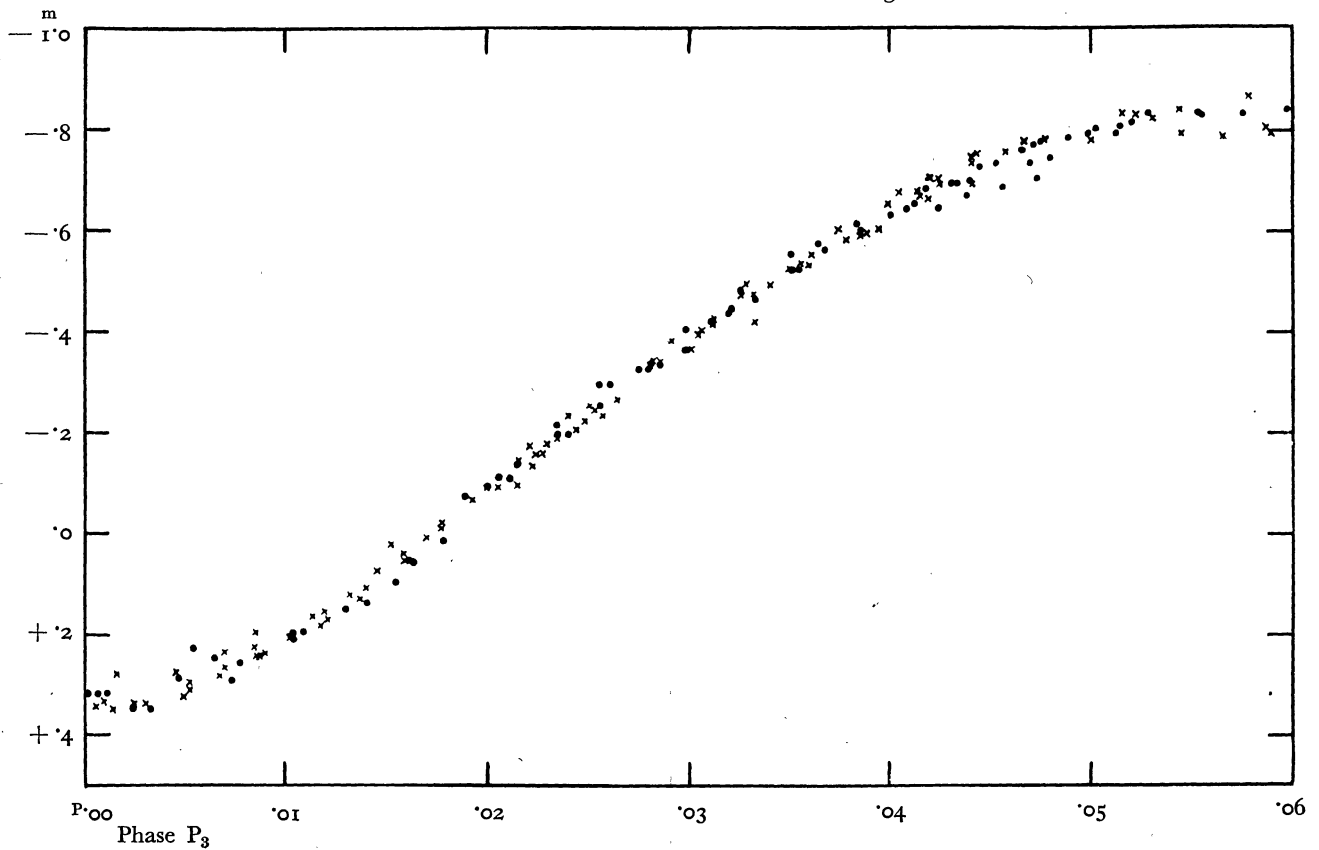


FIGURE 3.  
Means  $\Delta m_2$  of about 10 observations, ● descending branch, × ascending branch.



New phases  $P_3 = |P_2 - P_{.4171}|$  or reflected with respect to the primary minimum, have been computed. The observations on both branches of the primary minimum are given in Figure 3. Normal points representing 10  $\Delta m_2$ 's which follow each other in phase  $P_3$  have been derived (Table 13, last line mean of 11).

for the maximum  $m^2 \cdot 000485 = (\pm m \cdot 022)^2$  from 329 differences,  
 for the primary minimum  $\cdot 000499 = (\pm \cdot 022)^2$  „ 169 „  
 for the secondary minimum  $\cdot 000495 = (\pm \cdot 022)^2$  „ 82 „

The maximum magnitude of the combined light of both stars has been determined by BERGSTRAND (*Medd. Upsala* No. 57, 1933) to be  $8^m \cdot 41 \pm m \cdot 04$  (m.e.). Then the maximum brightness of WW Draconis is  $8^m \cdot 82 \pm m \cdot 04$  (m.e.).

TABLE 13.  
Normal points.

$P_3$	$\Delta m_2$	$P_3$	$\Delta m_2$	$P_3$	$\Delta m_2$	$P_3$	$\Delta m_2$
P	m	P	m	P	m	P	m
$\cdot 0015$	$+\cdot 328$	$\cdot 0483$	$-\cdot 768$	$\cdot 1859$	$-\cdot 832$	$\cdot 3907$	$-\cdot 796$
$\cdot 0054$	$+\cdot 280$	$\cdot 0528$	$-\cdot 823$	$\cdot 1934$	$-\cdot 835$	$\cdot 4095$	$-\cdot 808$
$\cdot 0086$	$+\cdot 237$	$\cdot 0588$	$-\cdot 823$	$\cdot 2030$	$-\cdot 842$	$\cdot 4219$	$-\cdot 804$
$\cdot 0122$	$+\cdot 155$	$\cdot 0650$	$-\cdot 823$	$\cdot 2126$	$-\cdot 832$	$\cdot 4307$	$-\cdot 793$
$\cdot 0158$	$+\cdot 055$	$\cdot 0706$	$-\cdot 837$	$\cdot 2210$	$-\cdot 824$	$\cdot 4411$	$-\cdot 804$
$\cdot 0197$	$-\cdot 081$	$\cdot 0799$	$-\cdot 842$	$\cdot 2320$	$-\cdot 833$	$\cdot 4529$	$-\cdot 790$
$\cdot 0226$	$-\cdot 164$	$\cdot 0880$	$-\cdot 838$	$\cdot 2451$	$-\cdot 830$	$\cdot 4628$	$-\cdot 759$
$\cdot 0251$	$-\cdot 244$	$\cdot 0939$	$-\cdot 834$	$\cdot 2601$	$-\cdot 836$	$\cdot 4738$	$-\cdot 745$
$\cdot 0284$	$-\cdot 343$	$\cdot 1010$	$-\cdot 845$	$\cdot 2763$	$-\cdot 819$	$\cdot 4833$	$-\cdot 738$
$\cdot 0313$	$-\cdot 427$	$\cdot 1086$	$-\cdot 841$	$\cdot 2844$	$-\cdot 825$	$\cdot 4873$	$-\cdot 723$
$\cdot 0341$	$-\cdot 495$	$\cdot 1181$	$-\cdot 832$	$\cdot 3029$	$-\cdot 814$	$\cdot 4905$	$-\cdot 730$
$\cdot 0371$	$-\cdot 576$	$\cdot 1362$	$-\cdot 841$	$\cdot 3325$	$-\cdot 810$	$\cdot 4938$	$-\cdot 712$
$\cdot 0403$	$-\cdot 645$	$\cdot 1515$	$-\cdot 829$	$\cdot 3504$	$-\cdot 813$	$\cdot 4983$	$-\cdot 716$
$\cdot 0427$	$-\cdot 689$	$\cdot 1622$	$-\cdot 832$	$\cdot 3653$	$-\cdot 821$		
$\cdot 0453$	$-\cdot 740$	$\cdot 1741$	$-\cdot 830$	$\cdot 3774$	$-\cdot 803$		

The mean result of the 18 photovisual exposures is given in the first line of Table 14. All these exposures have been taken during the maximum. They have been reduced in the same way as the photographic exposures with the exception that no photographic amplitude of the primary minimum photographic amplitude of the secondary minimum photovisual amplitude of the primary minimum

Dr. A. H. JOY has been so kind to place at my disposal the preliminary results derived from about Dr. JOY writes:

“Primary component of WW Draconis  
 secondary component of WW Draconis

The spectrum of the secondary is very much weaker than that of the primary, and it has been possible to measure only a few lines, but the velocity curve for the primary is well defined and I think fairly accurate. There seems to be no indication of

The asymmetry of the maximum and the secondary minimum is also shown by a comparison of the external mean errors of one  $\Delta m_2$ , derived when they are arranged respectively according to the phases  $P_2$  (see p. 128) or  $P_3$ . For this latter case the square of the mean external error is:

TABLE 14.  
Photovisual exposures.

Plate	$n$	J.D. - 2420000	$\Delta m_{pv}$	$P_3$	$\Delta m_{pg} - \Delta m_{pv}$
Leiden	18	—	m -1 <sup>m</sup> 079	maximum	m +2 <sup>m</sup> 54
L 17	7	8752 <sup>m</sup> 6806	-1 <sup>m</sup> 080	$\cdot 1809$	
L 40	18	8765 <sup>m</sup> 6703	- $\cdot 391$	$\cdot 0133$	+ $\cdot 519$
L 40	18	$\cdot 6767$	- $\cdot 350$	$\cdot 0119$	+ $\cdot 509$
L 40	18	$\cdot 6901$	- $\cdot 286$	$\cdot 0090$	+ $\cdot 498$
L 40	17	$\cdot 7034$	- $\cdot 144$	$\cdot 0061$	+ $\cdot 514$
L 41	18	$\cdot 7635$	- $\cdot 201$	$\cdot 0068$	+ $\cdot 465$
L 41	17	$\cdot 7703$	- $\cdot 208$	$\cdot 0083$	+ $\cdot 446$

rection for systematic errors has been applied. Prof. HERTZSPRUNG has been so kind to place at my disposal three photovisual plates of WW Draconis, which had been taken by him with the 36-inch refractor of the Lick Observatory. These plates, however, show a very irregular plate fog. The two plates L 40 and L 41 have been taken without a grating and were therefore reduced by the aid of a mean gradation derived from measurements of a series of similar plates by Mr. KOOREMAN. The mean internal error of one photovisual exposure is  $\pm m \cdot 07$ . The results of the Lick plates are also given in Table 14. The last column gives a comparison with the photographic light curve. For the middle of the primary minimum  $\Delta m_{pg} - \Delta m_{pv} = + m \cdot 500$  may be derived herefrom. The different amplitudes are:

$1^m \cdot 153 \pm m \cdot 006$  (m.e.),  
 $0 \cdot 111 \pm \cdot 003$  (m.e.),  
 $0 \cdot 907 \pm \cdot 021$  (m.e.).

twenty spectrograms taken at the Mt. Wilson Observatory.

Amplitude in radial velocity	Mass	Spectrum
95 km/sec	$3 \cdot 4 \odot$	gG5
135 „	$2 \cdot 4 \odot$	gG8

eccentricity in the orbit. The spectral type of the secondary is determined from plates taken at primary minimum on the assumption that the eclipse is total or nearly so.”



*Determination of orbital elements.*

The same method as used in the case of CV Carinae (B.A.N. No. 323, 1939; compare also VAN GENT, B.A.N. No. 215, 1931) has been followed. The real variation of the intensity during the maximum could not be determined on account of systematical errors as mentioned above. Therefore the reflection effect has been neglected and also the components have been assumed to be spherical, though it may be possible to make some theoretical assumptions. As found by Dr. JOY the orbit is circular. The two extreme assumptions about limb darkening, the *U* and *D* hypotheses, have been made.

*U hypothesis.*

From the amplitudes of both minima,  $1-l$  (min I) = .6542 and  $1-l$  (min II) = .0972, the ratio of the

surface brightnesses  $J_1 : J_2 = 6.73 = -2^{m.07}$  is derived. The smaller component is the brighter one.

Suppose:

- $L_1, L_2$  = the intensities of both components,
- $k$  = the ratio of the radii,  $k \leq 1$ ,
- $r_1, r_2$  = the radii in units of the radius of the orbit,
- $\delta$  = the projected distance of the centres of both stars in the same unit,
- $\alpha_1, \alpha_2$  = the loss of light in units of the intensity of the eclipsed component,
- $\theta$  = the orbital longitude counted from mid-eclipse,
- $i$  = the inclination of the orbit.

For different values of  $k$  the intensities  $L_1$  and  $L_2$  are given in Table 15 as derived from

$$L_2 = \frac{1}{6.73 k^2 + 1}, \quad L_1 + L_2 = 1.$$

TABLE 15.

Results of the different theoretical light curves.

$k$	$L_1$	$L_2$	$\Delta m$	m.e. of a single normal point	$r_2$	m.e.	$i$	m.e.
U	.53	.6540	.3460	.691	$\pm .0088$	.2284	$\pm .0014$	$83.6 \pm .1$
	.58	.6936	.3064	.887	.0091	.2271	9	83.3 .1
	.63	.7276	.2724	1.067	.0143	.2227	14	83.4 .1
	.68	.7568	.2432	1.233	.0191	.2178	19	83.6 .2
	.73	.7820	.2180	1.387	.0231	.2128	22	83.9 .2
D	.50	.6560	.3440	.701	.0259	.2394	36	82.1 .2
	.60	.7157	.2843	1.002	.0075	.2350	8	82.0 .1
	.70	.7732	.2268	1.332	.0087	.2259	9	82.4 .1

In the fourth column the difference in magnitude between both components is given. Then  $\alpha$  for mid-eclipse is known for each  $k$ . The eclipse of the smaller, brighter component is total for  $k = .5302$ . A smaller value of  $k$  is not possible without varying the ratio of the surface brightnesses.

Following RUSSELL (*Ap. J.* 35, 315, 1912):

$$\left(\frac{\delta}{r_2}\right)^2 = \frac{1}{r_2^2} - \frac{1}{r_2^2} \cos^2 \theta \sin^2 i.$$

The quantities  $\alpha$  and  $\theta$  are known for each normal point,  $\frac{\delta}{r_2}$  has been tabulated by HETZER (*Beitrag zu*

TABLE 16.

O-C for different theoretical light curves.

Phase	$m-m_{\max}$	U hypothesis					D hypothesis		
		$k = .53$	.58	.63	.68	.73	.50	.60	.70
P	m	m	m	m	m	m	m	m	
.0483	.057	+ .009	- .003	- .010	- .016	- .021	+ .026	+ .005	- .007
.0453	.085	- 5	- 18	- 24	- 29	- 34	+ 17	- 6	- 18
.0427	.136	+ 3	- 9	- 15	- 20	- 23	+ 27	+ 3	- 7
.0403	.180	+ 2	- 10	- 15	- 18	- 21	+ 24	+ 3	- 6
.0371	.249	+ 2	- 7	- 10	- 12	- 14	+ 18	+ 3	- 3
.0341	.330	+ 10	+ 4	+ 3	+ 2	+ 2	+ 18	+ 10	+ 8
.0313	.398	+ 4	+ 2	+ 2	+ 3	+ 4	+ 2	+ 2	+ 4
.0284	.482	+ 4	+ 6	+ 9	+ 11	+ 13	- 8	0	+ 6
.0251	.581	+ 1	+ 8	+ 13	+ 17	+ 21	- 22	- 3	+ 7
.0226	.661	- 2	+ 10	+ 16	+ 22	+ 26	- 30	- 5	+ 7
.0197	.744	- 15	0	+ 7	+ 13	+ 18	- 46	- 18	- 4
.0158	.880	- 6	+ 12	+ 19	+ 25	+ 29	- 31	- 2	+ 9
.0122	.980	- 10	+ 6	+ 11	+ 14	+ 17	- 20	0	+ 5
.0086	1.062	- 10	- 3	- 3	- 4	- 5	- 2	+ 2	- 3
.0054	1.105	- 10	- 14	- 20	- 26	- 31	+ 9	- 2	+ 15
.0015	1.153	+ 16	- 2	- 14	- 23	- 31	+ 38	+ 15	- 4

H. N. RUSSELL's *Methodes* . . ., Diss., Leipzig 1931) for the case of the *U* hypothesis as a function of *k* and  $\alpha$ .

Suppose:

$$u = \left(\frac{\delta}{r_2}\right)^2; t = -\cos^2 \theta; A = \frac{1}{r_2^2}; B = \frac{1}{r_2^2} \sin^2 i.$$

Then for a fixed value of *k* each normal point gives an equation of condition of the usual form  $u = A + Bt$  for the determination of  $r_2$  and *i*.

The normal points used for such least squares solutions are given in Table 16. The weights for *u*,

which are not all the same, have been determined empirically by computing *u* also for  $m - m_{\max} + m \cdot 005$ . They are taken proportional to  $(\Delta u)^{-2}$ . Table 16 and Figure 4 give the O—C's for the various solutions. The resulting values of  $r_2$ , *i* and the mean error of a single normal point, as derived from the O—C's, are given in Table 15, while this mean error is plotted against *k* in Figure 5 (the right one of the dotted lines gives *k* for total eclipse).

The agreement between the observed and the theoretical light curves is best for  $k = .55$ . The resulting elements are given in Table 17. The mean errors have been determined by the method of PANNEKOEK and MISS VAN DIEN (*B.A.N.* No. 297, 1937). Here it should be mentioned that the whole derivation of the elements *k*,  $r_2$  and *i* is based on the ratio of surface brightnesses, for which quantity a fixed value has been adopted as derived from the depths of both minima. Thus the normal point near the middle of the primary minimum has received a

FIGURE 4.  
O—C's of the normal points.

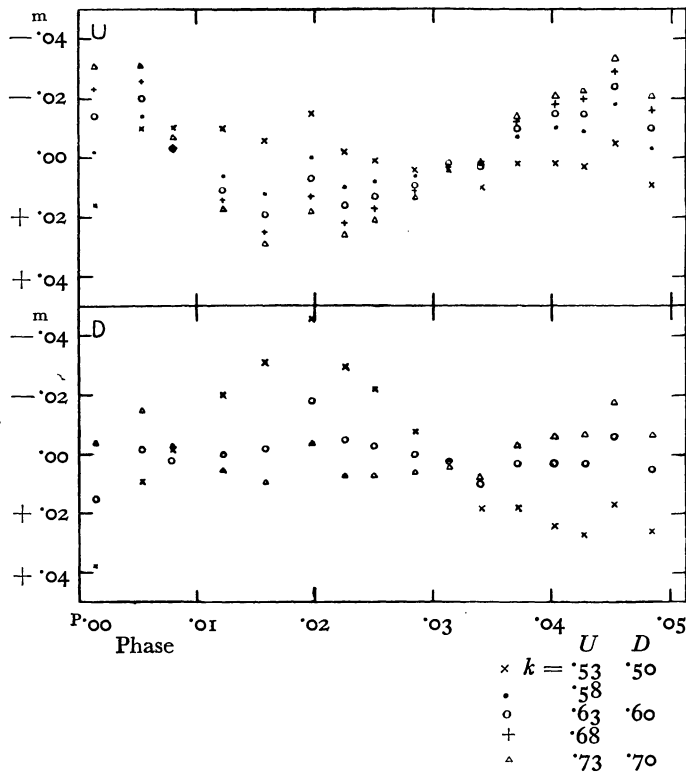


FIGURE 5.

Mean error of a single normal point for different theoretical light curves. • *U* hypothesis, × *D* hypothesis, the dotted lines give the *k* for total eclipse, right *U*, left *D* hypothesis.

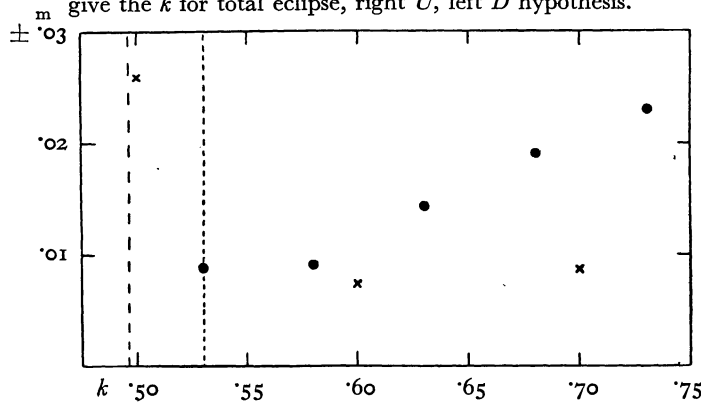


TABLE 17.

Definitive results.

	<i>U</i> hypothesis		<i>D</i> hypothesis	
		(m.e.)		(m.e.)
<i>k</i>	.55	± .03	.64	± .03
$r_2$	.228	± .003	.231	± .004
<i>i</i>	83°5	± °2	82°2	± °2
$J_1:J_2$	6.73	± .15	6.99	± .16
$\Delta m$	<sup>m</sup> .79	± .11	<sup>m</sup> 1.14	± .11
<i>D</i>	<sup>P</sup> .1094	± .0024	<sup>P</sup> .1162	± .0024

too large weight. The disadvantage of this fact can be avoided if both minima are observed with the same accuracy during the whole partial phases. In this case the ratio of the surface brightnesses can be determined by the aid of the whole light variation during the minima, as has been done by WESSELINK (l.c.).

*D hypothesis.*

For the middle of both eclipses:

$$\alpha_2 = \frac{\alpha_1 [1 - I(\text{min II})]}{\alpha_1 - [1 - I(\text{min I})]} = \frac{.0972 \alpha_1}{\alpha_1 - .6542}$$

Further, for both eclipses:

$\left(\frac{\delta}{r_2}\right)_1 = f_1(k, \alpha_1)$  resp.  $\left(\frac{\delta}{r_2}\right)_2 = f_2(k, \alpha_2)$ , while for corresponding phases at both eclipses  $f_1(k, \alpha_1) = f_2(k, \alpha_2)$ . Here the indices of  $\alpha$ ,  $\left(\frac{\delta}{r_2}\right)$  and *f* refer to the eclipses of the component 1 resp. 2. The function  $f_1$  has been tabulated by ZESSEWITSCH (*Poulkovo Circ.*

No. 24, 41, 1938), the function  $f_2$  by RUSSELL and SHAPLEY (*Aph. J.* 36, 39, 1912, Table I<sub>v</sub>).

Thus, for a fixed value of  $k$  there are two relations which connect  $\alpha_1$  and  $\alpha_2$  for the middle of the eclipses and these quantities can be determined. Table 15 gives the values of  $L_1$ ,  $L_2$  and  $\Delta m$  as derived from  $\alpha_1$  and  $\alpha_2$ . The eclipse is total for  $k = .4953$  (the left one of the dotted lines in Figure 5). The elements  $k$ ,  $r_2$  and  $i$  have been determined in the same way as in the case of the  $U$  hypothesis and the results and

the O—C's are given in the corresponding tables and figures. The difference in magnitude between the components,  $\Delta m = 1^m.14 \pm m.11$ , can scarcely be brought into agreement with the fact that the spectral lines of the fainter component have been measured on some of the Mt. Wilson spectrograms. The remark made above at the end of the computations for the  $U$  hypothesis holds also here.

No conclusion about the limb darkening can be made.

TABLE 18.  
Absolute dimensions.

$K_2$	masses		radius of the orbit	radii of the stars			
	$M_1$	$M_2$	$a$	$r_1$		$r_2$	
units: km/sec	$\odot$	$\odot$	$10^6$ km	$10^6$ km	$r_{\odot}$	$10^6$ km	$r_{\odot}$
$U \left\{ \begin{array}{l} 120 \\ 135 \\ 150 \end{array} \right.$	$\begin{array}{l} 2.72 \\ 3.51 \\ 4.42 \end{array}$	$\begin{array}{l} 2.16 \\ 2.47 \\ 2.80 \end{array}$	$\begin{array}{l} 13.8 \\ 14.7 \\ 15.7 \end{array}$	$\begin{array}{l} 1.73 \\ 1.84 \\ 1.97 \end{array}$	$\begin{array}{l} 2.49 \\ 2.65 \\ 2.83 \end{array}$	$\begin{array}{l} 3.14 \\ 3.35 \\ 3.58 \end{array}$	$\begin{array}{l} 4.52 \\ 4.82 \\ 5.15 \end{array}$
$D \left\{ \begin{array}{l} 120 \\ 135 \\ 150 \end{array} \right.$	$\begin{array}{l} 2.73 \\ 3.53 \\ 4.46 \end{array}$	$\begin{array}{l} 2.18 \\ 2.48 \\ 2.82 \end{array}$	$\begin{array}{l} 13.7 \\ 14.8 \\ 15.8 \end{array}$	$\begin{array}{l} 2.02 \\ 2.19 \\ 2.34 \end{array}$	$\begin{array}{l} 2.91 \\ 3.15 \\ 3.37 \end{array}$	$\begin{array}{l} 3.16 \\ 3.42 \\ 3.65 \end{array}$	$\begin{array}{l} 4.55 \\ 4.92 \\ 5.25 \end{array}$

$K_2$	photographic		bolometric		bolometric according to EDDINGTON, I.C.S.		parallax distance	
	1	2	1	2	1	2		
km/sec								
$U \left\{ \begin{array}{l} 120 \\ 135 \\ 150 \end{array} \right.$	$\begin{array}{l} +4.94 \\ +4.80 \\ +4.66 \end{array}$	$\begin{array}{l} +5.73 \\ +5.59 \\ +5.45 \end{array}$	$\begin{array}{l} +3.74 \\ +3.60 \\ +3.46 \end{array}$	$\begin{array}{l} +3.86 \\ +3.72 \\ +3.58 \end{array}$	$\begin{array}{l} +1.03 \\ +0.25 \\ -0.40 \end{array}$	$\begin{array}{l} +1.95 \\ +1.50 \\ +1.09 \end{array}$	$\begin{array}{l} .0138 \\ .0129 \\ .0121 \end{array}$	$\begin{array}{l} 72 \text{ ps} \\ 78 \\ 83 \end{array}$
$D \left\{ \begin{array}{l} 120 \\ 135 \\ 150 \end{array} \right.$	$\begin{array}{l} +4.60 \\ +4.43 \\ +4.28 \end{array}$	$\begin{array}{l} +5.74 \\ +5.57 \\ +5.42 \end{array}$	$\begin{array}{l} +3.40 \\ +3.23 \\ +3.08 \end{array}$	$\begin{array}{l} +3.87 \\ +3.70 \\ +3.55 \end{array}$	$\begin{array}{l} +1.02 \\ +0.23 \\ -0.43 \end{array}$	$\begin{array}{l} +1.92 \\ +1.48 \\ +1.07 \end{array}$	$\begin{array}{l} .0126 \\ .0114 \\ .0106 \end{array}$	$\begin{array}{l} 79 \\ 88 \\ 94 \end{array}$

#### Absolute dimensions.

The absolute dimensions can be derived from the combination of the spectroscopic and photometric observations in the usual manner. The amplitude  $K_2$  of the radial velocity of the fainter component is not accurately known. Therefore the computations have been made for the values  $K_2 = 120, 135$  and  $150$  km/sec. The results are given in Table 18. The absolute bolometric magnitudes are very sensitive to small changes in  $K_1$  and  $K_2$  when the mass-luminosity relation is used. Therefore the discrepancy between

these absolute magnitudes and those derived from the combination of the photometric and spectroscopic observations is probably not real.

With the assumption of a distance of 80 parsecs the projected distance between WW Draconis and component B of  $\Sigma 2092$  is  $1.0 \cdot 10^{11}$  km. This value multiplied with 1.13 yields the probable value of the semi-major axis of the orbit (HERTZSPRUNG, *B.A.N.* No. 25, 1922). With the assumption of  $9 \odot$  as the sum of the masses of the three stars the period of  $\Sigma 2092$  is of the order of 7000 years.