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SOLVING MATH PROBLEMS: WHERE AND WHY DOES THE SOLUTION PROCESS GO ASTRAY?

ABSTRACT. This article describes a model of adaptable learning which takes account of the interaction between affective and cognitive variables in learning situations. In addition, an instrument for measuring the relevant aspects of this model is introduced. We present some initial results of the application of this instrument in the domain of mathematics learning by students in the final year of primary education. Emphasis is placed on differences between boys and girls.

1. INTRODUCTION

1.1. *Background*

Mathematical problem solving can be seen as a process in which cognitive and affective variables interact. Schoenfeld (1992) has argued that doing mathematics can be considered as a social activity, with roots in the cultural and societal environment. Doing mathematics is also a social activity in the context of the classroom. Environmental variables interact with person variables to shape students' behavior as they work on mathematics tasks. Person variables may be cognitive or affective. The latter type include such constructs as self-concept, self-efficacy, confidence, anxiety, and causal attributions (cf. McLeod, 1992).

An important notion in current research is the distinction between attitudes and beliefs as (stable) products of cumulative experiences on the one hand, and cognitions and emotions that alter quickly in actual task situations, on the other. In the framework proposed by McLeod (1992), beliefs, attitudes, and emotions reflect the range of affective reactions involved in mathematics learning. These three types of affective reactions are distinct not only with respect to stability, but also with respect to their degree of cognitive loading. Beliefs have a very strong cognitive component; this cognitive loading decreases as one progresses from beliefs to attitudes to emotions.

Taking these distinctions into account, research can be divided into two main clusters. The first cluster takes beliefs and attitudes as a starting point and explores how these variables are related to such variables as achievement and enrollment in mathematics courses. This work is characterized by its emphasis on measurement issues, and its reliance on questionnaires and

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quantitative methods. The second cluster includes studies that emphasize how affective and cognitive variables interact to shape the actual behavior of an individual working on a mathematics task. This research paradigm is more in line with current research in cognitive psychology. The paradigm gives considerable attention to theoretical issues, and shows growing interest in qualitative methods of data collection (interviews and thinking-aloud protocols). McLeod and Adams (1989) present a number of examples of this new approach to the study of affect and cognition.

1.2. *The interaction between the cognitive and affective domains in the process of mathematical problem solving*

Within mathematical problem solving a distinction can be made between (meta)cognitive and affective aspects. Examples of cognitive aspects are operations on the knowledge base, which include such components as facts, definitions, algorithmic procedures, routine procedures, and problem solving (Schoenfeld, 1985). In the study of problem solving strategies, Polya's (1945) book 'How to solve it' has been extremely influential. In Polya's view problem solving is 'learning to grapple with new and unfamiliar tasks when the relevant solution methods (even if partly mastered) are not known' (Schoenfeld, 1992, p. 354). At the same time, however, the planning and execution of complex cognitive operations also demand executive control. Monitoring and control phenomena are mostly encompassed under the overarching term metacognition. In various studies it has been found that control strategies can be learned (Lester, Garofalo, and Kroll, 1989; Schoenfeld, 1989).

It is important to note that cognitive and noncognitive factors involved in problem solving are subject to monitoring and control, and that metacognitive aspects are crucial in problem solving. Doing mathematics requires not only knowledge of rules, facts and principles, but also an understanding of when and how to use that knowledge. These metacognitive aspects of problem solving are closely tied to affective phenomena. For example, in attempting to solve a problem, a student must make decisions about which strategy to apply and how long to keep on trying before stopping and selecting a new strategy. Schoenfeld (1983) has argued that the decisions that have to be made during problem solving can be influenced by all sorts of affective factors, such as expectations regarding success and failure, confidence in one's mathematical ability and the capacity to persist in the face of difficulties.

Although most research on problem solving has evolved out of the Polya tradition, and has devoted little or no attention to affective variables, a number of authors have stressed the relevance of affect in the analysis of cognitive processing (e.g., Lester, 1983; Charles and Lester, 1984; Schoenfeld, 1985; Silver, 1985). Mandler (1989) stressed the importance of the influence of emotions during problem solving. In his view, an important reason for the appearance of emotions during problem solving is the interruption of plans. These interruptions of planned sequences of thought or actions are called blockages, or discrepancies between what was expected and what is experienced. Especially during mathematical problem solving, the lack of a systematic plan may result in frequent interruptions. There are many reasons why an anticipated sequence of actions might not be completed as planned, and the individual's knowledge and beliefs about the math problem solving process play a significant role in the interpretation of these interruptions. For instance, students who believe that all mathematical problems can be solved by applying specific rules, may feel stuck after having tried in vain to apply a rule. When they think that no other heuristic is available to solve a specific problem or that the allotted time has almost passed, they may doubt that they can solve the problem, which may in turn lead them to experience anxiety or give up easily.

These affective responses may change during the problem solving process. According to Mandler (1989), any discrepancy in the course of problem solving represents a potential affective state. Beliefs and attitudes in relation to one's ability in math play a significant role here. For instance, students who believe that they are poor in solving math problems will already experience negative feelings in the beginning of the problem solving process. Some students may even express doubts about their ability to succeed on a task after having read the first item. That is, the affective response occurred prior to any attempt to solve the problem. It is also possible that a student might attack a problem confidently, losing confidence only when confronted with failure experiences in the course of problem solving.

Affective influences on problem solving will also vary depending on the heuristic strategies being used (McLeod and Adams, 1989). Consider, for example, a student who attempts to solve every problem through trial and error. A succession of errors may undermine confidence and pleasure in doing the task. If this student had more heuristic strategies at his or her disposal, the affective response might have been different.

1.3. *Gender-related Differences*

Gender-related differences in mathematical performance have been a subject of research for many years. A distinction can be made between research that examines the effect of environmental variables and that which explores person-related variables. There is evidence that some environmental variables exert a positive influence on the choices and behavior of males. For example, parental beliefs are a critical factor in determining students' attitudes toward mathematics, and it is believed that parents are often more encouraging of their sons' than their daughters' mathematical studies (Fennema and Sherman, 1977). Differences in patterns of teacher interactions with boys and girls also seem to affect math learning. For example, males tend to receive more encouragement and are more frequently praised for correct answers than females (Hart, 1989; Koehler, 1990; Leder, 1987). It is difficult, however, to estimate the effects of environmental variables on the math performance of boys and girls. The reported effects are often small and have to be considered in terms of a cumulative impact over time (Leder, 1992). Although acknowledging the importance of this type of research, we restrict ourselves to the effect of person-related variables on gender differences in math, without making statements about the causes of these differences.

Person related variables include both cognitive and affective variables. With regard to cognitive variables, most authors report that males are better in mathematics performance than females. Often a distinction is made between different types of mathematical problems for which gender-related differences in performance exist. Marshall (1984) reported that 6th grade girls performed better in computations than boys, whereas boys performed better than girls when story problems were involved. Gender-related differences in mathematics performance seem to be present from an early age onwards, and increase with age (Fennema and Carpenter, 1981; Hall and Hoff, 1988; Martin and Hoover, 1987). By the time they reach high school, boys often score higher on achievement tests, especially tests that entail problem solving (Eccles *et al.*, 1985; Kimball, 1989). Several authors reported that female students take fewer advanced math courses than male students and that they are grossly underrepresented in science and math professions (cf. Oakes, 1990). However, there is some evidence that differences between boys and girls are smaller than they were a decade ago (Friedman, 1989; Hyde, Fennema and Lamon, 1990).

As for affective variables, several theorists have demonstrated that differences between males and females in these variables favor males (Oakes, 1990; Leder, 1992). For example, girls tend to consider mathematics a masculine domain (Boswell, 1985; Fox, Brody and Tobin, 1985; Leder, 1986), and they show a less positive attitude towards mathematics than boys (Sherman, 1980). There is evidence that boys are more inclined to interpersonal competition, and that they more often than girls link success to capacity (Spence and Helmreich, 1983; Fennema, 1985). Boys also have higher perceptions of their own competence and higher performance expectations than girls, even if girls have equal or better results (Eccles *et al.*, 1985; Hyde, Fennema, and Lamon, 1990). Stipek and Gralinski (1991) reported that 3rd grade girls (between 8 and 9 years old) not only rated their math ability lower than did boys and expected to do less well, but they were also less likely than boys to attribute success to capacity and failure to bad luck. Moreover, they reported less pride following successful math performance, showed a stronger desire to hide their paper after failure and were less convinced that success could be achieved through effort. These achievement-related beliefs and attitudes are assumed to influence the student's choices and behavior, especially in mathematics learning (Eccles *et al.*, 1985).

With regard to the Dutch situation, Wijnstra (1988) reported clear differences in performance between boys and girls in the final years of primary education. Van de Werf (1988) indicated that although boys more frequently opt for math courses than girls and outperform them, the way in which the math curriculum is presented to students in the higher grades of secondary education (i.e., what content is covered at A-level and at O-level) influences their enrollment in math courses. A reorganization in the math curriculum which was introduced in a number of experimental schools led to a decrease in the difference between the sexes in enrollment in mathematics at O-level, but the difference in achievement level and in attitude towards mathematics remained. Girls who had successfully completed the math courses still viewed math as less personally relevant than did boys and they also perceived their competence to be lower.

The results reported above do not give insight into the processes that actually occur when boys and girls work on mathematics tasks, or in the interaction of attitudes and beliefs with task-specific factors. Important unanswered questions are: Do girls give up more easily than boys on mathematics tasks? And, if so, do certain beliefs and attitudes about mathematics play a role?

1.4. *The Model of Adaptable Learning*

Our research focuses on the interrelation of person characteristics and task-specific appraisals in actual learning situations. We set up the present study in an attempt to explore the mechanism by which gender differences in achievement-related beliefs and attitudes might influence the processes that occur during execution of mathematics tasks. The central question is how cognitive and affective variables interact when students work on an actual math task.

The model of adaptable learning as developed by Boekaerts (1991, 1992) was taken as a starting point. This model is based on the transactional theory of stress formulated by Lazarus and Launier (1978) and Lazarus and Folkman (1984). The model specifies that respondents, when confronted with a task, will use information from three main sources. The first source of information is the perception of the task and the physical, social, and didactic context in which it is embedded. The second source of information is activated domain specific knowledge and skills relevant to the task. The third source consists of manifest personality traits (including self-concept, goal-orientation, attitudes, beliefs, and personal goals). Information from these three main sources is used to dynamically appraise math tasks at task onset, during the task and at task cessation.

It is assumed that, when students perceive a learning situation, they may note a discrepancy between perceived task demands and perceived resources to meet these demands. Such appraisals, roughly comparable to what Mandler and Nakamura (1987) termed 'making sense' of the current situation, may be dominantly favorable or unfavorable at task onset and as such elicit positive or negative emotions. Intense emotions may influence upcoming and ongoing cognitive processes, not only because they draw the learner's attention away from the task, but also because toning down emotions may place demands on limited processing capacity (Bower, 1981; 1991). Unfavorable appraisals and negative emotions may be experienced upon confrontation with a math task, or they may develop while working on the task. For example, for some students being interrupted, or experiencing errors may in the past have been coupled with negative appraisals and emotions and they may signal to the learner that something has gone astray with the problem solving process.

It is theorized in the model of adaptable learning that, when the learner interprets learning situations or tasks as consequential for well-being, unfavorable appraisals and negative emotions (e.g., anxiety, anger, disappointment) may be dominant. The student's primary goal will then be to initiate activity in the 'coping mode' in order to restore well-being. On the other hand, when learning situations or tasks harbor the possibility of gains in competence for reasonable costs, favorable appraisals and positive emotions (e.g., joy, relaxation, excitement) will be dominant, leading to learning intention and to activity in the 'mastery mode'. The coping mode and the mastery mode co-exist and fight for priority in the individual's hierarchy of goals. This implies that, at any moment during the learning process, ongoing and upcoming appraisals and emotions may shift behavioral intentions from learning (the mastery mode) to stress reduction (the coping mode). For more details on this model, see Boekaerts, (1992) and (1993).

Boekaerts developed the On-line Motivation Questionnaire to estimate the values of a number of task-specific variables that are important components of the model of adaptable learning. This questionnaire is described elsewhere (Boekaerts, 1985; 1988). It was developed to obtain the learners' appraisals and emotions of the learning situation, and their learning intention, just before and just after the actual learning task. In one study (Seegers and Boekaerts, 1993), these task-specific variables were contrasted with trait measures of motivation (goal orientation, self-concept in math) to test the assumption underlying the model of adaptable learning that attitudes and beliefs concerning a specific subject area (e.g., math) would influence task-specific variables (subjective competence, pleasure in doing the task, and personal relevance). Willingness to invest effort in the task, emotional response, and task performance were considered outcome variables. It was found that willingness to invest effort, emotional state, and achievement were relatively independent outcomes of the appraisal process. Task-orientation was found to have an effect on estimated personal relevance and on pleasure in doing the task, while the latter two variables influenced willingness to invest effort. Tendency to attribute failure to lack of capacity had an influence on subjective competence, while the latter had an effect on both emotional response and task performance.

1.5. *Adaptable Learning and Gender-related Differences*

Seegers and Boekaerts (1995) reported data on gender differences in appraisals, learning intention, and performance on math tasks. They estimated mathematics ability for 6th graders (ages 11–12) with released items from a national assessment study (Wijnstra, 1988). They used the Online Motivation Questionnaire and an adapted version of Nicholls' (1983; 1984) Orientation Questionnaire to measure motivation. The results clearly confirmed gender-related differences, both for cognitive variables and for affective variables. It was found that boys outperformed girls in doing mathematical tasks. Differences were especially marked as item complexity increased. With regard to affective variables, they reported that boys made higher estimations of their ability in mathematics than did girls. This difference was not the result of better performance, because its effect remained when the influence of test results was controlled. Boys displayed a higher level of competitiveness than girls: Their scores on ego-orientation were significantly higher. No differences were found in task-orientation. The results also showed that girls were more inclined to ascribe failure to lack of capacity and that they were more inclined to cite effort as an explanation of their performance. Boys tended to emphasize capacity in explaining perceived success. These results regarding trait-characteristics are for the most part consistent with results found in other studies (e.g., Leder, 1992; Crombach, Boekaerts and Voeten, 1994).

The data on task-specific appraisals and learning intention show the advantageous pattern that is typical for boys. Boys made higher estimates of their competence at the task than did girls, while girls reported a higher level of willingness to invest effort. As willingness to invest effort was largely unrelated to achievement, learning intention had no positive effect on task results. This difference in reported willingness to invest effort may be an indication of readiness to put in quantitative effort as opposed to qualitative effort. Helmke (1990) made this distinction and showed that students who scored high on self-concept of math ability invested more qualitative effort in the learning process. Thus willingness to invest effort may be related with different forms of effort for boys and girls.

The results suggest that differences between boys and girls are due to mean differences in mathematics related cognitions rather than to differences in the pattern of the underlying relationships among the variables. Boys take a more competitive attitude, and evaluate their achievement in a more positive way. As a result, boys may have more favorable attributional beliefs, which may in turn affect their appraisals during task performance.

1.6. *Measuring Affective and Cognitive Processes On-line*

Task-specific variables, measured before and after working on a task, do not give specific information about the processes that occur during working on a task. As a next step in our research, we decided to investigate affective and cognitive variables during mathematical problem solving. By measuring task-specific motivation on-line during mathematical problem solving, we expect to gain more insight into individual differences in problem solving behavior. Emphasis is put on students' expectancies concerning successful goal attainment while they are working on a mathematics task, and their reactions to failure, when it occurs. We hypothesize that, when difficulties are anticipated or encountered, students will make an estimation of the extent to which they can (still) succeed on the task. Such 'confidence' or 'doubt' experienced in the different phases of the problem solving process is an important variable within our research. We agree with Carver and Scheier (1988) that the existence of certain emotions is less important than the way persons respond to these emotions. These authors argue that if a person, despite frustration, believes that he or she will be successful in attaining the desired goal, the result is continued striving, effective use of resources, and little or no impairment of performance. Even when frustrated, the person who is confident will continue to try. However, if the student is doubtful about the possibility of a good outcome, he or she experiences an impulse to disengage from the task, and this may cause a deterioration in performance.

The aim of our research is to describe how affective and cognitive variables interact in mathematical learning situations. The research fits within the broader tradition of the development of (efficient) learning environments by drawing on research tools of cognitive psychology. Our basic question is whether the instrument we developed is sensitive to individual differences in confidence and doubt during math problem solving. Emphasis is put on gender-related differences. Our research questions are:

Do the boys and girls in the study differ in mathematical problem solving ability and in solution time?

Do they differ in their task-specific cognitions, their emotions and learning intention before they work on mathematical problems?

Do they differ in task-specific attributions after doing math tasks?

Do they differ in their confidence and doubt while they attempt to solve mathematical problems?

What is the relation between affective and cognitive variables in actual problem solving situations?

2. METHOD

2.1. *Subjects*

Subjects were 30 sixth grade students (ages 11–12), 15 boys and 15 girls. These students came from eight schools situated in the urban region of Leiden. The selection procedure was as follows: Eight sixth grade teachers who agreed to take part in the study were asked to select boys and girls from their class who were average to good in mathematics. It was decided not to include students who were poor in mathematics in our sample, because our research questions require that students have minimal basic knowledge and understanding of mathematics. Namely, we are interested in motivational rather than cognitive sources of underperformance in mathematics. From the pool of provided students 30 were selected to take part in the study. No specific criteria were used apart from ease of making an appointment.

2.2. *The task*

The task consists of four mathematical problems: one algorithmic assignment and three word problems. The problems are given in Figure 1.

Tasks were given in an individual setting with only the experimenter present. The students started with the algorithmic problem and continued

Problem 1

5% of 46460=

Problem 2

In a school there are 350 students.
One day 28 students are ill.
What percentage is this?

Problem 3

About 1/5 of the people in the world live in China.
About 1/6 of the people in the world live in India.
What proportion of the people in the world live in China and India together?

Problem 4

From this piece of ground allotments are made.
These allotments are rented.
The whole piece of ground must bring in *f* 560,-.
How much rent must allotment D bring in?

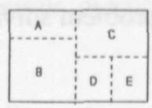

Fig. 1. The problems that were used in the mathematical task.

with the mathematical word problems. The latter problems are characterized by the fact that the sequence of steps which a student has to take in order to solve the problem is more ambiguous and less systematic than that required to complete an algorithmic problem.

2.3. *Instruments*

Besides the math task, two instruments were administered: the On-line Motivation Questionnaire (OMQ) and the Confidence and Doubt Questionnaire (CDQ). The OMQ consists of two parts. The first part is administered prior to the task and includes 7 items that refer to emotional state, followed by 17 items that cover task-specific cognitions and willingness to work on the task. The second part, which is filled in after the task, repeats the items on emotional state, followed by 7 items measuring invested effort, result assessment, and attribution of result. In both parts, subjects rate their position on four-point scales. Analyses of data from different samples of students, ranging in age from 10 to 14 years, served to distinguish clusters of items reflecting the student's appraisals in the following domains: confidence in doing the task – including success expectation, self-efficacy judgment, and perception of difficulty – (e.g., "How good are you doing these kind of tasks?"), perception of the attractiveness of the task (e.g., "How much do you like these kind of tasks?"), estimation of the personal relevance of the task (e.g., "How useful do you consider this task?"), willingness to invest effort (e.g., "How much effort are you going to invest in this task?"), emotional state (e.g., "Are you nervous?"), invested effort (e.g., "How hard did you work at this task?"), result assessment (e.g., "How well did you do on this task?"), and attribution of result (e.g., "I completed this task successfully because I put a lot of effort into it"). Cronbach's alphas for these scales varied from 0.71 to 0.86 (Seegers and Boekaerts, 1993).

The Confidence and Doubt Questionnaire is an instrument for registering on-line appraisals during mathematical problem solving. This instrument can be used to measure individual differences in (1) the use of problem solving heuristics and (2) the degree of confidence and doubt displayed at each step in the problem solving process. An important goal of the new instrument is to reveal why, and at what point in the problem solving process, students give up on a task. The different phases within the problem solving process, that are distinguished, are based on Polya's work. In the *orientation phase* the students' initial confidence or doubt, after they have read the problem, is measured. In the *execution phase*, the degree of confidence or doubt in relation to the various steps taken in the solution process is measured. In the *verification phase*, the estimated confidence and doubt about the correctness of the answer is measured.

In order to investigate these processes on-line, a special notation system was developed. While working on the task, students had to indicate to what extent they thought that their strategy would lead to the right solution. In the left margin of the working paper features five faces, ranging from very sad to very happy in their expression. They symbolize the degree of doubt or confidence a student has while working on the problem (see Figure 2).

The students are asked to put a mark under one of the faces (1) after having read the problem (the orientation phase), (2) at every solution step that is written down (the execution phase), and (3) after having found an answer (the verification phase). Marked faces are translated to scores ranging from 1 (very doubtful) to 5 (very confident). This notation system

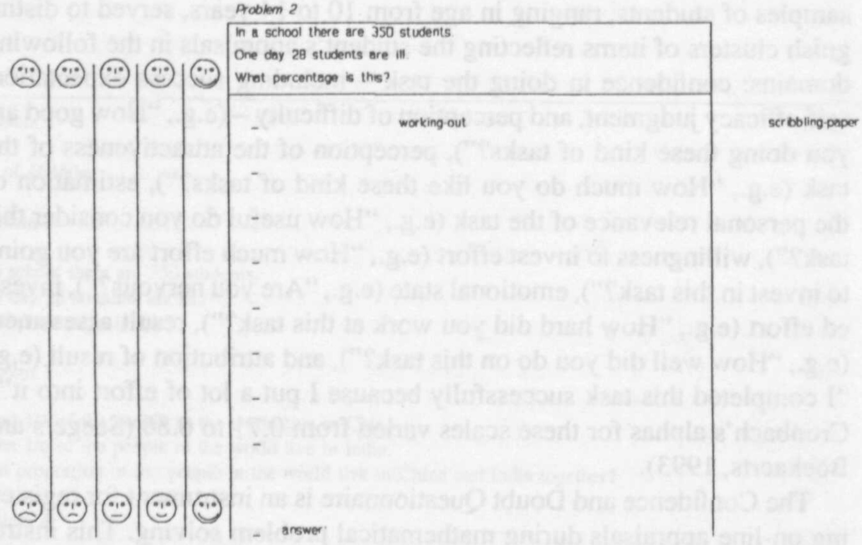

Fig. 2. An example of the Confidence and Doubt Questionnaire

provides at least three scores of confidence and doubt. In the first place an initial indication of confidence and doubt (smile1). Next an indication of the confidence and doubt experienced during the solution process (smile 2a, 2b, 2c,..., 2n). When a lot of solution steps are specified these scores may either be aggregated or scored individually. And finally, an indication of confidence and doubt in relation to the solution (smile3).

2.4. *Procedure*

Students were tested individually while they worked on the task. First, they filled in the first part of the OMQ. Two problems were given prior to testing in order to familiarize the students with the special notation system of the CDQ. The students were instructed to write down their solution process and calculations in as detailed a manner as possible. During the task, the students filled in the CDQ while they worked on the problems. After having given an answer, the students were told if it was correct or not. If their answer was incorrect, they were asked whether they wanted to try the assignment again. If students gave up, they were instructed to go on to the next assignment. In this study, no help was given to the students, because our main interest is in the individual problem solving behavior of students, comparable to working in test situations. Working time was

TABLE I

Means and standard deviations of math score and solution time for the total group and for boys and girls separately

	total group (n=30)	s.d.	boys (n=15)	s.d.	girls (n=15)	s.d.
score (1–4)	2.69	1.00	2.71	0.99	2.67	1.05
solution time (in minutes)	10.00	3.04	10.21	3.17	9.78	3.00

recorded. After the task, the students filled in the second part of the OMQ. Afterwards, some of the students were interviewed about their problem solving behavior and their confidence and doubt in relation to the different problems.

3. RESULTS

3.1. *Quantitative Measures*

As a first step, we investigated whether there were gender differences in scores on the mathematical problems or in solution time. In Table I the means and standard deviations for these variables are given. The results show that there were no statistically significant differences between boys and girls in either mean scores or in solution time.

3.2. *On-line Motivation Questionnaire*

Next, we investigated whether the gender differences in task-specific cognitions and emotion that have been found in prior research are corroborated in our sample. Our sample is small, so the results should be considered with care. The results from the pre-task items of the OMQ are given in Table II. Ranges of scores are from 1 to 4.

Boys display higher scores than girls on all subscales at task onset. These differences in scores are significant for four of the subscales: subjective competence, task attraction, learning intention, and emotional state. Differences between boys and girls in subjective competence at task onset are in line with findings from earlier research. However, the gender differences in learning intention are not: Crombach, Boekaerts and Voeten (1994) and Seegers and Boekaerts (1995) found that girls reported a higher level of willingness to invest effort than boys. Unequal task situations

TABLE II

Means scores and standard deviations when comparing the subscales of the On-line Motivation Questionnaire at task onset, for boys and girls separately

Subscale	Total group (n=30)	s.d.	Boys (n=15)	s.d.	Girls (n=15)	s.d.
subjective competence	2.83	0.37	2.99	0.27	2.67*	0.40
task attraction	2.96	0.44	3.16	0.35	2.76**	0.43
personal relevance	2.65	0.53	2.70	0.59	2.60	0.47
learning intention	3.34	0.46	3.58	0.34	3.11**	0.45
emotional state	3.27	0.44	3.50	0.30	3.05**	0.12

* t-test significant at $p < 0.05$
** t-test significant at $p < 0.01$

might have contributed to these unexpected findings: In this study testing took place in an individual setting, whereas in the other studies the mathematics test was administered in a group setting.

After completing the task, the students filled in the second part of the OMQ. No significant gender differences were found in scores on the post-task OMQ subscales: result assessment, effort expenditure, and emotions. A possible explanation for this is that all the students received feedback on the correctness of their answers following completion of each problem, which might have influenced their subjective appraisals of subsequent problems. However, differences between boys and girls were found in attribution of result. One item of the OMQ concerned attributions for success or failure on the task. Because only a small number of the students reported that they had failed on the task, we will discuss only the results for those students who believed that they had performed adequately. To test for differences between boys and girls, a Mann-Whitney U-test was applied to the data. The results of this are given in Table III.

Boys attribute their (positive) result more frequently to capacity and to invested effort, but only the difference between boys and girls in the degree to which positive results were attributed to *pleasure* was statistically significant. Boys more frequently mentioned that the pleasure they experienced in working on the task explained their positive result.

3.3. *Confidence and Doubt Questionnaire*

Table IV displays the mean confidence and doubt scores expressed during the mathematical problem solving task. Smile1 is an indicator of the mean

TABLE III

Results of the Mann-Whitney U-tests: differences between boys and girls in attributions for perceived (positive) task result

Attribution	Boys (n=13)	Girls (n=10)
capacity	12.85[1]	0.90
effort	12.33[1]	0.50
pleasure	15.12	7.95*
easiness	12.12	11.85
luck	12.19	11.75

* significant at $p < 0.01$

TABLE IV

Means and standard deviations of degree of confidence after first reading a problem (smile1) and after finishing a problem (smile3), for the total group and for boys and girls separately

	total group	s.d.	boys	s.d.	girls	s.d.
smile1 (1–5)	3.72 (n=30)	0.61	3.96 (n=15)	0.54	3.50* (n=15)	0.60
smile3 (1–5)	4.06 (n=20)	0.88	4.39 (n=9)	0.56	3.80 (n=11)	1.03

* t-test significant at $p < 0.05$

degree of confidence students expressed after having read the four respective problems. Smile3 represents the mean degree of confidence expressed after having worked on the four problems. These measures correspond respectively to the degree of confidence in the orientation phase, and to the degree of confidence in the verification phase. Ranges for these scores are from 1 (very doubtful) to 5 (very confident). The degree of confidence or doubt during the execution phase (smile2) will be further described in the section on qualitative measures. Table IV only includes data corresponding to the students' first attempts to solve a problem; data on smile3 were not collected for students who gave up before they reached a solution on one or more of the problems.

Gender differences were found in confidence displayed, both after first reading a problem (smile1) and after finding a solution (smile3). In the case

TABLE V

Means and standard deviations of degree of confidence after first reading a problem (smile1), and after finishing a problem (smile3), for those students who succeeded in the first attempt

	Total group smile1	s.d.	Total group smile3	s.d.	Boys smile1	s.d.	Boys smile3	s.d.	Girls smile1	s.d.	Girls smile3	s.d.
Problem 1	4.48	0.73	4.55	0.60	4.67	0.65	4.55	0.69	4.27	0.79	4.55	0.52
	(n=23)		(n=22)		(n=12)		(n=11)		(n=11)		(n=11)	
Problem 2	3.62	0.96	4.15	1.07	3.86	0.69	4.14	0.90	3.33	1.21	4.17	1.33
	(n=13)		(n=13)		(n=7)		(n=7)		(n=6)		(n=6)	
Problem 3	3.75	1.07	4.05	1.13	3.70	1.25	4.30	1.06	3.80	0.92	3.78	1.20
	(n=20)		(n=19)		(n=10)		(n=10)		(n=10)		(n=9)	
Problem 4	3.68	0.78	4.45	0.86	4.11	0.78	4.89	0.33	3.38*	0.65	4.15*	0.99
	(n=22)		(n=22)		(n=9)		(n=9)		(n=13)		(n=13)	

* t-test significant at $p < 0.05$

of smile1, the difference is significant: After initial reading of a problem, boys showed more confidence than girls. The difference remains for smile3, but does not surpass the level of statistical significance. Given the small sample size this is not surprising. In Table V, the mean confidence scores after first reading the problem (smile1) and after finishing the problem (smile3) are given for those students who in fact succeeded in solving a problem in the first attempt.

As can be seen from the number of students who succeeded in solving a problem in the first attempt (see Table V), every problem was solved by about as many boys as girls. It is striking, however, that more girls than boys succeeded on problem 4 in the first attempt, whereas boys expressed more confidence. These differences are statistically significant and are found both before (smile1) and after (smile 3) working on the problem. In this stage of our research, we can only speculate about the reasons for these differences. Considering the number of students who solved the problem in the first attempt, problem 4 does not appear to be more difficult to solve than the other problems.

3.4. *Qualitative Measures*

The confidence or doubt displayed by the students during the *execution stage* (smile2) is not necessarily consistent with the (in)effectiveness or (in)correctness of the applied strategies. In this section, we pay special attention to this interaction of subjective confidence and strategy use. A qualitative description of the solution behavior of students solving mathematical problems is given. Classifying students according to the pattern of interaction between cognitive and affective factors, we distinguished three types of students. Students who are *optimistic* about their solution process show confidence in their solution strategy, even though this strategy is not an effective one. Students who are *pessimistic* about their solution process express doubt about their solution strategy, even though their strategy is an effective one. Students have a *realistic* view of their solution process when the degree of confidence/doubt is in congruence with the (in)effectiveness of the solution strategy. More specifically, an effective strategy is connected with a high degree of confidence, or an ineffective strategy is connected with a high degree of doubt. To illustrate this, a few examples of the problem solving behavior of the students, in solving problem 2, will be given. In these descriptions, the 5-point scale for level of confidence and doubt will be referred to in the following terms as: very doubtful (1), doubtful (2), not doubtful/not confident (3), confident (4), very confident (5).

3.5. *Student 1*

After having read the problem, this student is confident that he can solve the problem. The first solution step he writes down as: 28 of 100 = 28%. At this step, he indicates that he is very confident. In the second step, he multiplies 28% by 3, and adds 14%. This gives: 84 + 14 = 98. He is again very confident that this step will lead to the solution of the problem. He writes 98% down as the solution to the problem and expresses high confidence in its correctness. Upon hearing that this answer is not right, he is very surprised, and starts again. In the second attempt, he indicates that he is doubtful. He starts by 'clever calculating': 100 = 28%, 200 = 14%, 300 = ? At this point, he is stuck, looks back at the assignment, and changes his strategy: $3 \times 28 = 84$. Here, he indicates that he is not confident/not doubtful. However, when writing down the solution, $85\frac{1}{2}\%$, he expresses confidence in the correctness of the solution.

This is an example of a student who is too optimistic about his solution process. The student does not show any signs of reflecting on the problem before he starts trying to solve it. He hopes to arrive at a correct answer by 'number crunching'. Stated differently, the student applies algorithms in order to find an answer, without orienting himself to the problem or estimating the range of possible solutions. But still, this student is very confident about his solution strategy. When he attempts to solve the problem for the second time, he is less confident. This could be due to the feedback he received.

3.6. *Student 2*

This student is very confident after having read the problem. As a first step, he calculates 1% of 350 = 3,5. He is very confident about this step. As a second step, he writes down 28 × 3,5 = 98,0, after he has made the calculation 28 × 3,5 on the scrap paper. He expresses doubt about this step, and puts it between brackets. Then, he writes down 28 : 350 = and begins with a 'long division', but stops in the middle of it. He indicates then that he is very doubtful. He gives up, putting a question mark in the spot where the answer should be.

This is an example of a student who is algorithm-dependent. He tries out alternative algorithms in a fairly random order. He is realistic about his own solution process and stops when he sees no way out.

3.7. *Student 3*

After having read the problem, she is doubtful. As a first step, she writes down: 1% = 3,5, indicating that she is now confident. The next step is: 3,5 : 28 = 8 (she means, 28 : 3,5). She indicates that she is confident about this step. She has calculated this by trying different numbers: first 6 times 3,5, then 8 times 3,5. Her answer is 8%, but she is doubtful about this answer.

This student was interviewed after the task. She said that she found the assignment difficult at first sight. She first tried to divide 28 : 3,5, but this did not lead to a solution. Then she started to calculate mentally: 9 times 3,5 appeared to be too much, so she tried 8 times 3,5 and this worked out. To the question why she thought her answer was wrong, she answered: "I just thought it was not right".

This is an example of a student who is unrightly pessimistic about her solution process. She demonstrates metacognitive skills and uses an effective strategy, and yet she is not confident that she has found the right answer.

4. DISCUSSION

In the present study no differences were found between boys and girls in their mathematical performance and solution time. However, we did find gender-related differences in the way boys and girls make sense of actual mathematical learning situations. Hence differences in affective factors were found: *Before* starting with the math task, boys displayed more confidence, more pleasure, more positive emotions, and a higher learning intention than girls. It is interesting to note that no differences were found in perceived personal relevance of math tasks. *After* having worked on one of the problems (problem 4) boys also showed more confidence than girls. Because of the small sample size that was investigated, we can not yet draw conclusions about the qualitative gender-related differences in actual problem solving behavior. However, the differences in affective factors displayed so far suggest that the relations among trait-affective variables reported in the literature are paralleled at the task-specific level. The results at this level confirm that boys and girls tune in differently when processing mathematical problems. For boys a relevant aspect of the mathematical learning environment is the challenge and competition it elicits. Their constructive attributional beliefs call for favorable scenarios which generate confidence rather than doubt. Girls may more than boys believe that doing math is applying a set of rules. When they are not sure that they know 'the' necessary rule they may want to protect their ego by lowering their affects and expectations.

At present a large scale study is in progress in which the insights gained in this study are being empirically tested. We expect that doubt expressed by girls will not necessarily coincide with less efficient solution strategies or with less effective performance (cf. student 3), and that high confidence in boys (cf. student 1) may sometimes be linked with experimentation with alternative algorithms in a fairly random order, without any sign of orientation or reflection. The casual observer might remark that one need not worry about girls' futures in relation to mathematics if it were to be found that they equal boys in the use of self-regulatory skills and display equivalent performance. We believe, however, that such a conclusion would be unwarranted, because underconfidence may cause girls to chronically underestimate their math ability, leading to gender differences in attitudes and beliefs about mathematics, and in girls, to low qualitative effort and underenrollment in math-oriented educational programs. (cf. Eccles *et al.*, 1985).

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