Some remarks on the system of galactic globular clusters

Kort, J.J.M.A. de

Some remarks on the system of galactic globular clusters, by *J. de Kort.*

Summary.

The mean motion of the stars near the sun around the galactic axis can be considered as the sum of the mean circular motion of these stars relative to the system of globular clusters and the axial rotation of that system. As the latter is not directly known, I have tried, following a suggestion by Professor OORT, to find out what amount of axial rotation of the cluster system is compatible with its apparently spherical distribution in space. It is found that the outward appearance of the system is insensitive even to a considerable rotation.

The gradient of the space density of globular clusters at the sun's distance from the galactic centre agrees well with theory, but the total number of globular clusters in the galactic system cannot be estimated from the data considered here.

Introduction.

According to the theory [1]) developed by OORT an ellipsoidal distribution of velocities in the case of dynamical equilibrium will have the following form

$$\pi^{-3/2}\sqrt{c_1 c_4 (c_2 \varpi^2 + c_1)}\ .\ \nu\ .\ e^{-c_1 \Pi^2 - (c_2 \varpi^2 + c_1)(\Theta - \Theta_0)^2 - c_4 Z^2},$$

where Π, Θ, Z denote the linear velocity components in cylindrical coordinates, ϖ the distance from the axis, ν the density, the remaining symbols being constants, cf. equations (21) and (18) l.c. A reasonable simplification is afforded by assuming a spherical distribution of peculiar velocities. We thus assume that $c_2 = 0$ and $c_4 = c_1$. In this case equation (22) becomes $c_3/c_1 = \Theta_0/\varpi$, the system rotates with uniform angular velocity as a solid body. Let us denote this angular velocity by ω.

The equations (24) [1]) and (25) can then be integrated as follows:

$$\ln (\nu/\nu_0) = c_1\,\omega^2\,(\varpi^2 - \varpi_0^2) + 2\,c_1 V - 2\,c_1 V_0,$$

where ν_0 denotes the density averaged over a sphere around the centre with radius 8 kps and V_0 the gravitational potential at a point (ϖ_0, z_0) where $\nu = \nu_0$.

A distance of 8 kps of the sun from the galactic centre and a centripetal force corresponding to a circular velocity of 271 km sec^{-1} at the sun's distance are used throughout the following calculations.

The model of the galaxy used in these calculations is, in the main, similar to that used and described in the preceding article by OORT and VAN WOERKOM. For simplification the number of separate spheroids was, however, reduced. The dimensions of the "outer" ellipsoids are shown in Table 1. For the oblateness of the larger spheroids, which is not known by observation, a somewhat smaller value was adopted. The unit of density is ·087 ⊙ ps^{-3} for the first spheroid and 2/3 × ·087 ⊙ ps^{-3} for the surrounding shells.

TABLE 1.

a (kps)	c (kps)	c/a	relative star density in shell
10·0	·5	·05	·72
10·6	1·6	·15	·22
13·2	5·0	·38	·020
36·3	14·5	·40	·0012

The well-known expression [2]) for the gravitational potential of a homogeneous ellipsoid can, for our case, be written in the form

$$V = 2\pi f \rho c^2 \operatorname{cosec} 2\psi\,[2\varphi_0 - (2\varphi_0 - \sin 2\varphi_0)\,\varpi^2/2c^2 \operatorname{tg}^2\psi - (2 \operatorname{tg}\varphi_0 - 2\varphi_0)\,z^2/c^2 \operatorname{tg}^2\psi].$$

Here f is the constant of gravitation, $\psi = \arccos c/a$, ψ being the eccentricity angle of a meridian section of the spheroid. If the potential is to be found for a point outside the spheroid, φ_0 is the eccentricity angle of the confocal spheroid passing through that point; in the case of an internal point φ_0 is equal to ψ. In most cases I have determined the eccentricity angles of the confocal spheroids graphically.

For the central mass which exerts, together with the spheroids of the model, the necessary centripetal force, a mass-point of $1·159 \times 10^{11}$ ⊙ or a spheroid with semi-axes 4·2 and 1·4 kps and density $11·57 \times$ ·087 ⊙ ps^{-3} have been assumed alternately.

The average peculiar velocity and the observed radial velocities.

Of 26 globular clusters the radial velocities have been published [3]). From repeated observations an

[1]) The first term in the numerator of the right hand side of equation (24) should read $-c_2^2\,\varpi^3$, instead of $-c_2\,\varpi^3$ as erroneously printed in *B.A.N.* No. 159.

[1]) *B.A.N.* No. 159, 276 (1928).

[2]) Compare e.g. J. C. KAPTEYN, *M.W.C.* No. 230; *Ap. J.* **55**, 302 (1922) Appendix, and O. HEYMANN, *A.N.* **256**, 181 (1935).

[3]) MAYALL, who has observed a considerable number of additional clusters, finds an average peculiar velocity of ± 112 km sec^{-1} from 43 clusters (result communicated privately to Prof. OORT), which agrees beautifully with the value assumed in the present article.

average error of observation of 38 km sec^{-1} is found, which is in accordance with HUMASON's estimated uncertainty of 50 km sec^{-1} [1]). The velocities of EDMONDSON's list [2]) were adopted, with the exception of the velocity of N.G.C. 5024, which was taken as — 185 km sec^{-1} (compare *Lick Publ.* **18**, 217, 1932). We now introduce a system of coordinates having the galactic centre as origin, the axis of z being directed towards the northern galactic pole at $18^h\,40^m$, $+ 28°$ (1900), the axes of x and y towards $l = 325°$ and $l = 55°$ respectively.

Considering the velocity components in the direction of y only and omitting the velocities of N.G.C. 6093 and N.G.C. 6402 as being less ceitain, I find 237 km sec^{-1} for the velocity of the sun. The residuals fit a normal frequency curve with an average residual velocity of 107·6 km sec^{-1}. The resulting average peculiar velocity component is, therefore,

$$\sqrt{107{\cdot}6^2 - 38^2} = 100{\cdot}5 \text{ km sec}^{-1},$$

with a relative probable error of 12%.

The quantity c_1, which enters as a factor in ln ν, is then $3{\cdot}14 \times 10^{-5}$ km^{-2} sec^2 = $2{\cdot}98 \times 10^{28}$ kps^{-2} sec^2, the relative probable error being 24%.

We shall now make two different assumptions regarding the axial rotation of the system of globular clusters. First, let us assume the cluster system to be at rest, so that the circular velocity of the stars near the sun is equal to the systematic velocity with respect to the clusters. In the second case the system is supposed to rotate, the linear velocity of rotation, Θ_0, being 100 km sec^{-1} at $\varpi = 8$ kps. The values of $c_1\omega^2$ are 0 and $4{\cdot}91 \times 10^{-3}$ kps^{-2} in these two cases.

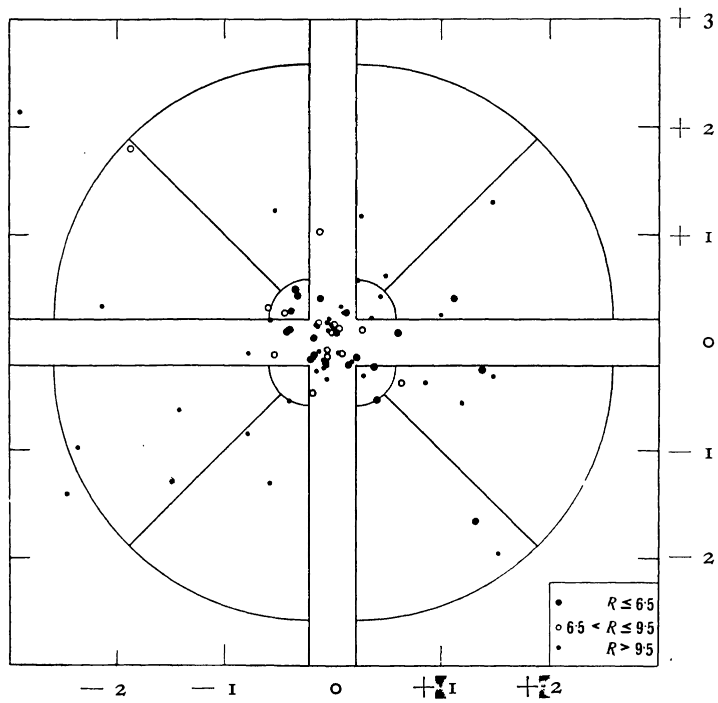
The rotational flattening of the system and the distribution on the sky.

In Figure 1 the globular clusters are shown projected against the yOz-plane. A few clusters beyond the limits of the diagram have been omitted. The unit of length is the distance of the sun to the plane.

[1]) *P.A.S.P.* **46**, 357 (1934).
[2]) *A.J.* **45**, 1 (1935), Table I.

A graph in which the number of clusters in the projection is plotted as a function of the distance from the horizontal axis shows a maximum near ·14, due to an absorbing layer near the galactic plane [1]). If we omit the clusters within a distance ·214 from the horizontal axis and similarly those within the same distance from the vertical axis, excluding also the regions close to the centre, which are less appropriate for a comparison with theory because they are too sensitive to the model adopted, we may obtain a usable measure of the flattening as follows:

FIGURE 1.



Distribution of clusters on the sky.
Abscissa: — tg (l — 325°), ordinate: tg b sec (l — 325°).

Denote by A_1, A_2, B_2, B_1, C_1, C_2, D_2, D_1 the eight areas bounded by the two pairs of lines parallel to the axes, by four pairs of circular arcs with radii

[1]) Cf. H. SHAPLEY, *Star Clusters*, p. 21 (1930).

TABLE 2.

	Central mass	ω	Number of clusters: Areas A_1, B_1, C_1, D_1 combined	Number of clusters: Areas A_2, B_2, C_2, D_2 combined	Ratio of numbers
First assumption	Mass-point	0 km sec^{-1}/kps	$4520\,\nu_0 = 22$	$4130\,\nu_0 = 20$	1·1
Second „	„	100/8	$4560\,\nu_0 = 22$	$3580\,\nu_0 = 17$	1·3
Third „	Spheroid	0	$4790\,\nu_0 = 23$	$3340\,\nu_0 = 16$	1·4
Fourth „	„	100/8	$4770\,\nu_0 = 23$	$2900\,\nu_0 = 14$	1·6
Observation			14	9	1·6

TABLE 3.

Density	1st assumption			2nd assumption			3rd assumption			4th assumption		
	k (kps)	l (kps)	l/k	k (kps)	l (kps)	l/k	k (kps)	l (kps)	l/k	k (kps)	l (kps)	l/k
ν_0	8·32	7·55	·90	9·15	7·36	·80	8·40	7·13	·85	9·45	6·90	·73
10 ν_0	5·35	5·00	·94	5·48	4·88	·89	5·52	4·36	·79	5·55	4·12	·74
100 ν_0	3·96	3·80	·96	3·90	3·52	·90	4·06	2·85	·70	3·97	2·75	·69
1000 ν_0	3·08	3·02	·98	2·97	2·69	·91	3·00	1·76	·59	3·08	1·72	·56

·366 and 2·366 and by the lines bisecting the quadrants. The number of clusters in the combined low-latitude areas A_1, B_1, C_1, D_1 being 14, against 9 in the areas A_2, B_2, C_2, D_2, a small flattening may be present.

The formulae and constants considered above permit a calculation of the theoretical cluster densities in the space volumes which are seen projected on the eight areas. The theoretical total numbers are given in Table 2 for the pyramidal volumes extending from the sun up to 11·5 kps in the direction of the galactic centre. The reason why ν_0 is assumed to be ·0048 per kps^3 will appear in the following section.

No final choice between rotation or rest of the system of globular clusters can be made on the basis of these measures of flattening.

The effect of the various assumptions upon the flattening may also be shown by giving the equidensity surfaces. These are practically ellipsoids of revolution; the axes are given in Table 3. It appears again that the shapes of the curves in the intermediate regions, which are accessible to density observation, are little influenced by the various assumptions.

The observed space distribution and the total number of globular clusters in the galactic system.

The distances of globular clusters determined by SHAPLEY have been recomputed by VAN DE KAMP [1]), who rejected the apparent diameters as measures of distances. His distances, based on photometric data only, are free from the systematic errors probably inherent in the estimates of diameters and can be unambiguously corrected by an absorption factor. VAN DE KAMP's list has been supplemented by some cases where better information has meanwhile been obtained. These "photometric" distances are given in Table 4 under the heading R_1.

Subsequently distances R have been calculated, in which space absorption is accounted for by a factor $10^{-\cdot 2A} = 10^{-1 \cdot 6E}$, where A is the photographic absorption and E the colour excess in the scale used by STEBBINS and his collaborators for the B-stars; a ratio $A : E = 8$ was adopted.

[1]) *A.J.* **42**, 101 (1932), Table 4.

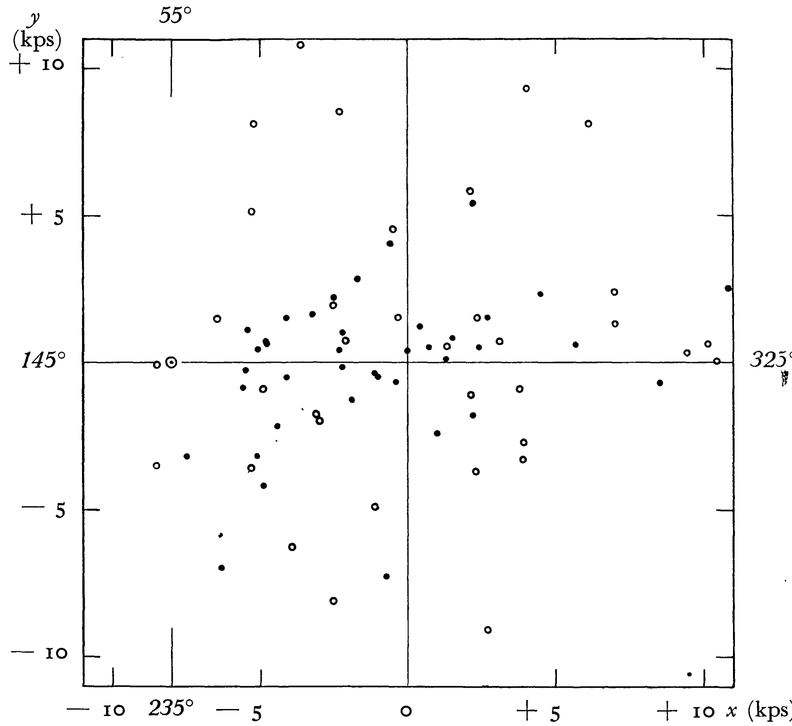
TABLE 4.

N.G.C. No.	R_1 (kps)	R (kps)
104	7·3	6·4*
288	15·0	15·0
362	12·8	11·0*
1261	28·6	24·7*
1851	15·7	12·6*
1904	21·4	15·9*
2298	35·3	18·9*
2419	70·5	*122·5*
2808	17·1	7·3*
3201	8·7	3·3*
4147	20·0	18·6
4590	15·1	12·1*
4833	14·9	5·3*
5024	20·3	20·3
5053	17·2	15·2*
5139	8·5	4·5*
5272	12·2	10·2
5286	25·3	10·5*
5466	7·3	14·4
5634	41·7	39·8
5694	39·3	39·0
I.C. 4499	28·1	18·1*
5824	37·3	31·0
5897	16·7	13·9
5904	10·0	8·3
5927	24·7	4·2*
5986	16·2	11·2
6093	17·9	12·4
6101	26·3	14·6*
6121	5·9	2·6
6139	39·4	9·4
6144	18·2	10·5
6171	15·9	3·2
6205	10·2	7·6
6218	10·6	6·5
6229	29·9	23·1
6235	29·9	18·5
6254	10·9	5·4
6266	18·7	6·9
6273	16·1	7·7
6284	28·6	17·7
6287	37·2	10·6
6293	28·8	18·5

TABLE 4 (*continued*).

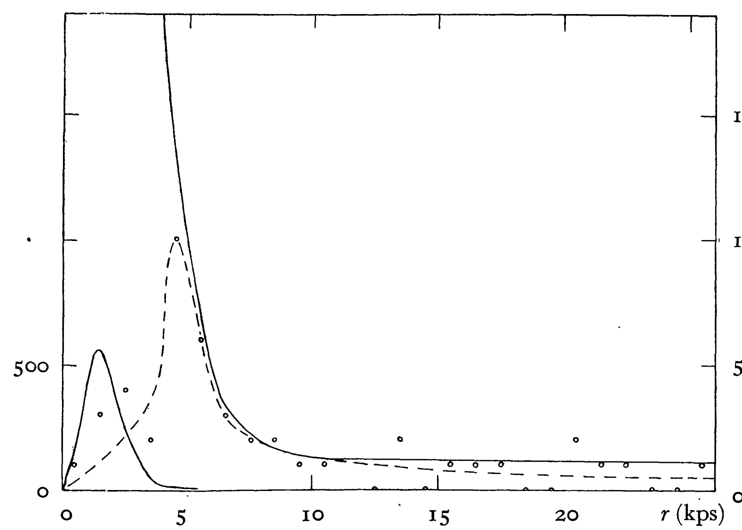
N.G.C. No.	R_1 (kps)	R (kps)
6304	29·4	5·8
6316	39·6	9·4
6325	60·3	13·8
6333	20·8	8·6
6341	9·7	*10·4*
6342	55·0	10·9
6356	50·3	15·4
6362	14·5	8·6*
6366	32·2	4·3
6388	19·0	6·3*
6402	22·9	7·1
6397	5·4	2·6*
6426	19·0	9·1
6440	53·0	3·2
6441	27·4	7·0
6453	50·1	16·6
6496	25·4	10·5*
6517	63·1	6·0
6522	40·6	10·4
6528	55·5	8·8
6535	26·7	16·5
6539	56·3	2·8
6541	8·6	4·0*
6553	29·0	19·1
6569	34·0	8·1
6584	21·5	12·8*
6624	24·4	9·7
6626	17·0	5·9
6637	20·1	5·8
6638	30·9	12·8
6652	25·6	11·4
6656	6·7	3·3
6681	20·1	15·5
6712	27·5	8·5
6715	19·0	10·9
6723	11·0	9·8
6752	8·1	6·0*
6760	34·7	20·7
6779	19·6	15·1
6809	8·1	*8·7*
6864	47·9	27·5
6934	20·9	16·2
6981	23·8	18·4
7006	61·1	56·9
7078	13·2	13·2
7089	14·2	12·7
7099	14·6	*17·6*
7492	26·8	*56·0*

FIGURE 2.



System of clusters projected on galactic plane. Circles: clusters with $|z| > 2$ kps; dots: clusters with $|z| < 2$ kps. At (–8,0) position of sun.

FIGURE 3.



Abscissa: distance from galactic centre. Ordinate: number of clusters in spherical half shells of 1 kps thickness. Circles and broken line: observed numbers. Full line (which is on a 100 times smaller scale in the central part): computed numbers.

STEBBINS and WHITFORD have given accurate colours of 68 globular clusters, from which colour excesses may be formed [1]). For the remaining clusters interpolated colour excesses have been derived from a colour excess-latitude diagram. Distances obtained by means of this relation are marked with an asterisk in Table 4. Distances printed in italics denote cases of negative colour excess. For individual distances one should better adopt R_1 in such cases. For the purpose of cal-

[1]) *M.W.C.* No. 547; *Ap.J.* **84**, 132 (1936). Compare also SEARES' discussion, *M.W.C.* No. 620, p. 6; *Ap.J.* **91**, 13 (1940).