QUANTUM POINT CONTACTS

The quantization of ballistic electron transport through a constriction demonstrates that "conduction is transmission."

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Punctuated equilibrium, the notion that evolution in nature is stepwise rather than continuous, sometimes applies to evolution in science as well. The seed of a scientific breakthrough may slumber for a decade or even longer without generating much interest. The seed may be a theoretical concept without clear predictions to test experimentally, or an intriguing but confusing experiment without a lucid interpretation. When the seed finally germinates, an entire field of science can reach maturity in a few years.

Although we did not know it at the time, the slumbering-seed process was well under way when, a decade ago, the two of us, as newly hired PhDs at Philips Research Laboratories in Eindhoven, ventured into the field of quantum ballistic transport. Together with Bart van Wees, then a graduate student at Delft University of Technology, we were confronted with some pretty vague challenges.

On the experimental side, there was the search for a quantum-size effect on conductance: a clear-cut manifestation of the quantum mechanical wave character of conduction electrons. Experiments on narrow silicon transistors at Yale University and AT&T Bell Laboratories had come close, but suffered from irregularities owing to disorder. (These irregularities would become known as universal conductance fluctuations; see PHYSICS TODAY, December 1988, page 36.) We anticipated that the electron motion would need to be ballistic—that is, without scattering by impurities. Moty Heiblum of IBM in Yorktown Heights, New York, had demonstrated ballistic transport of hot electrons, high above the Fermi level. For a quantum-size effect, one needs ballistic motion at the Fermi energy. Our colleague Thomas Foxon at Philips Research in Redhill, England, could provide us with heterojunctions of gallium arsenide and aluminum gallium arsenide, containing at the interface a thin layer of highly mobile electrons. Such a "two-dimensional electron gas" seemed an ideal system for ballistic transport.

On the theoretical side, there was debate over whether a wire without impurities could have any resistance at all.[1] Ultimately, the question was: What is measured when you measure a resistance? The conventional point of view (held in the classical Drude–Sommerfeld or quantum mechanical Kubo theories) is that the electrical current density is determined by the local velocity distribution, which deviates from equilibrium in linear response to the local electric field. An alternative point of view had been put forward in 1957 by Rolf Landauer of IBM in Yorktown Heights, New York; he had proposed that "conduction is transmission" between reservoirs that are maintained at different electrochemical potentials.[2]

Landauer's formula, a relationship between conductance and transmission probability, evolved into two versions. One gave infinite conductance (zero resistance) in the absence of impurity scattering (transmission probability one), and the other gave a finite answer. Although the origin of the difference between the two versions was understood by at least one of the theorists involved in the debate,[3] the experimental implications remained unclear.

Looking back ten years later, we find that the seed planted by Landauer in the 1950s has developed into a sophisticated theory that forms the basis of the entire field of quantum ballistic transport. The breakthrough can be traced back to experiments on an elementary conductor: a point contact. In this article, we present a brief account of these developments. (For a more comprehensive and detailed discussion, we direct readers to the reviews cited in the references.)

Quantized conductance

The history of ballistic transport goes back to 1965, when Yuri Sharvin at the Academy of Sciences of the USSR in Moscow used a pair of point contacts to inject and detect a beam of electrons in a single-crystalline metal.[4] In such experiments the quantum mechanical wave character of the electrons does not play an essential role, because the Fermi wavelength ($\lambda_F \approx 0.5$ nm) is much smaller than the opening of the point contact. The two-dimensional electron gas in a GaAs–AlGaAs heterojunction has a Fermi wavelength that is a hundred times larger than in a metal. This property makes it possible to study a constriction with an opening comparable to the wavelength (and much smaller than the mean free path for impurity scattering). Such a constriction is called a quantum point contact.

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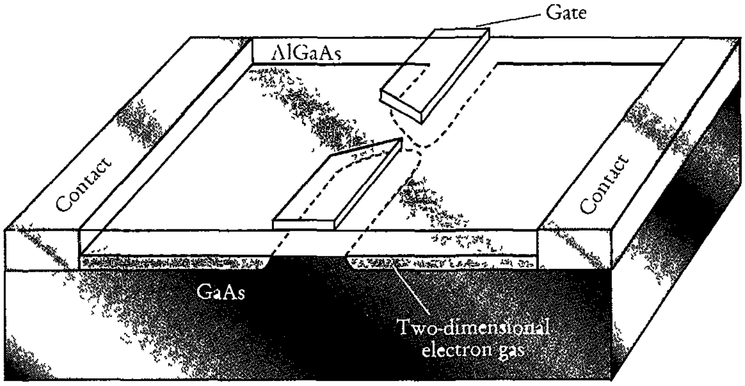

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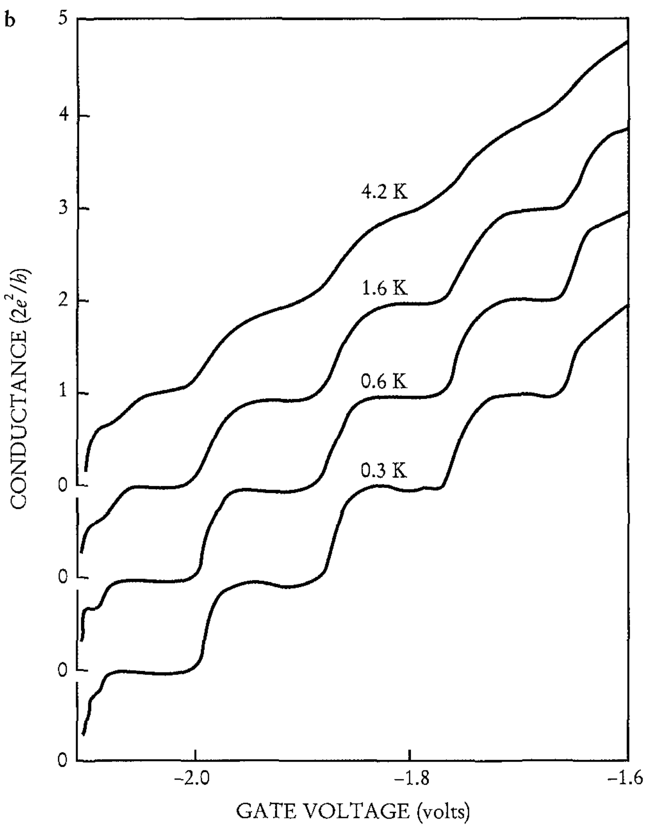


QUANTUM POINT CONTACT scheme (a) and conductance quantization (b). The contact is defined in a high-mobility two-dimensional electron gas at the interface of a GaAs–AlGaAs heterojunction. The point contact is formed when a negative voltage is applied to the gate electrodes on top of the AlGaAs layer. Transport measurements are made by employing contacts to the two-dimensional electron gas at either side of the constriction. The graph in **b** shows the conductance quantization of a quantum point contact in units of $2e^2/h$. As the gate voltage defining the constriction is made less negative, the width of the point contact increases continuously, but the number of propagating modes at the Fermi level increases stepwise. The resulting conductance steps are smeared out when the thermal energy becomes comparable to the energy separation of the modes. (Adapted from ref. 6.) FIGURE 1

In a metal a point contact is fabricated simply by pressing two wedge- or needle-shaped pieces of material together. A quantum point contact requires a more complicated strategy because the two-dimensional electron gas is confined at the GaAs–AlGaAs interface in the interior of the heterojunction. A point contact of adjustable width can be created in this system using the split-gate technique developed by the groups of Michael Pepper at the University of Cambridge and Daniel Tsui at Princeton University.[5] The gate is a negatively charged electrode on top of the heterojunction, which depletes the electron gas beneath it. (See figure 1.) In 1988 the Delft–Philips and Cambridge groups reported the discovery of a sequence of steps in the conductance of a constriction in a two-dimensional electron gas, as its width, W, was varied by means of the voltage on the gate.[6,7] (See PHYSICS TODAY, November 1988, page 21.) As shown in figure 1, the steps are near-integer multiples of $2e^2/h$, or about $1/13\text{k}\Omega$, after correction for a small series resistance independent of gate voltage.

An elementary explanation of the quantization views the constriction as an electron waveguide, through which a small integer number, $N \approx 2W/\lambda_F$, of transverse modes can propagate at the Fermi level. The wide regions at opposite sides of the constriction are reservoirs of electrons in local equilibrium. A voltage difference, V, between the reservoirs induces a current, I, through the constriction, equally distributed among the N modes. This equipartition rule is not immediately obvious because electrons at the Fermi level in each mode have different group velocities, v_n. However, the difference in group velocity is canceled by the difference in density of states, $\rho_n = 1/hv_n$. As a result, each mode carries the same current, $I_n = Ve^2\rho_n v_n = Ve^2/h$. Summing over all modes in the waveguide, one obtains the conductance, $G = I/V = Ne^2/h$. The experimental step size is twice e^2/h because spin-up and spin-down modes are degenerate.

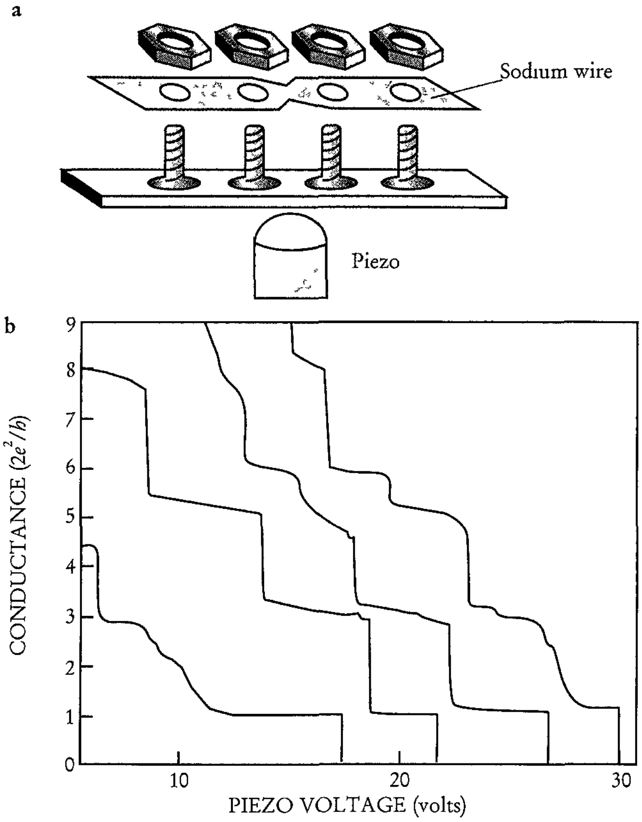
The electron waveguide has a nonzero resistance even though there are no impurities, because reflections occur when a small number of propagating modes in the waveguide is matched to a larger number of modes in the reservoirs. A thorough understanding of this mode-matching problem is now available, thanks to the efforts of many investigators.[5]

The quantized conductance of a point contact provides firm experimental support for the Landauer formula

$$G = \frac{2e^2}{h} \sum_n t_n$$

for the conductance of a disordered metal between two electron reservoirs. The numbers t_n, between 0 and 1, are

ATOMIC-SCALE POINT CONTACT technique (a) and conductance quantization (b). The contact is made by breaking a sodium wire at a notch cut across it, whereafter the two parts are brought into contact mechanically at 4.2 K. The width of the point contact is adjusted by increasing the force of contact through a piezo element. Electrical measurements are made using four miniature brass bolts connected to the wire. The graph in **b** shows quantized steps in the sodium atomic-size point contact. The experiment is not fully reproducible, as shown by the four representative single measurements, because of varying atomic rearrangements in the contact region. (Adapted from ref. 10.) FIGURE 2



the eigenvalues of the product $\mathbf{tt}^\dagger$ of the transmission matrix, $\mathbf{t}$, and its Hermitian conjugate. For an "ideal" quantum point contact, N eigenvalues are equal to 1 and all others are 0. Deviations from exact quantization in a realistic geometry are about 1%. This value can be contrasted with the quantization of the Hall conductance in strong magnetic fields, where an accuracy better than 1 part in 10^7 is obtained routinely. One reason why a similar accuracy cannot be achieved in zero magnetic field is the series resistance from the wide regions, whose magnitude cannot be determined precisely.

Another source of excess resistance is quantum mechanical reflection at the entrance and exit of the constriction, due to the abrupt widening of the geometry. A magnetic field perpendicular to the electron gas suppresses this backscattering, improving the accuracy of the quantization.

Suppression of backscattering by a magnetic field is the basis of the theory of the quantum Hall effect developed by Marcus Buttiker of IBM in Yorktown Heights, New York.[8] Buttiker's theory uses a multireservoir generalization of the two-reservoir Landauer formula. The propagating modes in the quantum Hall effect are the magnetic Landau levels interacting with the edge of the sample. (Classically, these magnetic edge states correspond to the skipping orbits discussed later.) There is a smooth crossover from zero-field conductance quantization to the quantum Hall effect, corresponding to the smooth crossover from zero-field waveguide modes to magnetic edge states.

When 1 mode = 1 atom

Because the conductance quantum, e^2/h, contains only constants of nature, the conductance quantization might be expected to occur in metals as well as in semiconductors. A quantum point contact in a semiconductor is a mesoscopic object, on a scale intermediate between the macroscopic world of classical mechanics and the microscopic world of atoms and molecules. This separation of length scales exists because of the large Fermi wavelength in a semiconductor. In a metal, on the contrary, the Fermi wavelength is of the same order of magnitude as the atomic separation. A quantum point contact in a metal is therefore necessarily of atomic dimensions.

If the initial contact between two pieces of metal is formed by a single atom, the conductance will be of the order of $2\,e^2/h$. This was first observed in 1987 by James K. Gimzewski and R. Moller of IBM's Zurich Research Laboratory, in experiments in which the iridium tip of a scanning tunneling microscope was pressed onto a silver surface.[9] Upon making contact, the conductance jumped from an exponentially small value to $1/16\text{k}\Omega$. Later work, using mechanically more stable devices, showed that further jumps of order $2\,e^2/h$ in the conductance will occur as the contact area is increased.

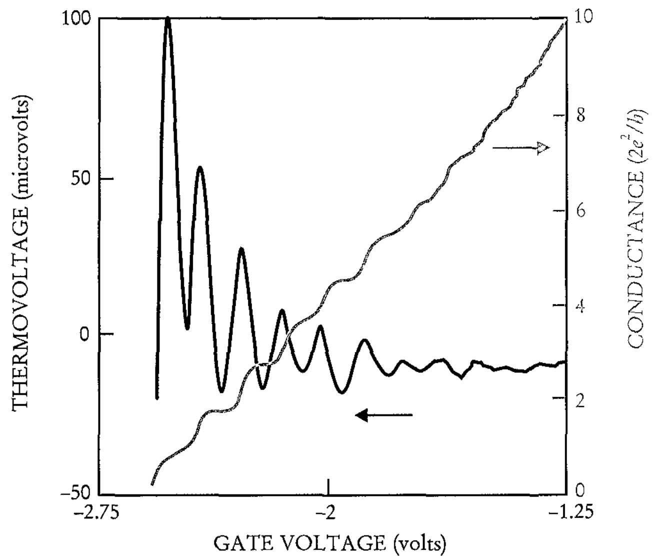
Figure 2 shows experimental data for a sodium point contact measured by Martijn Krans and his collaborators at the Kamerlingh Onnes Laboratory in Leiden.[10] An adjustable contact of atomic dimensions, with high mechanical stability, is made by bolting a notched wire of sodium onto a flexible substrate. As the substrate is bent, the wire breaks at the notch. The contact area can be controlled down to the atomic scale simply by bending the substrate more or less.

A statistical analysis of a large number of samples shows that, as the contact area is increased, steps in the conductance appear near one, three, five and six times $2\,e^2/h$. (Figure 2 shows the conductance steps for representative single measurements.) The absence of steps at two and four times $2\,e^2/h$ is significant, and has a neat explanation: In a cylindrically symmetric potential, the second and third transverse modes are degenerate, as are the fourth and fifth, while the first and sixth are nondegenerate.

The energy separation of transverse modes in a point contact of atomic dimensions is so large that the conductance steps are visible at room temperature. Nicolás García and his group at the Autonomous University of Madrid have made use of this property to develop a classroom experiment on quantized conductance. (See PHYSICS TODAY, February, page 9.)

Photons and Cooper pairs

The interpretation of conduction as transmission of electrons at the Fermi level suggests an analogy with the transmission of monochromatic light. The analog of the conductance is the transmission cross section, σ, defined as the transmitted power divided by the incident flux. The transmission cross section of a slit of variable width was measured by Edwin Montie and his coworkers at



THERMOPOWER OSCILLATIONS in a quantum point contact. The peaks in the thermovoltage, which is proportional to the thermopower, coincide with the steps in the conductance. (Adapted from ref. 14.) FIGURE 3

Philips.[11] Steps of equal height were observed in the transmission cross section whenever the slit width, W, equaled half the 1.55-μm wavelength of the light. Because σ equals W for large slit widths, the step height is also equal to $\lambda/2$. Two-dimensional isotropic illumination was achieved by passing the light through a random array of glass fibers parallel to the slit. The isotropy of the illumination mimics the reservoirs in the electronic case, and is crucial for the effect. The two-dimensionality is not essential, but was chosen because a diaphragm of variable area on the order of λ^2 is difficult to fabricate. (For a diaphragm, the steps in σ are $\lambda^2/2\pi$.)

It is remarkable that this optical phenomenon, with its distinctly 19th-century flavor, was not noticed prior to the discovery of its electronic counterpart. There is an interesting parallel in the history of the discovery of the two phenomena. In the electronic case, the Landauer formula was already known before the quantized conductance of a point contact was discovered. Yoseph Imry of the Weizmann Institute of Science in Israel had made the connection with Sharvin's work on point contacts.[3] The reason the conductance quantization came as a surprise was that the relation $\sum_n t_n = N$ for ballistic transport was regarded as an order-of-magnitude estimate. To have quantization, the relative error in this estimate must be smaller than $1/N$, which is not obvious. The equivalent of the Landauer formula for the transmission cross section has long been familiar in optics,[12] but also in this field it was not noticed that the relation $\sum_n t_n = N$ holds with a relative accuracy of better than $1/N$.

One can speak of the optical analog as a quantum point contact for photons. Can the analog be extended toward a quantum point contact for Cooper pairs? The answer is yes: The maximal supercurrent through a narrow and short, impurity-free constriction in a superconductor is an integer multiple of $e\Delta/\hbar$, where Δ is the energy gap of the bulk superconductor.[13] A superconducting quantum point contact has been realized by Hideaki Takayanagi and collaborators at Nippon Telegraph and Telephone Corp in Japan,[13] but the superconducting analog of the quantized conductance remains to be observed experimentally.

Thermal analogs

The conductance is the coefficient of proportionality between current and voltage. The additional presence of a small temperature difference, δT, across the point contact gives rise to a matrix of coefficients:

$$\begin{pmatrix}\text{electrical current}\\ \text{heat current}\end{pmatrix} = \begin{pmatrix} G & L \\ L' & K\end{pmatrix} \cdot \begin{pmatrix}-V\\ \delta T\end{pmatrix}$$

The thermal conductance, K, relates heat current to temperature difference. The thermoelectric cross-phenomena are described by coefficients L and L'. As first deduced by William Thomson (Lord Kelvin), time-reversal symmetry requires that $L' = -LT$ at a temperature T.

The two new transport coefficients K and L can be expressed in terms of the transmission probabilities, just like the electrical conductance, G. Approximately, $K \propto t$ and $L \propto \mathrm{d}t / \mathrm{d}E_F$, where $t = \sum_n t_n$ is the total transmission probability at the Fermi energy, E_F. (The proportionality of K to t, and hence to G, is the Wiedemann–Franz law of solid-state physics.) The stepwise energy dependence of the transmission probability through a quantum point contact implies two types of quantum-size effects: steps in K and peaks in L. Both effects have been observed by Laurens Molenkamp and his coworkers at Philips.[14]

The thermal conductance, K, of a quantum point contact exhibits steps when the gate voltage is varied, aligned with the steps in the electrical conductance. Each step signals the appearance of a Fermi level mode that can propagate through the constriction. A step in the transmission probability leads to a peak in the thermoelectric transport coefficient, L. Pavel Středa[14] at the Institute of Physics in Prague has calculated that, at zero temperature, the height of the peaks in L is approximately k/e times the conductance quantum e^2/h. The unit k/e, which is about 50 μV/K, is the entropy production per Coulomb of charge transferred through the point contact, or $1/e$ times the entropy carried by a single conduction electron, which is on the order of the Boltzmann constant, k.

Figure 3 shows measurements of the thermopower $S = -L/G$ of a quantum point contact.[14] (The thermopower is proportional to the voltage produced by a temperature difference for zero electrical current.) The coincidence of peaks in the thermopower with steps in the conductance (measured for the same point contact) is clearly visible. Joule heating was used to create a temperature difference across the point contact in this work. A more recent experiment used local heating by a focused beam of far-infrared radiation.[15]

Shot noise

The electrical current through a point contact is not constant in time, but fluctuates. The conductance determines only the time-averaged current. The noise power $P = 2\int \mathrm{d}t\, \langle \delta I(0)\delta I(t)\rangle \cos \omega t$ at frequency ω is the Fourier transform of the correlator of the time-dependent fluctuations $\delta I(t)$ in the current at a given voltage V and temperature T. One distinguishes between equilibrium thermal noise ($V = 0$, $T \neq 0$) and nonequilibrium shot noise ($V \neq 0$, $T = 0$). Both types of noise have a white power spectrum—that is, the noise power does not depend on frequency over a very wide frequency range. Thermal noise is directly related to the conductance through the fluctuation–dissipation theorem ($P_{\text{thermal}} = 4kTG$). Therefore, the thermal noise of a quantum point contact does not give any new information.

Shot noise is more interesting because it contains information on the temporal correlation of the electrons, which is not contained in the conductance. Maximal shot noise ($P_{max} = 2eI$) is observed when the stream of electrons is fully uncorrelated. A typical example is a tunnel diode. Correlations reduce the noise power below P_{max}. One source of correlations, operative even for noninteracting electrons, is the Pauli principle, which forbids multiple occupation of the same single-particle state. A typical example is a ballistic point contact in a metal, where $P = 0$ because the stream of electrons is completely correlated by the Pauli principle in the absence of impurity scattering.

A quantum point contact in a two-dimensional electron gas has a different behavior. Using a Landauer-type formula, Gordey Lesovik at the Solid State Physics Institute in Chernogolovka, Russia, has predicted peaks in the shot noise at the steps in the conductance.[16] The peak height $P_{peak} = eI$ is half the maximal value for uncorrelated electrons. The shot noise vanishes in between the steps. Michael Reznikov and his collaborators at the Weizmann Institute of Science in Rehovot, Israel, recently presented a convincing demonstration of this quantum-size effect in the shot noise.[16] (See figure 4.) By going to microwave frequencies of 8–18 GHz, they avoided the "$1/f$ noise" ubiquitous at lower frequencies.

Solid-state electron optics

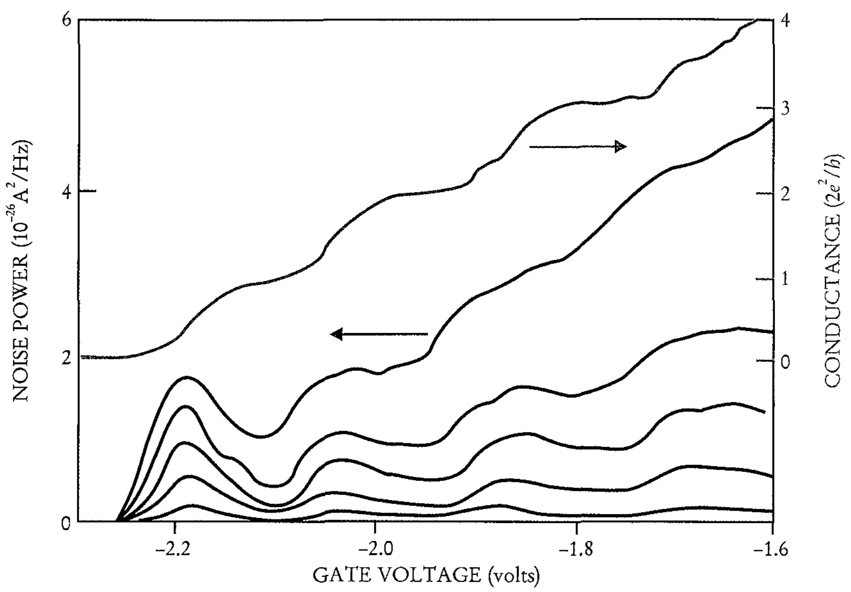
The effects discussed so far refer to properties of the quantum point contact itself. A wealth of new phenomena has been discovered using a quantum point contact as a spatially coherent point source and detector, and specially formed electrodes as mirrors, prisms or lenses.

Figure 5 shows the basic experiment[17] on coherent electron focusing. A point contact injects electrons with the Fermi momentum p_F into the two-dimensional electron gas, in the presence of a perpendicular magnetic field B. The electrons follow a "skipping orbit" along the boundary, moving in circular arcs of cyclotron diameter $d_c = 2p_F / eB$. Some of the electrons are collected at a second point contact, at a separation L from the first. The voltage measured at the collector is proportional to the transmission probability between the two point contacts. Valery S. Tsoi at the Solid State Physics Institute in Russia first used this focusing technique in a metal.[5] The magnetic field acts as a lens, bringing the divergent trajectories at the injector together at the collector. The collector is at a focal point of the lens when L is a multiple of d_c, and hence when B is a multiple of $2p_F /eL$ (arrows in figure 5). For reverse magnetic fields the injected electrons are deflected away from the collector, so that no signal is generated.

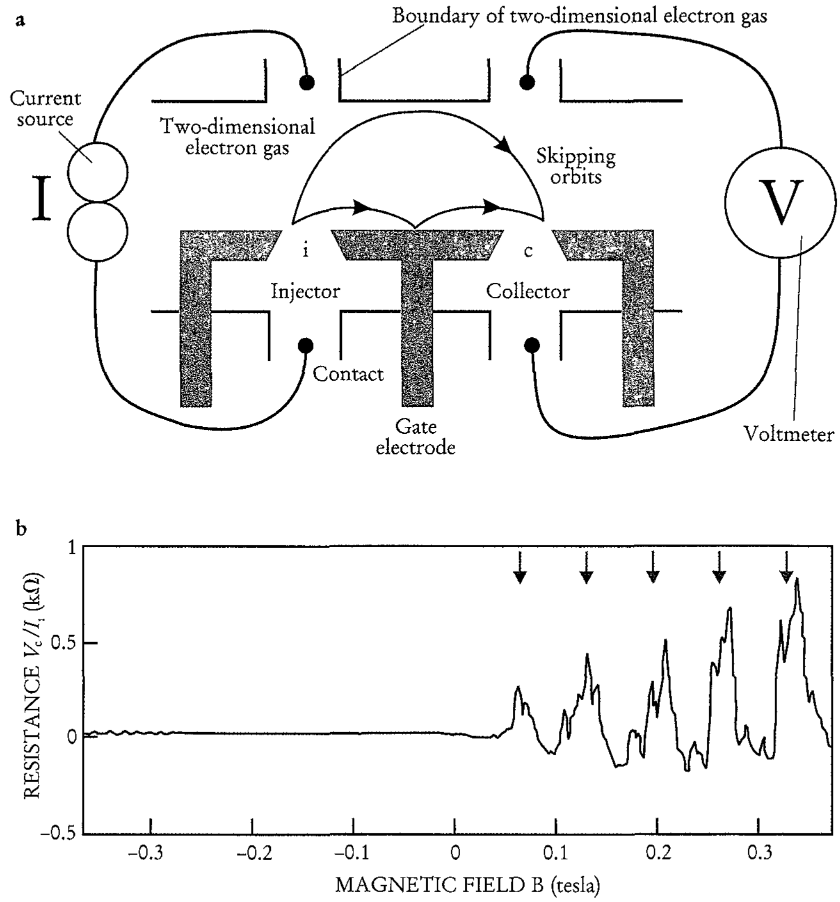
Observation of peaks at the expected positions demonstrates that a quantum point contact acts as a monochromatic point source of ballistic electrons, and that the reflections at the boundary of the two-dimensional electron gas are specular. The fine structure on the focusing peaks is due to quantum interference of trajectories between the two point contacts. Such fine structure does not appear in metals. It demonstrates that the quantum point contact is a spatially coherent source and that the phase coherence is maintained over a distance of several micrometers to the collector.

Several research groups have used magnetic focusing to obtain information on the dynamics and scattering of quasiparticles in the two-dimensional electron gas. An intriguing application in the regime of the fractional quantum Hall effect is the focusing of composite fermions,[18] which can be thought of as electrons bound to an even number of flux quanta.

In the regime of the integer quantum Hall effect, the geometry of figure 5 has been used to selectively populate and detect the magnetic edge states mentioned earlier.[6] The observation of plateaus in the Hall conductance at anomalously quantized values provides support for the



PERIODIC SUPPRESSION of the shot-noise power of a quantum point contact, measured with applied voltages of 0.5, 1, 1.5, 2 and 3 mV (black curves, from bottom to top). (Adapted from ref. 16.) FIGURE 4



MAGNETIC FOCUSING. **a:** Quantum point contacts are building blocks of solid-state electron optics. An example is electron focusing by a magnetic "lens." Electrons injected through one point contact (i) follow skipping orbits over a distance of 3 μm to a second point contact (c) acting as a collector. The two-dimensional electron gas boundary acts as a mirror, producing specular reflection. **b:** Magnetic focusing in a two-dimensional electron gas at 50 mK. The arrows indicate the positions of the focusing peaks expected when the point contact separation is a multiple of the cyclotron diameter. The fine structure on the peaks is due to quantum interference. (Adapted from ref. 17.) FIGURE 5

edge-state theory of the quantum Hall effect.

Electrostatic focusing, by means of the electric field produced by a lens-shaped electrode, provides an alternative technique to focus the beam of electrons injected by a point contact. Instead of focusing the beam, one can also deflect it—by means of either a magnetic field or a prism-shaped electrode. By now, the building blocks of electron optics in the solid state have all been realized.[5]

Ultimate confinement

A quantum point contact that is nearly pinched off (so that its conductance is less than $2e^2/h$) is a tunnel barrier of adjustable height for electrons near the Fermi level. This property has been used to inject and detect electrons in a small confined region of a two-dimensional electron gas, called a quantum dot. A quantum dot coupled to the outside by a pair of quantum point contacts has provided an ideal model system for the investigation of the effects of Coulomb repulsion on resonant tunneling. (See PHYSICS TODAY, January 1993, page 24.)

The zero-dimensional quantum dot forms the logical end to the reduction of dimensionality of the two-dimensional electron gas. As we have seen, the one-dimensional quantum point contact has played an important role in the conceptual development started by Landauer four decades ago. The concept of electrical conductance was conceived in the 19th century, even before the electron was discovered. It is amusing that it required the sophisticated microelectronics technology of the late 20th century to demonstrate experimentally that "conduction is transmission."

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