The secular perturbations of Pallas by Jupiter

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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

The Secular Perturbations of Pallas by Jupiter, by *R. T. A. Innes.* *)

Our knowledge of the motion of this minor planet is succinctly given by A. O. LEUSCHNER in *Bulletin* N°. 25 of the U. S. Am. National Research Council (1922).

As is well known GAUSS gave much attention to this planet and he believed that commensurability with Jupiter existed in the relation

$$18\,n_1 - 7\,n = 0.$$

GAUSS's work was published after a lapse of about a century, and has been used by G. STRUVE in his Inaugural Dissertation "Die Darstellung der Pallasbahn durch die GAUSS'sche Theorie für den Zeitraum 1803 bis 1910." (Berlin 1911). In *B. A. N.* 28 J. WOLTJER points out that in case of commensurability the integers must add up to zero. In other words, in place of $18\,n_1 - 7\,n = 0$ we must have some such form as

$$18\,n_1 - 7\,n - j_1\frac{d\varpi_1}{dt} - j\frac{d\varpi}{dt} - i\frac{d\Omega}{dt} = 0$$

in which

$$j_1 + j + i = 18 - 7 = 11.$$

The most remarkable case of commensurability is that connecting the longitudes of three of Jupiter's satellites, in which

$$n - 3\,n_1 + 2\,n_2 = 0.$$

The satellites of Saturn furnish several examples, the most striking being that of Titan and Hyperion, namely

$$4\,l - 3\,l_1 - \varpi = 180^\circ + \text{librational terms}.$$

In all these cases the factors j_1, j and i are zero or $\pm$ 1.

In the case of Pallas and Jupiter, the only admissible values would appear to be

j_1	j	i
0	+ 11	0
0	+ 10	+ 1
+ 1	+ 10	0

and the very size of the j almost prohibits a real librational effect.

It is not easy to determine the mean motion of Pallas. LE VERRIER used

$$n_1 = 109256'', \quad n = 280711'',$$

consequently

$$18\,n_1 - 7\,n = 1631''.$$

We may consider Jupiter's mean motion n_1 as known exactly. Its daily value is 299''·1283. If the relation $18\,n_1 - 7\,n = 0$ holds, then $n = 769''\cdot187$. In the table given by LEUSCHNER, the value of n varies from

$$768''\cdot45 \text{ to } 770''\cdot75.$$

These are osculating values; GAUSS's mean value was 769''·1507, whilst G. STRUVE finds 769''·13850.

It is evident that the difference $18\,n_1 - 7\,n$ is very small. If we accept G. STRUVE's mean motion then

$$18\,n_1 - 7\,n = +0''\cdot3412 \text{ in a day,}$$
or
$$= 124''\cdot6 \text{ in a Julian year.}$$

Can we now find some likely combination, such that

$$j_1\frac{d\varpi_1}{dt} + j\frac{d\varpi}{dt} + i\frac{d\Omega}{dt} = +\,124''\cdot6.$$

We know $\frac{d\varpi_1}{dt}$, the motion of the perihelion of Jupiter, with great precision; it is + 7''·70. This motion is too small to be of any importance so we turn to the motions of ϖ and Ω.

GAUSS gave values for these, which reduced by G. STRUVE to a more modern value of the mass of Jupiter, are

*) The investigation by Dr. INNES contained in this communication arose out of a conversation which he had with Dr. WOLTJER and myself during his visit to Leiden in December 1924. It is with reference to these circumstances that Dr. INNES gracefully considers it as a work of the Leiden Observatory. This explains why this paper is published as a communication from Leiden in the *B. A. N.* W. DE SITTER

$$\frac{d\varpi}{dt} = -11''\cdot0301 \qquad \frac{d\Omega}{dt} = -36''\cdot0642$$

N. B. This value of $\frac{d\varpi}{dt}$ corresponds to $-10''\cdot8260$ and not to $-10''\cdot8960$ as printed on page 35 of STRUVE's paper.

We notice that the motion of the perihelion of Pallas is negative and this is unusual. Thus the action of Jupiter on the four interior planets (NEWCOMB, *A. P.* V.) gives

Planet	$e\frac{d\varpi}{dt}$
Mercury	$+0''\cdot317$
Venus	$+0\cdot046$
Earth	$+0\cdot117$
Mars	$+1\cdot164$

In the cases of Ceres and Iris C. J. MERFIELD found $\frac{d\varpi}{dt} = +55''\cdot9$ and $+31''\cdot94$ respectively (*M. N.* LXVII p. 560 and *Ast. Nach.* 4337). If in the case of Pallas $\frac{d\varpi}{dt}$ was positive, it would fit in well as $11\frac{d\varpi}{dt} = 121''\cdot3$, almost exactly equal to $18n_1 - 7n$. Was it possible that GAUSS had made an error of sign? It was to settle this doubt that I determined to compute the secular perturbations of Pallas through the action of Jupiter by GAUSS's Elliptical Ring method. It was only when my calculation was practically finished that I found out that P. BRUCK had made the computation in 1895 (*Bull. Astr.*, XII p. 289) and found $\frac{d\varpi}{dt} = -9''\cdot76$. The osculating values of ϖ given in LEUSCHNER's table also indicated that the motion of the perihelion was negative and might even be as large as $-21''$. My surmise that the sign given by GAUSS was in error thus falls to the ground.

Thus it is fully proved that the possible values of

$$j_1\frac{d\varpi_1}{dt} + j\frac{d\varpi}{dt} + i\frac{d\Omega}{dt}$$

are adverse to any near approach to $18n_1 - 7n$.

By far the most probable argument is

$$18n_1 - 7n - 11\frac{d\varpi}{dt} = 0.$$

If we now take $\frac{d\varpi}{dt} = -10''$, and adopt

$$18n_1 - 11\frac{d\varpi}{dt} = 5494'' = 7n,$$

we get

$$n = 785'',$$

which is a quite inadmissable value. The idea of a libration may therefore be dismissed.

It may nevertheless be worth while to give the results of my computation and the steps by which they were reached. In the *Monthly Notices* for May 1907, I gave a process for GAUSS's method based on the HALPHEN-ARNDT investigation, which evades the solution of a cubic equation, but the process was intended mainly for logarithmic work and made use of the true anomaly of the disturbed planet. This solution has been partly revised so as to eliminate the true anomaly and to make more use of the calculating machine.

The elements adopted are from G. STRUVE's paper

Pallas	Jupiter
$\Omega = 172^\circ\ 52'\ 52''\cdot5$	$\Omega_1 = 99^\circ\ 26'\ 36''\cdot3$
$i = 34\ \ 42\ \ 53\cdot8$	$i_1 = 1\ \ 18\ \ 31\cdot4$
$\omega = 309\ \ 15\ \ 39\cdot6$	$\omega_1 = 273\ \ 16\ \ 39\cdot2$
$e = 0\cdot2379962$	$e_1 = 0\cdot0483348$
$\log a = 0\cdot442645$	$\log a_1 = 0\cdot716217$
$n = 769''\cdot1385$	

$$\log\alpha = 9\cdot7264278$$

Putting

J = mutual inclination of orbits.

π and π_1 = angular distances of the perihelia from the ascending node of the disturbing planet on the orbit of the disturbed, and solving directly, we have

$$\sin\pi\ \sin J = -\sin i\cos i_1\sin\omega + \cos i\sin i_1\cos(\Omega_1-\Omega)\sin\omega - \sin i_1\sin(\Omega_1-\Omega)\cos\omega$$
$$\cos\pi\ \sin J = -\sin i\cos i_1\cos\omega + \cos i\sin i_1\cos(\Omega_1-\Omega)\cos\omega + \sin i_1\sin(\Omega_1-\Omega)\sin\omega$$
$$\sin\pi_1\sin J = \cos i\sin i_1\sin\omega_1 - \sin i\cos i_1\cos(\Omega_1-\Omega)\sin\omega_1 - \sin i\ \sin(\Omega_1-\Omega)\cos\omega_1$$
$$\cos\pi_1\sin J = \cos i\sin i_1\cos\omega_1 - \sin i\cos i_1\cos(\Omega_1-\Omega)\cos\omega_1 + \sin i\ \sin(\Omega_1-\Omega)\sin\omega_1$$

The introduction of auxiliary angles lengthens the work.

We get

$$J = 34^\circ\ 21'\ 42''\cdot9$$
$$\pi = 127\ \ 2\ \ 17\cdot3$$
$$\pi_1 = 18\ \ 0\ \ 30\cdot8$$

We now compute

$$a_c = \alpha(\cos\pi_1\cos\pi + \sin\pi_1\sin\pi\cos J) \qquad b_c = \alpha\cos\varphi_1(\sin\pi_1\sin\pi + \cos\pi_1\cos\pi\cos J)$$
$$a_s = \alpha(\cos\pi_1\sin\pi - \sin\pi_1\cos\pi\cos J) \qquad b_s = -\alpha\cos\varphi_1(\sin\pi_1\cos\pi - \cos\pi_1\sin\pi\cos J)$$

and get

$a_c = -0\cdot1966065$	$e\,a_c = -0\cdot0467916$	$\cos\varphi\,a_c = -0\cdot1909572$
$a_s = +0\cdot4862153$	$e\,a_s = +0\cdot1157174$	$\cos\varphi\,a_s = +0\cdot4722444$
$b_c = -0\cdot1202819$	$e\,b_c = -0\cdot0286266$	$\cos\varphi\,b_c = -0.1168257$
$b_s = +0\cdot4324572$	$e\,b_s = +0\cdot1029232$	$\cos\varphi\,b_s = +0\cdot4200310$

At this stage it is necessary to decide on the number of divisions of the circumference to be used, and 16 was chosen. For each of these, we now compute

$$r_0 = \alpha - e\,\alpha \cos E$$
$$= 0{\cdot}5326327 - {\cdot}1267646 \cos E$$
$$r_0^2 = 0{\cdot}2917322 - {\cdot}1350379 \cos E + {\cdot}0080346 \cos 2E$$
$$r_0 A_c = a_c \cos E - \cos\varphi\, a_s \sin E - e a_c$$
$$r_0 B_c = -a_s \cos E - \cos\varphi\, a_c \sin E + e a_s$$
$$r_0 A_s = b_s \cos E + \cos\varphi\, b_c \sin E - e b_s$$
$$r_0 B_s = b_c \cos E - \cos\varphi\, b_s \sin E - e b_c$$
$$D = r_0 A_c \cos^2\varphi_1 - e_1 r_0^2 = +\,0{\cdot}0325815$$
$$- {\cdot}1896201 \cos E - {\cdot}4711411 \sin E - {\cdot}0003884 \cos 2E$$

In computing these, it is best, in spite of some repetition, to make a table giving the 16 values of cos E, sin E, etc. This table is reproduced.

The sums of the even and of the odd divisions for these quantities must equal 8 times the constant quantities and this affords a check. This holds good until we come to the quantities involving the angle ι, and even then, if the number of divisions of the circumference is large enough, the two sums ought to approach equality. In the tables these sums are indicated by Σ and Σ'.

Then

$$P_1 = \tfrac{2}{3}(1 - e_1^2 + r_0^2 + 2\, e_1 r_0 A_c)$$
$$P_2 = \tfrac{2}{3}(r_0^2 A_c^2 + r_0^2 A_s^2 + 2e_1 D + e_1^2 r_0^2)$$
$$P_3 = 4e_1^2 (r_0 A_s)^2$$

N.B. $\sqrt{P_3} = 2\, e_1 r_0 A_s$ is also wanted.

$$G_2 = P_1^2 - 2P_2 \qquad G_3 = P_1^3 - 3 P_1 P_2 - P_3$$
$$\lambda = \tfrac{9}{4} G_2 \qquad \cos\iota = G_3 \,/\, G_2^{\frac{3}{2}}$$
$$\Phi_R = (r_0 A_c D + r_0 A_s\, r_0 A_s)\frac{1}{r_0} - r_0 (P_1 - e_1 r_0 A_c)$$
$$\Phi_S = (r_0 B_c D + r_0 B_s\, r_0 A_s)\frac{1}{r_0}$$
$$\Phi_W = \sin\pi_1 D + \cos\pi_1 \cos\varphi_1\, r_0 A_s$$
$$\chi_R - \tfrac{1}{4} G_2 r_0 = \tfrac{1}{2}(P_1 \Phi_R + P_2 r_0)$$
$$\chi_S = \tfrac{1}{2}(P_1 \Phi_S - \sqrt{P_3}\, r_0 \cos\varphi_1 \cos J)$$
$$\chi_W = \tfrac{1}{2}(P_1 \Phi_W - \sqrt{P_3}\,\{r_0 A_c \cos\varphi_1 \cos\pi_1 - r_0 A_s \sin\pi_1\})$$

We must now find the values of the two series

$$F\left(\frac{1}{6}, \frac{5}{6}; 2; \sin^2\frac{\iota}{2}\right)$$

and

$$F\left(-\frac{1}{6}, \frac{7}{6}; 2; \sin^2\frac{\iota}{2}\right)$$

Their logarithms are tabulated for each degree of ι at the end of this paper. From $\iota = 0°$ to $90°$ the table is taken from FRANK ROBBINS's paper in the *Monthly Notices*, 1907 May; and from 90° to 180° from C. J. MERFIELD's in the *Monthly Notices*, 1908 June, with the kind permissions of the authors. The properties of these series are given in the *Proceedings of the R. S. Edinburgh*, XXVII part IV, pp. 360—361.

Putting

$$F_+ = \frac{15}{8} F\left(\frac{1}{6}, \frac{5}{6}; 2; \sin^2\frac{\iota}{2}\right) \Big/ \left(\lambda^{\frac{7}{4}} \cos^2\frac{\iota}{2}\right)$$

and

$$F_- = \frac{7}{8} F\left(-\frac{1}{6}, \frac{7}{6}; 2; \sin^2\frac{\iota}{2}\right) \Big/ \left(\lambda^{\frac{5}{4}} \cos^2\frac{\iota}{2}\right)$$

we have

$$\frac{1}{\alpha^2}\frac{a}{r} R_0 = F_-\, \Phi_R + F_+ (\chi_R - \tfrac{1}{4} G_2 r_0)$$
$$\frac{1}{\alpha^2}\frac{a}{r} S_0 = F_-\, \Phi_S + F_+ \chi_S$$
$$\frac{1}{\alpha \sin J} \times \frac{a}{r} W_0 = [F_-\, \Phi_W + F_+ \chi_W]\, r_0.$$

Having the 16 values of each of these quantities, which we may designate as

$R_0, R_1 \ldots\ldots R_{15}$; $S_0, S_1 \ldots\ldots S_{15}$ and $W_0, W_1 \ldots\ldots W_{15}$

respectively, we compute

$$A_{c,0} = \tfrac{1}{16}[R_0 + R_1 \ldots\ldots + R_{15}]$$
$$\tfrac{1}{2} A_{c,1} = \tfrac{1}{16}[R_0 + R_1 \cos 22\tfrac{1}{2}° \ldots\ldots + R_{15} \cos 337\tfrac{1}{2}°]$$
$$\tfrac{1}{2} A_{c,2} = \tfrac{1}{16}[R_0 + R_1 \cos 45° \ldots\ldots + R_{15} \cos 315°]$$

$\tfrac{1}{2} A_{s,1}$ and $\tfrac{1}{2} A_{s,2}$: — Replace cos by sin in $\tfrac{1}{2} A_{c,1}$ and $\tfrac{1}{2} A_{c,2}$ respectively.

The expressions for $B_{c,0}$, $\tfrac{1}{2} B_{c,1} \ldots\ldots C_{c,0}$, $\tfrac{1}{2} C_{c,1}$ etc. are similar.

N.B. $\tfrac{1}{2} A_{s,2}$ is not required.

The above formulae can be consolidated, but this is not advisable as it complicates the check by the addition of even and odd divisions.

The equation

$$\sin\varphi\, \tfrac{1}{2} A_{s,1} + \cos\varphi\, B_{c,0} = 0$$

affords a useful check. In the present case, the first five significant figures agree.

The final equations are

$$e\frac{d\varphi}{dt} = m_1 n \alpha^2 \left[-\left(1 + \frac{e^2}{2}\right) B_{c,0} + 2e\tfrac{1}{2} B_{c,1} - \frac{e^2}{2}\tfrac{1}{2} B_{c,2}\right]$$
$$e\frac{d\chi}{dt} = m_1 n \alpha^2 \left[\, e\cos\varphi\, A_{c,0} - \cos\varphi\, \tfrac{1}{2} A_{c,1} + (2 - e^2)\,\tfrac{1}{2} B_{s,1} - \frac{e}{2}\tfrac{1}{2} B_{s,2}\right]$$
$$\frac{di}{dt} = \frac{m_1 n \alpha \sin J}{\cos\varphi}\left[(\tfrac{1}{2} C_{c,1} - e\, C_{c,0})\cos\omega - \tfrac{1}{2} C_{s,1} \sin\omega\right]$$
$$\sin i \frac{d\Omega}{dt} = \frac{m_1 n \alpha \sin J}{\cos\varphi}\left[(\tfrac{1}{2} C_{c,1} - e\, C_{c,0})\sin\omega + \tfrac{1}{2} C_{s,1} \cos\omega\right]$$
$$-2\frac{r}{a} R_0 = \alpha^2 \left[-(2 + e^2) A_{c,0} + 4e\tfrac{1}{2} A_{c,1} - e^2 \tfrac{1}{2} A_{c,2}\right]$$

Taking the values of the A, B and C from the tables which follow and using

$$m_1 n \alpha^2 = 76\cdot095$$
$$m_1 n \alpha \sin J = 80\cdot634$$

we obtain the secular perturbations due to the first order of Jupiter's mass. With our results we include for comparison BRUCK's and the GAUSS-STRUVE figures.

GAUSS was able to extract from his work the secular variations and these are given by G. STRUVE (p. 17). STRUVE adds that he attempted to recompute the secular variations by means of the Lund Tables (1900) but that the attempt was unsuccessful because e and i are not sufficiently small quantities and that he did not go further. On page 35 he repeats GAUSS's values reduced to the mass of Jupiter adopted (1/1047·35) and these are given below.

BRUCK's figures are reduced to the values of α and $m_1 n$ used by STRUVE. It should be remarked that BRUCK used 32 divisions and that the lack of equality between Σ and Σ' from ι onwards in my results shows that 16 divisions as used by me are really insufficient although the resulting error may be very small.

The comparison is as follows:

	This calculation	GAUSS G. STRUVE p. 35	BRUCK
$\frac{de}{dt}$	$-10''\cdot591$		$-10''\cdot531$
$\frac{d\varphi}{dt}$	$-10''\cdot904$	$-10''\cdot998$	
$\frac{d\varpi}{dt}$	$-9''\cdot871$	$-11''\cdot030$	$-9''\cdot756$
$\frac{di}{dt}$	$+5''\cdot094$	$+5''\cdot054$	$+4''\cdot926$
$\frac{d\Omega}{dt}$	$-35''\cdot354$	$-36''\cdot064$	$-35''\cdot647$
$-2\frac{r}{a}R_o$	$-0\cdot06094$		$-0\cdot05940$
$\frac{d\chi}{dt}$	$-3''\cdot5781$		
$\frac{d\varepsilon}{dt}$	$-22''\cdot7408$	$-22''\cdot7264$	$-22''\cdot3871$

With this value of $d\varepsilon/dt$ we are now prepared to calculate the semi-axis major with accuracy. If this is determined by the Keplerian relation

$$a^3 n^2 = k^2 (1 + m) \qquad \log k = 3\cdot5500066$$

it requires the correction

$$+\frac{1}{6}\frac{a}{n}\frac{d\varepsilon}{dt}$$

LE VERRIER indicates a correction four times larger (see for example, *Annales de l'Obs. de Paris*, Mems. X. p 189) but this has ultimately to be reduced by the inclusion of the constant terms in the perturbations of the radius-vector. If therefore the perturbations are going to be developed analytically and by powers of the masses it is best to start with the semi-axis major

$$a\left(1 + \frac{d\varepsilon}{dt} / 6 n\right). \qquad (1)$$

If however the perturbations of the radius-vector are determined by mechanical integrations so that the non-periodical parts are included in the integrations, the equation leading to the semi-axis major should be

$$a\left(1 + 2\frac{d\varepsilon}{dt} / 3 n\right). \qquad (2)$$

In computing the secular perturbations form (1) should be used. Actually the value given by form (2) was used.

G. STRUVE's mean motion $n = 769''\cdot1385$ gives the Keplerian value

$$\log a = 0\cdot4426680.$$

The correction to this by (1) is

$$[8\cdot85963]\frac{d\varepsilon}{dt}/n = -0\cdot00000587$$

The effect due to the action of the other planets may be estimated at $-1/33$ of Jupiter's; and accepting this, the correction is

$$-0\cdot00000567 \quad \text{by} \quad (1)$$

and

$$-0\cdot00002268 \quad ,, \quad (2)$$

giving

$$\log a = 0\cdot4426623 \text{ or } \log a = 0\cdot4426453$$

the latter being the value given by G. STRUVE.

The first value may be called the mean value because it only suffers strictly periodic variations.

References.

A good list is given by ERIC DOOLITTLE in his Memoir published in the *Transactions of the American Philosophical Society*, XXII part 2, (Philadelphia 1912). Others are:

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E	N°.	cos E	sin E	cos $2E$	sin $2E$	r_0	r_0^2	$r_0 A_c$	$r_0 B_c$	$r_0 A_s$
0°	0	1·0000000	0·0000000	1·0000000	0·0000000	0·4058681	0·1647289	— 0·1498150	— 0·3704979	+ 0·3295339
22½	1	0·9238795	0·3826834	0·7071068	0·7071068	·4155174	·1726548	— ·3155692	— ·2604108	·2519078
45	2	0·7071068	0·7071068	0·0000000	1·0000000	·4429966	·1962460	— ·4261574	— ·0930616	+ ·1202620
67½	3	0·3826834	0·9238795	— 0·7071068	0·7071068	·4841220	·2343738	— ·4647432	+ ·1060723	— ·0453619
90	4	0·0000000	1·0000000	— 1·0000000	0·0000000	·5326327	·2836976	— ·4254527	·3066746	— ·2197489
112½	5	— 0·3826834	0·9238795	— 0·7071068	— 0·7071068	·5811434	·3377274	— ·3142671	·4782054	— ·3763502
135	6	— 0·7071068	0·7071068	0·0000000	— 1·0000000	·6222688	·3872184	— ·1481136	·5945507	— ·4913248
157½	7	— 0·9238795	0·3826834	0·7071068	— 0·7071068	·6497480	·4221727	+ ·0477123	·6379979	— ·5471688
180	8	— 1·0000000	0·0000000	1·0000000	0·0000000	·6593973	·4348047	·2433983	·6019327	— ·5353804
202½	9	— 0·9238795	— 0·3826834	0·7071068	0·7071068	·6497480	·4221727	·4091525	·4918456	— ·4577542
225	10	— 0·7071068	— 0·7071068	0·0000000	1·0000000	·6222688	·3872184	·5197407	·3244964	— ·3261084
247½	11	— 0·3826834	— 0·9238795	— 0·7071068	0·7071068	·5811434	·3377274	·5583264	+ ·1253625	— ·1604846
270	12	0·0000000	— 1·0000000	— 1·0000000	0·0000000	·5326327	·2836976	·5190359	— ·0752398	+ ·0139025
292½	13	0·3826834	— 0·9238795	— 0·7071068	— 0·7071068	·4841220	·2343738	·4078504	— ·2467706	·1705038
315	14	0·7071068	— 0·7071068	0·0000000	— 1·0000000	·4429966	·1962460	·2416968	— ·3631159	·2854785
337½	15	0·9238795	— 0·3826834	0·7071068	— 0·7071068	·4155174	·1726548	.0458709	— ·4065630	·3413224
	Σ					4·2610616	2·3338576	+ ·3743330	+ ·9257392	— ·8233856
	Σ′					4·2610616	2·3338574	+ ·3743330	+ ·9257393	— ·8233857

N°.	$r_0 B_s$	P_1	P_2	P_3	D	Φ_R	Φ_S	Φ_W	$\chi_R - \frac{1}{4} G_2 r_0$
0	— 0·0916553	+ 0·7652734	+ 0·0774691	+ 0·0010148	— 0·1574271	+ 0·0121275	+ 0·0692903	+ 0·2643539	+ 0·0203587
1	— ·2432383	·7598751	·0881356	·0005930	— ·3231774	+ ·0760807	+ ·0550764	+ ·1393737	·0472169
2	— ·3534323	·7684755	·1030095	·0001352	— ·4346474	+ ·1012164	— ·0046402	— ·0201386	·0617076
3	— ·4054613	·7914075	·1151165	·0000192	— ·4749858	+ ·0662104	— ·0660790	— ·1899357	·0540649
4	— ·3914044	·8268220	·1250696	·0004513	— ·4381711	— ·0106840	— ·0908042	— ·3442039	·0288912
5	— ·3134015	·8700075	·1395368	·0013236	— ·3298569	— ·0923233	— ·0684697	— ·4594729	+ ·0003843
6	— ·1833280	·9137093	·1654323	·0022559	— ·1664837	— ·1454653	— ·0143176	— ·5181788	— ·0149848
7	— ·0209864	·9496322	·2035236	·0027978	+ ·0271952	— ·1527417	+ ·0443765	— ·5113472	— ·0064047
8	+ ·1489085	·9706651	·2455556	·0026786	·2218134	— ·1157320	+ ·0815801	— ·4399814	+ ·0247908
9	·3004917	·9729257	·2769457	·0019581	·3877910	— ·0526188	+ ·0818501	— ·3149308	·0643753
10	·4106857	·9567501	·2837984	·0009938	·4998103	+ ·0086373	+ ·0454121	— ·1552488	·0924314
11	·4627145	·9262430	·2603613	·0002407	·5406981	+ ·0411905	— ·0111422	+ ·0147180	·0947298
12	·4486576	·8876909	·2126577	·0000018	·5041108	+ ·0321544	— ·0595003	+ ·1690567	·0709058
13	·3706547	·8476432	·1561337	·0002717	·3955691	— ·0075201	— ·0710910	+ ·2842552	·0346067
14	·2405810	·8115163	·1085113	·0007616	·2316465	— ·0439686	— ·0348396	+ ·3427914	·0061945
15	·0782394	·7831685	·0817505	·0010887	·0374184	— ·0399925	+ ·0276570	+ ·3357900	·0013239
Σ	+ 0·2290128	+ 6·9009026	+ 1·3215035	+ 0·0082930	+ 0·2606517	— ·1617143	— ·0078194	— ·7015495	+ ·2902952
Σ′	+ 0·2290128	+ 6·9009027	+ 1·3215037	+ 0·0082928	+ 0·2606517	— ·1617148	— ·0078219	— ·7015497	+ ·2902971

N°.	χ_S	χ_W	G_2	G_3	$(\log 1/\lambda^{\frac{1}{4}})$	ι	log $F(\frac{1}{6}, \frac{5}{6}; 2; \sin^2 \frac{\iota}{2})$	log $F(-\frac{1}{6}, \frac{7}{6}; 2; \sin^2 \frac{\iota}{2})$
0	+ 0·0211828	+ 0·1050409	+ 0·4307051	+ 0.2693073	0·0034093	17° 41′ 1″·4	0·0007180.	9·9989941
1	+ ·0167541	+ ·0575518	·4011388	·2372499	0·0111306	20 57 42·6	·0010088	·9985864
2	— ·0039061	— ·0051688	·3845355	·2162107	0·0157203	24 56 19·8	·0014277	·9979986
3	— ·0252725	— ·0760954	·3960924	·2223476	0·0125050	26 52 56·8	·0016587	·9976743
4	— ·0328749	— .1458686	·4334953	·2545619	0·0027083	26 53 12·7	·0016592	·9976735
5	— .0210681	— ·2031863	·4778393	·2930022	9·9921339	27 29 43·0	·0017351	·9975670
6	+ ·0056435	— ·2364663	·5040001	·3070966	9·9863467	30 52 33·3	·0021876	·9969312
7	+ ·0352394	— ·2371234	·4947544	·2737645	9·9883570	38 7 28·8	·0033336	·9953186
8	+ ·0536629	— ·2032711	·4510795	·1968163	9·9983910	49 29 4·0	·0056098	·9921055
9	+ ·0516704	— ·1414718	·3926934	·1106560	0·0134409	63 16 39·0	·0091539	·9870756
10	+ ·0298113	— ·0648961	·3477744	·0602152	0·0266304	72 55 34·2	·0121332	·9828216
11	— ·0014433	+ ·0113150	·3372037	·0709340	0·0299812	68 45 40·2	·0107972	·9847320
12	— ·0267041	+ ·0747066	·3626797	·1331714	0·0220735	52 25 52·1	·0062953	·9911352
13	— ·0334196	+ ·1177151	·4062311	·2117216	0·0097611	35 8 33·4	·0028331	·9960232
14	— ·0191769	+ ·1371402	·4415356	·2694935	0·0007129	23 17 9·2	·0012448	·9982553
15	+ ·0051778	+ ·1325122	·4498521	·2871970	9·9986869	17 50 58·5	·0007315	9·9989752
Σ	+ ·0276385	— ·3387832	3·3558052	1·7068729		298 30 46·7	·0312756	9·9559150
Σ′	+ ·0276382	— ·3387828	3·3558052	1·7068728		298 29 42·3	·0312519	9·9559523

N°.	F_+	log F_+	F_-	log F_-	$\frac{1}{\alpha^2}\frac{a}{r}R_0$	$\frac{1}{\alpha^2}\frac{a}{r}S_0$	$\frac{1}{\alpha \sin J}\frac{a}{r_0}W_0$
0	2·0322087	0·3079683	0·9298922	9·9684326	+ 0·0526504	+ 0·1074804	+ 0·1859502
1	2·3256212	·3665389	1·0253262	0·0108620	·1878161	+ ·0954350	+ ·1149933
2	2·5422258	·4052145	1·0948413	0·0393508	·2676906	— ·0150105	— ·0155886
3	2·4339966	·3863198	1·0625088	0·0263318	·2019429	— ·1317227	— ·1873670
4	2·0785000	·3177500	0·9491929	9·9773545	+ ·0499092	— ·1545213	— ·3355074
5	1·7576016	·2449204	0·8423475	9·9254913	— ·0770929	— ·0947047	— ·4324615
6	1·6275510	·2115346	0·7990824	9·9025916	— ·1406274	— ·0022559	— ·4971483
7	1·7531512	·2438194	0·8474171	9·9280972	— ·1406643	+ ·0993854	— ·5516604
8	2·2438094	·3509858	1·0226014	0·0097063	— ·0627219	·2038333	— ·5974318
9	3·2809731	·5160025	1·3678840	0·1360493	+ ·1392372	·2814905	— ·5814944
10	4·5790320	·6607746	1·7668830	0·2472093	·4385085	+ ·2167453	— ·3556074
11	4·5754969	·6604382	1·7515300	0·2434174	·5055822	— ·0261197	+ ·0450681
12	3·3735944	·5280926	1·3733850	0·1377923	·2833677	— ·1718055	+ ·2579058
13	2·4303258	·3856645	1·0674295	0·0283392	+ ·0760784	— ·1571051	+ ·2853940
14	1·9828730	·2972949	0·9159809	9·9618864	— ·0279915	— ·0699377	+ ·2595615
15	1·8841741	·2751218	0·8810458	9·9449985	— ·0327407	+ ·0341230	+ ·2266742
Σ	20·4597943		8·8518591		+ ·8607856	+ ·1145281	— 1·0978660
Σ′	20·4413405		8·8454889		+ ·8601589	+ ·1007817	— 1·0808537

$$A_{c,0} = +0{\cdot}1075590$$
$$\tfrac{1}{2}A_{c,1} = +\ {\cdot}0100774$$
$$\tfrac{1}{2}A_{s,1} = -\ {\cdot}0549149$$
$$\tfrac{1}{2}A_{c,2} = -\ {\cdot}0458926$$

$$B_{c,0} = +0{\cdot}0134569$$
$$\tfrac{1}{2}B_{c,1} = -\ {\cdot}0377854$$
$$\tfrac{1}{2}B_{s,1} = -\ {\cdot}0115546$$
$$\tfrac{1}{2}B_{c,2} = +\ {\cdot}0805150$$
$$\tfrac{1}{2}B_{s,2} = +\ {\cdot}0320310$$

$$C_{c,0} = -0{\cdot}1361700$$
$$\tfrac{1}{2}C_{c,1} = +\ {\cdot}1942004$$
$$\tfrac{1}{2}C_{s,1} = -\ {\cdot}1091055$$

$$\sin\varphi\ \tfrac{1}{2}A_{s,1} = -0{\cdot}0130695$$
$$\cos\varphi\ \ B_{c,0} = +\ {\cdot}0130702$$
$$\Sigma = +\ {\cdot}0000007$$

The following tables of $F(\frac{1}{6}, \frac{5}{6}; 2; \sin^2\iota)$ and $F(-\frac{1}{6}, \frac{7}{6}; 2; \sin^2\frac{\iota}{2})$ can by checked by the equation

$$F_1(\iota)\ F_2(180^\circ - \iota) + F_1(180^\circ - \iota)\ F_2(\iota) = \frac{216}{35\ \pi} = 1{\cdot}964427,$$

where we have put $F_1(\iota) = F(\frac{1}{6}, \frac{5}{6}; 2; \sin^2\frac{\iota}{2})$, and $F_2(\iota) = F(-\frac{1}{6}, \frac{7}{6}; 2; \sin^2\frac{\iota}{2})$.

t	log $F(\frac{1}{6}, \frac{5}{6}; 2; \sin^2\frac{t}{2})$	Δ_1	Δ_2	log $F(-\frac{1}{6}, \frac{7}{6}; 2; \sin^2\frac{t}{2})$	Δ_1	Δ_2
0°	0·0000000	+ 23	+ 46	0·0000000	− 32	− 64
1	0·0000023	+ 69	+ 46	9·9999968	− 97	− 65
2	0·0000092	+ 115	+ 46	9·9999871	− 160	− 63
3	0·0000207	+ 160	+ 45	9·9999711	− 225	− 65
4	0·0000367	+ 207	+ 47	9·9999486	− 290	− 65
5	0·0000574	+ 253	+ 46	9·9999196	− 354	− 64
6	0·0000827	+ 299	+ 46	9·9998842	− 418	− 64
7	0·0001126	+ 344	+ 45	9·9998424	− 482	− 64
8	0·0001470	+ 390	+ 46	9·9997942	− 547	− 65
9	0·0001860	+ 437	+ 47	9·9997395	− 611	− 64
10	0·0002297	+ 482	+ 45	9·9996784	− 675	− 64
11	0·0002779	+ 528	+ 46	9·9996109	− 740	− 65
12	0·0003307	+ 574	+ 46	9·9995369	− 804	− 64
13	0·0003881	+ 620	+ 46	9·9994565	− 869	− 65
14	0·0004501	+ 666	+ 46	9·9993696	− 933	− 64
15	0·0005167	+ 711	+ 45	9·9992763	− 997	− 64
16	0·0005878	+ 758	+ 47	9·9991766	− 1062	− 65
17	0·0006636	+ 803	+ 45	9·9990704	− 1126	− 64
18	0·0007439	+ 849	+ 46	9·9989578	− 1191	− 65
19	0·0008288	+ 896	+ 47	9·9988387	− 1255	− 64
20	0·0009184	+ 941	+ 45	9·9987132	− 1320	− 65
21	0·0010125	+ 987	+ 46	9·9985812	− 1384	− 64
22	0·0011112	+ 1032	+ 45	9·9984428	− 1449	− 65
23	0·0012144	+ 1079	+ 47	9·9982979	− 1513	− 64
24	0·0013223	+ 1124	+ 45	9·9981466	− 1578	− 65
25	0·0014347	+ 1170	+ 46	9·9979888	− 1642	− 64
26	0·0015517	+ 1215	+ 45	9·9978246	− 1707	− 65
27	0·0016732	+ 1261	+ 46	9·9976539	− 1771	− 64
28	0·0017993	+ 1307	+ 46	9·9974768	− 1837	− 66
29	0·0019300	+ 1353	+ 46	9·9972931	− 1901	− 64
30	0·0020653	+ 1399	+ 46	9·9971030	− 1965	− 64
31	0·0022052	+ 1444	+ 45	9·9969065	− 2031	− 66
32	0·0023496	+ 1490	+ 46	9·9967034	− 2095	− 64
33	0·0024986	+ 1535	+ 45	9·9964939	− 2160	− 65
34	0·0026521	+ 1581	+ 46	9·9962779	− 2224	− 64
35	0·0028102	+ 1627	+ 46	9·9960555	− 2290	− 66
36	0·0029729	+ 1672	+ 45	9·9958265	− 2354	− 64
37	0·0031401	+ 1718	+ 46	9·9955911	− 2419	− 65
38	0·0033119	+ 1763	+ 45	9·9953492	− 2484	− 65
39	0·0034882	+ 1808	+ 45	9·9951008	− 2549	− 65
40	0·0036690	+ 1854	+ 46	9·9948459	− 2613	− 64
41	0·0038544	+ 1900	+ 46	9·9945846	− 2679	− 66
42	0·0040444	+ 1945	+ 45	9·9943167	− 2744	− 65
43	0·0042389	+ 1990	+ 45	9·9940423	− 2808	− 64
44	0·0044379	+ 2035	+ 45	9·9937615	− 2874	− 66
45	0·0046414	+ 2081	+ 46	9·9934741	− 2938	− 64
46	0·0048495	+ 2125	+ 44	9·9931803	− 3004	− 66
47	0·0050620	+ 2171	+ 46	9·9928799	− 3068	− 64
48	0·0052791	+ 2217	+ 46	9·9925731	− 3134	− 66
49	0·0055008	+ 2261	+ 44	9·9922597	− 3199	− 65
50	0·0057269	+ 2306	+ 45	9·9919398	− 3264	− 65
51	0·0059575	+ 2351	+ 45	9·9916134	− 3328	− 64
52	0·0061926	+ 2396	+ 45	9·9912806	− 3394	− 66
53	0·0064322	+ 2441	+ 45	9·9909412	− 3459	− 65
54	0·0066763	+ 2486	+ 45	9·9905953	− 3525	− 66
55	0·0069249	+ 2531	+ 45	9·9902428	− 3589	− 64
56	0·0071780	+ 2575	+ 44	9·9898839	− 3655	− 66
57	0·0074355	+ 2620	+ 45	9·9895184	− 3719	− 64
58	0·0076975	+ 2665	+ 45	9·9891465	− 3785	− 66
59	0·0079640	+ 2709	+ 44	9·9887680	− 3850	− 65
60	0·0082349	+ 2753	+ 44	9·9883830	− 3915	− 65
61	0·0085102	+ 2798	+ 45	9·9879915	− 3980	− 65
62	0·0087900	+ 2842	+ 44	9·9875935	− 4045	− 65
63	0·0090742	+ 2886	+ 44	9·9871890	− 4110	− 65
64	0·0093628	+ 2930	+ 44	9·9867780	− 4175	− 65
65	0·0096558	+ 2974	+ 44	9·9863605	− 4240	− 65
66	0·0099532	+ 3019	+ 45	9·9859365	− 4305	− 65
67	0·0102551	+ 3062	+ 43	9·9855060	− 4370	− 65

t	log $F(\frac{1}{6}, \frac{5}{6}; 2; \sin^2\frac{t}{2})$	Δ_1	Δ_2	log $F(-\frac{1}{6}, \frac{7}{6}; 2; \sin^2\frac{t}{2})$	Δ_1	Δ_2
68°	0·0105613	+ 3105	+ 43	9·9850690	− 4435	− 65
69	0·0108718	+ 3150	+ 45	9·9846255	− 4500	− 65
70	0·0111868	+ 3192	+ 42	9·9841755	− 4565	− 65
71	0·0115060	+ 3236	+ 44	9·9837190	− 4629	− 64
72	0·0118296	+ 3279	+ 43	9·9832561	− 4694	− 65
73	0·0121575	+ 3323	+ 44	9·9827867	− 4759	− 65
74	0·0124898	+ 3366	+ 43	9·9823108	− 4823	− 64
75	0·0128264	+ 3408	+ 42	9·9818285	− 4889	− 66
76	0·0131672	+ 3451	+ 43	9·9813396	− 4952	− 63
77	0·0135123	+ 3494	+ 43	9·9808444	− 5016	− 64
78	0·0138617	+ 3536	+ 42	9·9803428	− 5081	− 65
79	0·0142153	+ 3578	+ 42	9·9798347	− 5145	− 64
80	0·0145731	+ 3621	+ 43	9·9793202	− 5209	− 64
81	0·0149352	+ 3662	+ 41	9·9787993	− 5273	− 64
82	0·0153014	+ 3704	+ 42	9·9782720	− 5336	− 63
83	0·0156718	+ 3746	+ 42	9·9777384	− 5400	− 64
84	0·0160464	+ 3787	+ 41	9·9771984	− 5464	− 64
85	0·0164251	+ 3829	+ 42	9·9766520	− 5526	− 62
86	0·0168080	+ 3869	+ 40	9·9760994	− 5590	− 64
87	0·0171949	+ 3909	+ 40	9·9755404	− 5652	− 62
88	0·0175858	+ 3951	+ 42	9·9749752	− 5716	− 64
89	0·0179809	+ 3991	+ 40	9·9744036	− 5778	− 62
90	0·0183800	+ 4032	+ 41	9·9738258	− 5840	− 62
91	0·0187832	+ 4072	+ 40	9·9732418	− 5902	− 62
92	0·0191904	+ 4110	+ 38	9·9726516	− 5964	− 62
93	0·0196014	+ 4149	+ 39	9·9720552	− 6025	− 61
94	0·0200163	+ 4188	+ 39	9·9714527	− 6086	− 61
95	0·0204351	+ 4226	+ 38	9·9708441	− 6146	− 60
96	0·0208577	+ 4265	+ 39	9·9702295	− 6208	− 62
97	0·0212842	+ 4303	+ 38	9·9696087	− 6268	− 60
98	0·0217145	+ 4342	+ 39	9·9689819	− 6328	− 60
99	0·0221487	+ 4379	+ 37	9·9683491	− 6387	− 59
100	0·0225866	+ 4416	+ 37	9·9677104	− 6447	− 60
101	0·0230282	+ 4452	+ 36	9·9670657	− 6505	− 58
102	0·0234734	+ 4488	+ 36	9·9664152	− 6563	− 58
103	0·0239222	+ 4523	+ 35	9·9657589	− 6620	− 57
104	0·0243745	+ 4559	+ 36	9·9650969	− 6678	− 58
105	0·0248304	+ 4593	+ 34	9·9644291	− 6734	− 56
106	0·0252897	+ 4627	+ 34	9·9637557	− 6790	− 56
107	0·0257524	+ 4662	+ 35	9·9630767	− 6845	− 55
108	0·0262186	+ 4696	+ 34	9·9623922	− 6901	− 56
109	0·0266882	+ 4729	+ 33	9·9617021	− 6955	− 54
110	0·0271611	+ 4762	+ 33	9·9610066	− 7008	− 53
111	0·0276373	+ 4793	+ 31	9·9603058	− 7061	− 53
112	0·0281166	+ 4825	+ 32	9·9595997	− 7113	− 52
113	0·0285991	+ 4856	+ 31	9·9588884	− 7165	− 52
114	0·0290847	+ 4886	+ 30	9·9581719	− 7215	− 50
115	0·0295733	+ 4914	+ 28	9·9574504	− 7263	− 48
116	0·0300647	+ 4943	+ 29	9·9567241	− 7312	− 49
117	0·0305590	+ 4972	+ 29	9·9559929	− 7360	− 48
118	0·0310562	+ 4999	+ 27	9·9552569	− 7405	− 45
119	0·0315561	+ 5025	+ 26	9·9545164	− 7451	− 46
120	0·0320586	+ 5051	+ 26	9·9537713	− 7495	− 44
121	0·0325637	+ 5075	+ 24	9·9530218	− 7539	− 44
122	0·0330712	+ 5099	+ 24	9·9522679	− 7580	− 41
123	0·0335811	+ 5121	+ 22	9·9515099	− 7621	− 41
124	0·0340932	+ 5144	+ 23	9·9507478	− 7660	− 39
125	0·0346076	+ 5165	+ 21	9·9499818	− 7697	− 37
126	0·0351241	+ 5185	+ 20	9·9492121	− 7733	− 36
127	0·0356426	+ 5204	+ 19	9·9484388	− 7767	− 34
128	0·0361630	+ 5222	+ 18	9·9476621	− 7800	− 33
129	0·0366852	+ 5238	+ 16	9·9468821	− 7830	− 30
130	0·0372090	+ 5253	+ 15	9·9460991	− 7859	− 29
131	0·0377343	+ 5266	+ 13	9·9453132	− 7885	− 26
132	0·0382609	+ 5279	+ 13	9·9445247	− 7910	− 25
133	0·0387888	+ 5290	+ 11	9·9437337	− 7934	− 24
134	0·0393178	+ 5300	+ 10	9·9429403	− 7954	− 20
135	0·0398478	+ 5307	+ 7	9·9421449	− 7971	− 17

ι	log $F(\frac{1}{6}, \frac{5}{6}; 2; \sin^2\frac{\iota}{2})$	Δ_1	Δ_2	log $F(-\frac{1}{6}, \frac{7}{6}; 2; \sin^2\frac{\iota}{2})$	Δ_1	Δ_2
136°	0·0403785	+ 5314	+ 7	9·9413478	— 7986	— 15
137	0·0409099	+ 5319	+ 5	9·9405492	— 7999	— 13
138	0·0414418	+ 5323	+ 4	9·9397493	— 8008	— 9
139	0·0419741	+ 5323	+ 0	9·9389485	— 8015	— 7
140	0·0425064	+ 5322	— 1	9·9381470	— 8018	— 3
141	0·0430386	+ 5318	— 4	9·9373452	— 8018	0
142	0·0435704	+ 5313	— 5	9·9365434	— 8014	+ 4
143	0·0441017	+ 5305	— 8	9·9357420	— 8008	+ 6
144	0·0446322	+ 5295	— 10	9·9349412	— 7997	+ 11
145	0·0451617	+ 5283	— 12	9·9341415	— 7983	+ 14
146	0·0456900	+ 5267	— 16	9·9333432	— 7963	+ 20
147	0·0462167	+ 5248	— 19	9·9325469	— 7938	+ 25
148	0·0467415	+ 5227	— 21	9·9317531	— 7908	+ 30
149	0·0472642	+ 5202	— 25	9·9309623	— 7874	+ 34
150	0·0477844	+ 5174	— 28	9·9301749	— 7834	+ 40
151	0·0483018	+ 5143	— 31	9·9293915	— 7789	+ 45
152	0·0488161	+ 5108	— 35	9·9286126	— 7737	+ 52
153	0·0493269	+ 5068	— 40	9·9278389	— 7682	+ 55
154	0·0498337	+ 5025	— 43	9·9270707	— 7617	+ 65
155	0·0503362	+ 4977	— 48	9·9263090	— 7544	+ 73
156	0·0508339	+ 4925	— 52	9·9255546	— 7465	+ 79
157	0·0513264	+ 4867	— 58	9·9248081	— 7376	+ 89
158	0·0518131	+ 4803	— 64	9·9240705	— 7280	+ 96
159°	0·0522934	+ 4735	— 68	9·9233425	— 7173	+ 107
160	0·0527669	+ 4659	— 76	9·9226252	— 7057	+ 116
161	0·0532328	+ 4576	— 83	9·9219195	— 6929	+ 128
162	0·0536904	+ 4487	— 89	9·9212266	— 6792	+ 137
163	0·0541391	+ 4389	— 98	9·9205474	— 6641	+ 151
164	0·0545780	+ 4283	— 106	9·9198833	— 6476	+ 165
165	0·0550063	+ 4168	— 115	9·9192357	— 6296	+ 180
166	0·0554231	+ 4042	— 126	9·9186061	— 6102	+ 194
167	0·0558273	+ 3906	— 136	9·9179959	— 5891	+ 211
168	0·0562179	+ 3758	— 148	9·9174068	— 5660	+ 231
169	0·0565937	+ 3595	— 163	9·9168408	— 5407	+ 253
170	0·0569532	+ 3418	— 177	9·9163001	— 5133	+ 274
171	0·0572950	+ 3224	— 194	9·9157868	— 4833	+ 300
172	0·0576174	+ 3011	— 213	9·9153035	— 4502	+ 331
173	0·0579185	+ 2773	— 238	9·9148533	— 4140	+ 362
174	0·0581958	+ 2510	— 263	9·9144393	— 3737	+ 403
175	0·0584468	+ 2216	— 294	9·9140656	— 3289	+ 448
176	0·0586684	+ 1882	— 334	9·9137367	— 2782	+ 507
177	0·0588566	+ 1497	— 385	9·9134585	— 2201	+ 581
178	0·0590063	+ 1035	— 462	9·9132384	— 1514	+ 687
179	0·0591098	+ 428	— 607	9·9130870	— 624	+ 890
180	0·0591526			9·9130246		