Provisional scheme for the determination of fundamental declinations from azimuth observations

Sitter, W. de; Oort, J.H.

BULLETIN OF THE ASTRONOMICAL INSTITUTES
OF THE NETHERLANDS.

1925 July 4 Volume III. No. 81.

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Provisional scheme for the determination of fundamental declinations from azimuth observations, by *W. de Sitter* and *J. H. Oort*.

The systematic errors of our existing fundamental systems of declination are known to be large compared with the accidental errors of observation with modern meridian instruments. Practically all our declinations have been derived from observations of zenith distance in the meridian, and are thus subject to the systematic errors affecting the latitudes, the refraction and the flexure of the telescope and the circles. It has for some time been the desire of the Leiden Observatory to draw up a programme of observations of azimuth for the determination of a fundamental system of declinations covering the whole sky, and free from the sources of error affecting the observation of zenith distances.

A special instrument will be built for this purpose. It will consist of a horizontal telescope of about 90 mm aperture and about 1 m focal length, of the „broken" type, the prism, or mirror, being at one end near the object glass, and the eyepiece at the other end of the horizontal tube. This tube will rotate on its optical axis in two V's and will be reversible, a striding level being supplied to test the horizontality of the axis. The azimuth will be read off from a horizontal circle of at least 50 cm diameter by two or four microscopes firmly connected to the movable frame carrying the V's. The circle must be rotatable in its plane, so as to use different zero-points on different nights. The instrument will be of very solid and stable construction, a special feature being that the vertical distance between the telescope and the circle will be made as small as possible. The vital parts of the instrument are the division of the horizontal circle, and the striding level of the axis. Beyond these it need not satisfy any very high requirements of centring, etc.

In order to cover the whole sky, the observations should be made from three different stations, one on or near the equator, and one in each hemisphere at a latitude somewhere between 35° and 50°.

Let us consider the equatorial station first. In 1917 Mr. C. SANDERS has proposed a method by which declinations can be measured free from errors in the vertical refraction. *) It consists in the measurement of differences of azimuth and zenith distances, east and west, from a station near the equator. Tentative observations along these lines were carried out by Mr. SANDERS at his observing station in Matuba (latitude 5° 17′ south) and were discussed in these pages. **) Notwithstanding rather bad observing conditions and a relatively small instrument the results showed a satisfactory agreement between the various observations of different stars in the same zone of declination.

The present plan for the equatorial station will be essentially that followed by Mr. SANDERS; the measurement of the zenith distance is however to be replaced by a determination of time: the star being followed by the slow motion in azimuth, a time signal will be given when it is near the centre of the field and the azimuth readings will be made at that point. Each star will be measured in both positions of the instrument and also both east and west of the meridian at equal altitudes. By this procedure it is hoped to eliminate personal errors in the azimuths as well as in the time recording. In order to make the conditions at the western observations exactly equal to those at the eastern observations we intend to use a reversing prism and an artificial reduction of the light with the non-prism observations with at least part of our programme. It will be advantageous to measure at rather large zenith distances because the influence of errors in the hour angle as well as in the latitude of the station is there considerably reduced. As it appears impossible to tell beforehand at which zenith distance the loss due to poorer condition of the atmosphere will begin to outweigh the gain, it would

*) *The Observatory*, 40, 271.
**) *B. A. N.*, Vol II, 76, p. 201, 1925.

seem wise to perform measures at various zenith distances.

The coefficients, with which the errors in the measured quantities enter into the declinations computed from them, are found by differentiating the equation

$$(1) \qquad \tan\delta = \tan\varphi \cos t - \cot A \frac{\sin t}{\cos\varphi}$$

We find

$$(2) \qquad d\delta = -\cot q \cos\delta\, dt + \frac{\sin z}{\sin q}\, dA + \frac{\cos z \cos\delta}{\cos\varphi}\, d\varphi$$

in which q represents the parallactic angle.

The coefficients of dt and $d\varphi$ are tabulated below for a station at the equator:

TABLE I.

Coefficients with which uncertainties in t and φ enter into the declination.

z \ δ	Coefficients of dt					Coefficients of $d\varphi$				
	0°	20°	40°	60°	70°	0°	20°	40°	60°	70°
50°	0.00	0.30	0.76	—	—	0.64	0.60	0.49	0.32	0.22
60°	.00	.20	.42	∞	—	.50	.47	.38	.25	.17
70°	.00	.12	.24	0.41	∞	.34	.32	.26	.17	.12
80°	.00	.06	.11	.16	0.19	.17	.16	.13	.09	.06

If measuring at a zenith distance of 70° the declinations of stars between + 60° and — 60° declination will be easily obtained with sufficient accuracy; the coefficient of dt varies from 0.00 to 0.41, the average of about 0.20 allowing a rather considerable inaccuracy in the time determination. Although it is not necessary that the observing station is situated exactly at the equator, conditions would become less favourable very rapidly with increasing latitude. Already at 7° or 8° the difference of factors for the northern and the southern stars becomes very considerable, so that practically observations can only be made on stars whose declination has the same sign as the latitude.

The coefficients of dt being equal but of opposite sign at equal zenith distances east and west of the meridian, any errors in the right ascensions of the stars will be eliminated if we observe them in both positions. In order to get rid of the remaining systematic errors in the clock it will be advisable to choose the programme in such a way that in each declination zone the stars can be observed in pairs, one east and one west, with only a short interval between the two observations. We shall then only need to trust to the rate of our clock during this interval.

In the instance considered the coefficient of $d\varphi$ ranges from about one third to one fifth. The *average* error in the assumed latitude can be eliminated by Talcott observations of the same stars as have been measured with the azimuth instrument. The *variation* of the latitude will be known well enough from the existing latitude stations, as the remaining local variations are always too small to affect our results. The variation of the longitude is eliminated by the observation of equal numbers of stars east and west.

In what follows we shall give an estimate of the precision with which the declinations can be expected to be determined from these observations.

We shall assume a probable error of ± 0."35 for the mean azimuth from four successive pointings on a star (two in each position of the instrument). This probable error includes uncertainties in level and collimation, which will be small compared with the errors of pointing. That a value of ± 0."35 is well inside the limits of possibility is sufficiently demonstrated by the observations of Mr. SANDERS quoted above. From his azimuth measures made at zenith distances between 55° and 80° a probable error of ± 0."35 was found for a night's observation of one star from an intercomparison of the results on different nights. A night's observation of a star consisted of 10 consecutive pointings, and, as the probable error of pointing (as found from a rough intercomparison of the different pointings on one night) amounts to about ± 0."5, the corresponding probable error of the azimuth from 4 pointings would be ± 0."40. This error includes level and collimation errors as well as those in zenith distance, though the latter only with a small coefficient; it is derived from observations made with an instrument of considerably smaller dimension than the one planned for the Leiden Observatory, and it appears quite likely therefore that with the larger instrument we may reach a value below 0."35.

To some astronomers it may seem surprising that in a tropical climate it is possible to make good observations at zenith distances as large as 75° or 80°. The quoted article by SANDERS and OORT gives some valuable information on this point. Selecting those stars from table 4 *) for which the average zenith distance exceeds 73° we find a probable error of ± 0."31 (± 0."05 p. e.) at an average zenith distance of 74°.4; in a similar way the stars with an average zenith distance below 64° give a probable error of ± 0."44 (± 0."05 p. e.) at an average zenith distance of 59°.2. These values seem to prove that up to a zenith distance of 75° the accuracy does not diminish with increasing zenith distance.

After the observer has got some practice along these lines the probable uncertainty in the record of the time at which the azimuth observation is made

*) *L. c.* page 206.

is not likely to exceed a few hundredths of a second. We shall certainly not overestimate the accuracy if we adopt a probable error of ± 0.″50 in arc of great circle for an average of four pointings. It is to be noted that the constant personal lag between the moment of the pointing and the time signal will again be eliminated from the combination of the east and west measures.

It will be necessary, in order to eliminate the errors of the clock, to observe a closed cycle of stars in each declination zone. Thus if n denotes the number of equal parts into which the hour circle is divided, and l the number of these divisions lying between the star observed east and the one observed west, we shall always observe at hour angles of $l/n \times 12^h$.

The above values for the probable errors of dA and dt have been inserted in formula (2) and the probable errors of δ have been computed. The probable errors given in Table 2 are those of the declination of one star, after elimination of the latitude from one azimuth determination of this star as well as of its Talcott companion and one Talcott observation of the pair. If four azimuth observations had been made of each star and only one Talcott measure, the probable errors would be very nearly halved.

A probable error of ± 0.″09 was adopted for the latitude from one Talcott observation.*)

TABLE 2.

Probable errors of declinations from a station at the equator.

δ	$l/n = {}^2/_5$ ($t = 4^h.8$)		$l/n = {}^1/_3$ ($t = 4^h.0$)		$l/n = {}^1/_4$ ($t = 3^h.0$)		$l/n = {}^1/_5$ ($t = 2^h.4$)		$l/n = {}^1/_6$ ($t = 2^h.0$)	
	r_δ	z	r_δ	z	r_δ	z	r_δ	z	r_δ	z
0°	±0.″41	72°	±0.″49	60°						
10	.41	72	.49	60						
20	.41	73	.48	61						
30	.41	74	.48	63	±0.″67	51°				
40	.41	75	.47	66	.61	56	±0.″77	52°		
50			.46	70	.58	62	.65	59	±0.″91	55°
60			.46	74	.57	68	.67	66	.92	63
70					.56	75	.72	73	1.16	71

No numbers have been inserted if the zenith distance came out smaller than 50° or larger than 75°.

As we shall see later on a practicable programme will consist of about 20 stars in each zone of 10° of declination, and four complete observations of each star. We find the probable error of the average declination in each zone by dividing the numbers of the table by $\sqrt{80}$. With suitable values of l/n these errors can be brought down to ± 0.″05.

If we want to be free from the assumption that north and south refractions are equal we may use only the two or four zones nearest to the zenith for eliminating the latitude. The probable errors will remain about the same, but the different zones of declination will be no longer independent of each other. Talcott measures of the other zones will still provide a check on the azimuth results, or — if one prefers to put it that way — on the assumed equality of the refraction in the two directions.

We now proceed to the other stations. If at these we want to get a determination of the latitude independent of the results of the equatorial station, we must select a latitude of at least 30°. On the other hand a low latitude will be decidedly advantageous for the determination of the declinations of stars culminating between the zenith and the pole from the measurement of their greatest elongations: the lower the latitude of the station the larger the zone overlapping the observations from the equatorial station and the smaller the factor with which uncertainties in the latitude enter into the declinations.

A latitude of 35° will be about the most favourable in all respects, and we shall adopt this value in the following discussion. The same conclusions will hold for a station at, say, 50° but for the quantitative disadvantages mentioned above.

As long as we are observing quite near the greatest elongation it will not be necessary to have any exact knowledge of either the time or the right acsension of the star. It should be easy to get the average time of the four pointings within one minute of the time of greatest elongation. For a star at declination 50°, ($\varphi = 35°$), the average coefficient of dt will then be less than ± 0.002. Thus we only need to know our time and right ascension within half a second of time.

The coefficients with which $d\varphi$ enters into the declination computed from the greatest elongation azimuth, as well as the zenith distances at greatest elongation, are shown in table 3.

TABLE 3.

Average declination	$\varphi = 35°$		$\varphi = 50°$	
	coeff. of $d\varphi$	z	coeff. of $d\varphi$	z
35°	1.00	0°	—	—
45	0.70	36	—	—
55	.49	45	0.83	21°
65	.33	51	.56	33
75	.19	54	.32	38
85	.06	55	.10	40

*) J. STEIN, *Leiden Annals*, Vol 9, p. 123.

Several methods are available to determine the latitude of the station from azimuth or Talcott observations of the stars measured at the equatorial station with fair enough accuracy to eliminate $d\varphi$ from the observations of the stars near the pole. As we propose to try a method not hitherto used, it will however be much better to follow a different course and to determine the latitude wholly from observations made at the station itself. The intercomparison of the overlapping zones, directly or by Talcott measures, will then decide whether the method and its results are trustworthy or not. In our opinion the advantage of this test quite outweighs the disadvantage of the extra amount of labour devoted to the determination of the latitude, even if it were very considerable.

Several methods have been proposed for determining the latitude free from systematic errors in star positions, time and vertical refraction. We shall consider that of KAPTEYN, *) which has been applied by COURVOISIER, **) and that of COURVOISIER himself. ***)

KAPTEYN'S method consists, briefly, of measurements of the times of transit through a certain vertical of stars whose southern zenith distances of culmination are about equal to the colatitude of the station, and of observations of the difference in meridian altitude between these stars and close circumpolar stars. KAPTEYN has very clearly proved that the programme can be chosen in such way that the result will be free from all errors in the positions of the stars as well as from the clock errors. It assumes, as has been assumed throughout the present discussion, that no considerable lateral refraction exists, but also that there is no difference between the vertical refraction north and south. A difficulty with the method is that, in order to find a practicable observing programme, one will have to go down to rather faint stars, especially if a Talcott instrument is used to measure the differences in meridian altitude. At our station at 35° latitude we shall have to confine ourselves to circumpolar stars within 10° of the pole because the zenith distances at which the azimuth measures are to be made would otherwise become larger than 75°. In this case we shall find about 10 suitable cycles of 5 circumpolar stars brighter than $8.^m0$ and an average of 1/30 of such a cycle of stars brighter than $7.^m0$. We might of course extend the limits of the regions in which the stars are to be situated so as to get a sufficient number of stars brighter than $7.^m0$, but by doing so we would probably get into serious difficulty with the elimination of the errors of the clock.

*) *Copernicus*, Vol. 3, p. 147, 1883.
**) L. COURVOISIER, Inaugural Dissertation, 1901.
***) *L. c.* p. 110.

COURVOISIER'S method does not put such restrictions on the programme, nor does it make use of the assumption that the vertical refraction is equal in different azimuths. He proposes to select sets of four stars whose declinations do not differ more than one degree from the latitude of the station and whose right ascensions differ by about six hours. Of each star the two transits through each of two verticals (situated symmetrically with respect to the meridian) are to be observed. Two of the transits occur near the zenith with only a short interval between them*), the other two occur at an hour angle of ± 6 hours corresponding with an average zenith distance of 71°. Neither star positions nor clock errors have a sensible influence on the results for the latitude. As far as the selection of the programme is concerned it will probably be possible to find a good observing list even among the stars brighter than $6^m.5$.

Let us now, with some simple assumptions, determine the number of observations which would be necessary to get a good determination of the latitude. We shall, somewhat arbitrarily, adopt the following probable errors which have been inferred from observational data given in the cited memoirs, and are considered to be about applicable to the instruments under consideration:

in A	±0″.20	(only including the uncertainty in the circle readings and possible differential sidewise bending).
in a difference of two times of transit	±0″.45	(11 threads; compare KAPTEYN, *l. c.* p. 174).
in the inclination of the horizontal axis	±0″.15	
in the difference of z from one Talcott observation	±0″.18	(compare p. 3 of the present paper).

For KAPTEYN's method the probable error of a latitude as determined from 4 transits and 2 Talcott

*) It is not strictly necessary to keep very near the zenith, except to avoid the errors introduced by an uncertainty in the rate of the clock between the two upper transits.

observations comes out $\pm 0''.24$ (taking $l/n = 2/5$), whereas COURVOISIER's method gives a probable error of $\pm 0''.49$ of the latitude as computed from 2 transits (which is about four times the probable error of the latitude obtained from two meridian circle altitudes of a circumpolar star). The difference in accuracy between the two methods appears to be small, 8 transits with the second method giving about the same results as 4 transits + 2 Talcott observations with the first one. COURVOISIER's method would thus appear preferable, as it does not assume equal refraction north and south. About 200 transits would be required to find the latitude with a probable error of $\pm 0''.05$. This will suffice to being the error of the mean declination in the zones from 50° to 90° down to below $\pm 0''.05$ (again assuming 20 stars in each zone and 4 measurements of the azimuth of greatest elongation of each). But also the mean declinations between 35° and 50° will be determined with such accuracy as to afford a valuable intercomparison with results from the equatorial station. By means of Talcott observations of part of the northern stars this check on the results may be extended to the other side of the zenith down to perhaps — 30° declination.

With regard to the *choice of the programme stars* the principal point to be considered is how many of them can be connected by Talcott observations.

We have already proposed that the sky should be divided into zones extending over 10° of declination each. If the zones contain 20 stars, and if each star is to get 4 observations, a total of about 1000 azimuth observations must be made at the equatorial station. A complete azimuth observation, including the measurement of the inclination of the horizontal axis, will take about 20 minutes. The azimuth programme will then be completed in about 80 nights of 4 hours' observing. Counting the Talcott and auxiliary observations this is about as much as can be expected to be finished in one year's stay at the equatorial station. As there are only half as many zones to be observed at the other stations we may either extend the number of stars in each zone or extend the observations by taking transits through the prime — or an arbitrary vertical of stars culminating south of the zenith.

Suppose that the limiting brightness of the azimuth instrument, without reversing prism and with a dark field, is $8^m.0$. The reversing prism will bring the limit down to say $7^m.0$, and we shall lose another half magnitude by atmospheric extinction when observing at 70° zenith distance. (This consideration makes it very desirable to select stations, if possible, at a great altitude above the sea, especially for the equatorial station). The stars to be measured with the prism should be brighter than $6^m.5$ for the zones observed at the equatorial station, and brighter than $7^m.0$ for the zones from ± 60° to ± 90°. A simple calculation shows that with these limits of brightness it will just be possible at each station to find a sufficient number of Talcott pairs fulfilling the conditions for the azimuth programme; this will be true for all zones if we omit the reversing prism in the zone from 80° to 90°. However very few stars will be found which can be observed by the Talcott method from more than one station. Thus, if we use Talcott observations to connect the stars from + 10° to + 35° with stars observed at greatest elongation at the higher latitude stations, we shall have to confine the Talcott observations at the equatorial station to the zones between + 10° and — 10°, for instance. The conditions become much more favourable if we omit the reversing prism and take the limits one magnitude fainter than in the preceding. In that case the programme can be made up in such a way that between + 70° and — 70° declination each star is part of two different Talcott pairs, so that at the equatorial station all programme stars from + 70° to — 70° can be connected, whereas at the northern station those between 0° and + 35° will again be observed in Talcott pairs with the programme stars from + 35° to + 70°, and similarly at the southern station.

The best plan would seem to have half the programme consist of bright stars, occurring in some fundamental catalogue if possible, and which can be observed through the reversing prism, while the other half consists of fainter stars to be connected by Talcott observations at all three stations.

Recapitulating the results of the present considerations we come to these conclusions:

Given an azimuth-instrument of sufficient power to observe stars down to $8.^m0$ in a dark field and a zenith telescope with a limiting magnitude of at least $7.^m0$, then one year observing at each of three stations will provide us with a system of declinations free from some of the principal systematic errors that still seem to influence the meridian circle observations. The probable error of the average declination of each zone of 10° will not be larger than $\pm 0.''05$. The most favourable positions for the stations would be at 0° and ± 35° latitude. The equatorial station furnishes us only with average declinations of sets of three or more stars, except for the stars near the equator, where the coefficient of dt is very small. At the stations of 35° latitude individual declinations are determined from the observations of greatest elongation. The latitude is eliminated at each station independently and we shall have various checks on the results:

a) by direct intercomparison of the declinations from 35° to 60°, (or possibly from 0° to 60°), which have been observed with the azimuth-instrument at two stations;

b) by comparison of the declinations from Talcott observations at the higher latitude station with those from azimuth measures at the equatorial station in the zones between 0° and 35°;

c) by connecting with the zenith telescope the different zones observed at the equatorial station.