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COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN.

The co-ordinates of the Sun's Apex from proper motions of stars of the third type,

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1. *The material.*

The proper motions used for the computations are those contained in Vol. XV, Part I of the *Annals of the Observatory* at Leiden.

I will recall here the most important data regarding that catalogue.

a. The proper motions depend on the BOSS system and have been derived by means of all the available meridian observations of older date and the new observations at Leiden of the epoch 1921.

b. The mean error of all the proper motions may be adopted as $\pm 0''\cdot012$.

c. The mean spectral type of all the stars is between III and III[1] in the notation of KRÜGER or about K5 in that of the *New Henry Draper Catalogue.*

d. The mean visual magnitude of all the stars of which proper motions have been derived is $8^{m}\cdot2$.

With the intention to investigate whether it was possible to find a systematic change in the co-ordinates of the sun's apex, especially of its declination, depending on the declination or the visual magnitude of the stars, I have first divided the material in two groups viz: the stars $< 8^{m}\cdot0$ and $> 8^{m}\cdot0$.

The first group contains 455, the second group 909 stars.

Each of these groups has been divided in 7 zones of declination viz: $< 15°$, 15°—25°, 25°—30°, 30°—35°, 35°—40°, 40°—45° and $> 45°$.

Within these groups hourly means of the proper motions have been formed, excluding the few large proper motions.

In order to make the results comparable with other investigations on the proper motions of the stars of the *P. G. C.*, these hourly means have been corrected by the amounts $+0^{s}\cdot00021 - 0^{s}\cdot00015 \sin\alpha \tan\delta$ for μ_α and $-0''\cdot0023 \cos\alpha$ for μ_δ as given on page XXVIII of the introduction to the *P. G. C.*

Tables 1 and 2 give these corrected hourly means, $\bar{\mu}_\alpha$ and $\bar{\mu}_\delta$, with the number of stars contributing to every mean. The $\bar{\mu}_\alpha$'s have been transformed to seconds of arc by multiplying by $15 \cos\delta$, δ being the mean declination of the zone.

The left-hand numbers refer to the stars $< 8^{m}\cdot0$, the right-hand numbers to those $> 8^{m}\cdot0$.

Least squares solutions have been made, zone by zone, for the groups $< 8^{m}\cdot0$ and $> 8^{m}\cdot0$ and for their weighted mean, giving equal weights to all the hourly means.

A second solution has been made, giving weights proportional to the numbers of stars contributing to every hourly mean.

2. *The method of solution.*

The method applied in the computation of the co-ordinates of the apex is that of AIRY.

Calling P the parallactic motion,
A the R. A. of the Apex,
D the Decl. „ „ „ ,
$X = P \cos A \cos D$,
$Y = P \sin A \cos D$,
$Z = P \sin D$,

the equations of condition have the form:

$$X \sin\alpha - Y \cos\alpha = \bar{\mu}_\alpha$$
$$X \cos\alpha \sin\delta + Y \sin\alpha \sin\delta - Z \cos\delta = \bar{\mu}_\delta$$

For the α I have substituted the angles 7°·5, 22°·5, 37°·5 etc.

For the δ I have substituted the angles 7°·5, 20°, 27°·5, 32°·5, 37°·5, 42°·5 and 48°.

The $\bar{\mu}_\alpha$ and $\bar{\mu}_\delta$ have been taken directly from the tables 1 and 2; the same is the case for the weights in the weighted solution.

From the $\bar{\mu}_\alpha$ equations only X and Y, and therefore A and $P \cos D$, can be solved.

From the $\bar{\mu}_\delta$ equations all the three unknowns can be determined.

Because of the intended purpose of this investigation viz: a derivation of the value of the KAPTEYN correction,

TABLE 1. Unit 0″·0001

$\overline{\mu}_\alpha$, separately for the groups < $8^{m}·0$ and > $8^{m}·0$

	< 15°		15°—25°		25°—30°		30°—35°		35°—40°		40°—45°		> 45°	
0^h	+ 35	—	+ 173	—	— 66	+ 37	+ 378	+ 144	— 51	+ 6	+ 180	+ 41	— 6	+ 253
	4	—	3	—	1	6	1	8	3	13	4	4	4	6
1^h	+ 39	—	+ 571	— 182	+ 782	+ 117	+ 123	— 8	+ 240	+ 65	+ 18	+ 165	+ 121	+ 51
	3	—	1	3	1	3	4	9	2	5	3	7	5	10
2^h	+ 33	+ 35	— 19	—	—	+ 15	+ 83	+ 74	+ 124	— 7	+ 2	+ 26	— 80	— 3
	5	2	2	—	—	2	5	9	2	8	5	10	5	14
3^h	+ 86	+ 55	— 110	— 61	—	—	+ 24	— 48	+ 240	+ 155	+ 47	+ 55	+ 240	— 13
	6	1	2	1	—	—	2	4	1	12	3	9	1	19
4^h	+ 255	— 89	+ 69	— 24	— 74	+ 258	— 98	— 16	+ 219	+ 187	+ 185	+ 47	+ 107	— 26
	4	1	2	4	1	3	3	2	5	7	3	15	4	14
5^h	+ 96	+ 216	+ 130	— 33	—	+ 270	+ 69	+ 60	+ 50	+ 92	— 145	+ 178	+ 101	+ 52
	2	1	2	7	—	3	3	5	5	7	1	11	2	9
6^h	— 74	— 34	+ 69	— 28	—	+ 164	— 35	+ 34	+ 114	+ 158	— 283	+ 42	+ 96	+ 98
	4	5	3	2	—	2	1	6	5	3	2	10	4	4
7^h	— 3	— 310	— 190	— 283	—	— 8	+ 218	+ 86	+ 45	+ 214	+ 108	+ 124	—	— 4
	7	1	2	1	—	1	8	8	6	2	1	2	—	1
8^h	— 153	— 28	+ 10	— 96	—	— 286	—	—	+ 205	— 105	—	—	—	—
	3	1	1	4	—	1	—	—	1	2	—	—	—	—
9^h	— 113	—	—	—	—	—	—	—	+ 267	— 55	— 75	— 97	—	—
	5	—	—	—	—	—	—	—	1	3	1	1	—	—
10^h	— 283	—	—	—	— 269	—	— 78	—	—	—	+ 190	—	—	+ 80
	4	—	—	—	1	—	1	—	—	—	3	—	—	2
11^h	— 124	—	—	—	—	—	—	—	— 494	—	—	—	—	+ 39
	2	—	—	—	—	—	—	—	2	—	—	—	—	1
12^h	—	— 290	— 163	—	—	—	—	— 23	—	—	—	—	—	—
	—	3	1	—	—	—	—	1	—	—	—	—	—	—
13^h	— 295	— 239	— 271	—	—	—	—	—	+ 71	— 95	— 241	— 26	— 308	— 164
	5	2	2	—	—	—	—	—	2	2	2	1	3	2
14^h	— 269	— 185	— 30	— 158	— 298	— 152	+ 386	— 39	— 401	+ 283	—	— 80	— 150	+ 1
	4	2	4	4	1	1	1	3	2	1	—	3	1	2
15^h	— 98	—	— 23	— 402	— 31	— 197	—	—	—	+ 274	—	— 53	— 471	—
	6	—	3	2	1	4	—	—	—	1	—	2	1	—
16^h	— 126	—	+ 217	—	— 56	— 43	—	— 479	+ 157	— 21	—	+ 29	—	— 28
	3	—	1	—	1	2	—	1	1	6	—	4	—	5
17^h	+ 84	+ 64	+ 45	— 225	+ 41	+ 297	—	— 19	+ 151	+ 65	+ 257	— 42	+ 42	— 28
	7	7	3	1	6	5	—	6	2	3	1	6	2	10
18^h	— 80	+ 14	— 69	+ 9	—	+ 47	+ 29	— 47	— 13	+ 64	— 92	+ 81	— 180	+ 30
	9	12	6	4	—	7	6	22	7	12	4	5	4	9
19^h	— 5	— 57	+ 2	— 10	— 40	+ 3	+ 52	+ 3	+ 124	+ 7	— 116	— 3	— 49	+ 162
	6	13	8	10	5	31	7	22	14	19	7	17	8	11
20^h	— 87	— 139	— 13	— 20	— 100	— 27	— 6	+ 5	+ 48	— 4	— 207	+ 80	— 4	— 35
	11	9	8	20	4	19	7	13	3	21	3	12	4	23
21^h	— 156	— 310	— 25	+ 35	— 258	— 25	+ 112	+ 77	— 74	+ 100	+ 44	+ 11	+ 103	+ 25
	8	3	6	10	2	6	3	6	3	11	1	5	4	25
22^h	+ 228	+ 108	+ 107	— 77	— 114	— 20	+ 21	+ 166	— 32	+ 199	— 26	+ 86	— 9	— 10
	4	6	7	8	2	22	3	5	2	11	2	4	3	14
23^h	— 76	+ 1	— 61	—	+ 694	+ 8	+ 170	+ 18	—	+ 104	+ 82	— 31	+ 334	+ 61
	3	1	7	—	1	14	3	10	—	11	6	7	4	8

TABLE 2. Unit 0″·0001

$\overline{\mu_\delta}$, separately for the groups < 8ᵐ·0 and > 8ᵐ·0

	< 15°		15°—25°		25°—30°		30°—35°		35°—40°		40°—45°		> 45°	
0ʰ	+ 25	—	− 55	—	+ 148	+ 16	− 22	− 121	− 235	− 166	− 104	− 297	− 82	− 124
	4	—	3	—	1	6	1	8	3	13	4	4	4	6
1ʰ	− 74	—	− 281	− 178	− 451	− 28	− 251	− 141	− 181	− 229	− 111	− 148	− 199	− 66
	3	—	1	3	1	3	4	9	2	5	3	7	5	10
2ʰ	− 266	− 48	− 133	—	—	− 58	− 156	− 129	− 88	− 96	− 72	− 147	− 38	− 170
	5	2	2	—	—	2	5	9	2	8	5	10	5	7
3ʰ	− 202	+ 176	− 184	− 64	—	—	− 144	− 34	− 214	− 153	− 197	+ 9	− 34	− 121
	6	1	2	1	—	—	2	4	1	12	3	9	1	19
4ʰ	− 327	+ 161	− 324	− 174	− 69	− 112	− 179	− 139	− 213	− 183	+ 134	− 116	− 17	− 248
	4	1	2	4	1	3	3	2	5	7	3	15	4	14
5ʰ	− 418	− 103	− 198	− 93	—	− 76	− 166	− 327	− 231	− 163	− 153	− 330	− 123	− 284
	2	1	2	7	—	3	3	5	5	7	1	11	2	9
6ʰ	− 310	− 41	− 57	− 17	—	− 77	− 87	− 214	− 191	− 140	− 1087	− 258	− 349	− 177
	4	5	3	2	—	2	1	6	5	3	2	10	4	4
7ʰ	− 225	+ 239	− 76	− 261	—	− 251	+ 25	− 191	− 266	− 166	− 231	− 191	—	− 171
	7	1	2	1	—	1	8	8	6	2	1	2	—	1
8ʰ	− 263	− 116	− 116	− 226	—	+ 24	—	—	− 396	− 306	—	—	—	—
	3	1	1	4	—	1	—	—	1	2	—	—	—	—
9ʰ	− 154	—	—	—	—	—	—	—	− 202	− 235	− 132	− 62	—	—
	5	—	—	—	—	—	—	—	1	3	1	1	—	—
10ʰ	− 34	—	—	—	− 79	—	− 289	—	—	—	− 269	—	—	− 124
	1	—	—	—	1	—	1	—	—	—	3	—	—	2
11ʰ	− 348	—	—	—	—	—	—	—	− 198	—	—	—	—	− 538
	2	—	—	—	—	—	—	—	2	—	—	—	—	1
12ʰ	—	− 118	− 118	—	—	—	—	− 8	—	—	—	—	—	—
	—	3	1	—	—	—	—	1	—	—	—	—	—	—
13ʰ	− 281	− 239	− 234	—	—	—	—	—	− 294	− 384	+ 66	− 619	− 59	− 199
	5	2	2	—	—	—	—	—	2	2	2	1	3	2
14ʰ	− 174	− 102	− 150	− 4	− 362	− 332	− 962	− 175	− 177	− 572	—	− 242	− 292	− 47
	4	2	4	4	1	1	1	3	2	1	—	3	1	2
15ʰ	− 103	—	− 373	− 233	− 236	− 146	—	—	—	− 386	—	− 226	− 136	—
	6	—	3	4	1	4	—	—	—	1	—	2	1	—
16ʰ	− 131	—	− 61	—	− 451	− 226	—	− 451	− 161	− 233	—	− 169	—	+ 83
	3	—	1	—	1	2	—	1	1	6	—	4	—	5
17ʰ	− 61	− 150	− 300	− 337	− 295	− 261	—	− 329	− 97	− 30	+ 183	− 2	− 187	− 88
	7	7	3	1	6	5	—	6	2	3	1	6	2	10
18ʰ	− 82	− 131	− 47	− 208	—	− 214	− 130	− 113	+ 30	− 326	− 128	− 317	− 73	− 108
	9	12	6	4	—	7	6	22	7	12	4	5	4	9
19ʰ	− 201	− 134	− 16	− 226	− 237	− 35	− 25	− 60	− 72	− 174	− 158	− 199	− 166	− 64
	6	13	8	10	5	31	7	22	14	19	7	17	8	11
20ʰ	− 39	− 30	+ 12	− 99	+ 24	− 69	− 95	− 195	− 164	− 157	− 1	− 54	+ 68	− 93
	11	9	8	20	4	19	7	13	3	21	3	12	4	23
21ʰ	− 109	− 281	− 100	− 69	− 208	− 68	+ 39	− 128	− 205	− 136	− 158	− 218	+ 32	+ 10
	8	3	6	10	2	6	3	6	3	11	1	5	4	25
22ʰ	− 321	− 101	− 95	− 117	− 236	− 55	− 108	− 103	− 6	− 134	− 241	− 206	− 111	− 69
	4	6	7	8	2	22	3	5	2	11	2	4	3	14
23ʰ	− 92	− 42	− 217	—	+ 148	− 26	− 205	− 89	—	− 95	− 134	− 195	− 237	− 71
	3	1	7	—	1	14	3	10	—	11	6	7	4	8

to the μ_δ's, it was very tempting to derive the declination of the sun's apex from the second set of equations only. However, the weight with which the three quantities X, Y and Z can be solved from these equations, differs too much for consecutive zones.

Assuming a symmetrical distribution of the stars over the sky, in which all the hours of R. A. are represented, many coefficients of the normal equations become zero; when the distribution is not symmetrical, as in our case, these coefficients are always small and the weight of the X, Y and Z solution still depends chiefly on the diagonal coefficients of these variables.

The principal part of the normal equations can thus be written:

$$\Sigma\,(\sin^2\alpha)\;X = \Sigma\overline{\mu}_\alpha \sin\alpha \qquad \sin\delta\,\Sigma\,(\cos^2\alpha)\;X = \Sigma\overline{\mu}_\delta \cos\alpha$$

$$\Sigma\,(\cos^2\alpha)\;Y = \Sigma\overline{\mu}_\alpha \cos\alpha \qquad \sin\delta\,\Sigma\,(\sin^2\alpha)\;Y = \Sigma\overline{\mu}_\delta \sin\alpha$$

$$24\cos\delta\,.\,Z = \Sigma\overline{\mu}_\delta$$

or:

$$12\,X = \Sigma\,\overline{\mu}_\alpha \sin\alpha \qquad 12\sin\delta\,.\,X = \Sigma\,\overline{\mu}_\delta \cos\alpha$$

$$12\,Y = \Sigma\,\overline{\mu}_\alpha \cos\alpha \qquad 12\sin\delta\,.\,Y = \Sigma\,\overline{\mu}_\delta \sin\alpha$$

$$24\cos\delta\,.\,Z = \Sigma\,\overline{\mu}_\delta$$

The mean error of $\overline{\mu}_\alpha$ and $\overline{\mu}_\delta$ depends on the mean error of the proper motions of the Leiden catalogue and the number of stars.

Let the mean error of $\overline{\mu}_\alpha$ and $\overline{\mu}_\delta$ be $\pm\,\varepsilon$, then:

$$\text{m. e. } \Sigma\overline{\mu}_\alpha \cos\alpha = \text{m. e. } \Sigma\overline{\mu}_\alpha \sin\alpha = \text{m. e. } \Sigma\overline{\mu}_\delta \cos\alpha = \text{m. e. } \Sigma\overline{\mu}_\delta \sin\alpha = \pm\,3{\cdot}5\,\varepsilon$$

$$\text{m. e. } \Sigma\overline{\mu}_\delta = \pm\,4{\cdot}9\,\varepsilon$$

In consequence of the varying factors $\sin\delta$ and $\cos\delta$ in the second set of normal equations the resulting mean error of X, Y and Z as derived from the proper motions in declination, is very different for the individual zones.

The following table gives the m. e. of X, Y and Z from the μ_α and μ_δ solutions of the seven zones:

	μ_α Solution		μ_δ Solution		
Zone	X	Y	X	Y	Z
$0°$—$15°$	$\pm\,0{\cdot}29\,\varepsilon$	$\pm\,0{\cdot}29\,\varepsilon$	$\pm\,2{\cdot}2\,\varepsilon$	$\pm\,2{\cdot}2\,\varepsilon$	$\pm\,0{\cdot}2\,\varepsilon$
15—25	,,	,,	0·9	0·9	0·2
25—30	,,	,,	0·6	0·6	0·25
30—35	,,	,,	0·5	0·5	0·25
35—40	,,	,,	0·5	0·5	0·3
40—45	,,	,,	0·4	0·4	0·3
$>$ 45	,,	,,	0·4	0·4	0·3

These mean errors for the solution of X and Y from both sets of equations correspond to relative weights of 56, 9, 4, 3, 3, 2 and 2 for the different zones.

The Z, however, can be solved from the μ_δ's with a weight, approximately identical with that of X and Y from the μ_α's.

In order to abbreviate the work of computation, the following method of solution has been used, which is sufficiently accurate:

The values of X and Y were solved from the μ_α equations; from the equations of condition given by the μ_δ's only the third normal equation was computed; in this equation the values of X and Y, as obtained from the μ_α's were substituted and then the value of Z was solved.

3. *The results.*

TABLE 3 gives the solution zone for zone, assigning the same weight to all hourly means, from all stars. The second column gives the number of stars in the zone.

TABLE 3. *Solution without weights, all stars.*

Zone	n	X	Y	A	$P\cos D$	$Z = P\sin D$	D	P
$<15°$	186	$+0''{\cdot}0056$	$-0''{\cdot}0124$	$294°$	$+0''{\cdot}0136$	$+0''{\cdot}0153$	$48°$	$0''{\cdot}0205$
15—25	157	− 9	− 77	264	+ 78	+ 159	64	177
25—30	159	+ 57	− 122	295	+ 135	+ 147	48	200
30—35	198	+ 51	− 70	306	+ 87	+ 212	68	229
35—40	229	+ 14	− 100	278	+ 101	+ 252	68	271
40—45	187	+ 47	− 48	314	+ 67	+ 245	74	254
$>$ 45	248	+ 59	− 78	307	+ 98	+ 218	66	239
total	1364	+ 0·0038	− 0·0089	293	+ 0·0097	+ 0·0189	63	0·0212
		± 9	± 11	± 6·7	± 10	± 17	± 3·9	± 14

TABLE 4 gives the solution zone for zone, applying weights proportional to the number of stars contributing to every hourly mean.

TABLE 4. *Solution with weights, all stars.*

Zone	n	X	Y	A	$P\cos D$	$Z=P\sin D$	D	P
$<15^\circ$	186	$+0''.0060$	$-0''.0090$	304°	$+0''.0108$	$+0''.0139$	52°	$0''.0176$
15—25	157	+ 32	— 49	303	+ 59	+ 147	68	158
25—30	159	+ 40	— 41	315	+ 57	+ 126	66	138
30—35	198	+ 37	— 60	301	+ 70	+ 175	68	188
35—40	229	+ 29	— 96	287	+ 100	+ 232	67	253
40—45	187	+ 44	— 52	310	+ 68	+ 237	74	246
>45	248	+ 18	— 55	288	+ 58	+ 176	72	185
total	1364	+0·0037	—0·0064	300	+0·0074	+0·0175	67	0·0190
		± 5	± 9	± 4·0	± 8	± 16	± 1·6	± 16

From both tables we may conclude:

1° that there is no essential difference between the results of the weighted and unweighted solutions.

2° that there is no marked difference in the co-ordinates of the apex, as derived from the stars in the individual zones; perhaps a maximum value of Z and consequently of D is indicated in the neighbourhood of 40°.

The second conclusion justifies a solution of the co-ordinates of the sun's apex, dividing the stars in the two groups $\leq 8^m.0$ and $> 8^m.0$, taking all the zones of declination together.

TABLE 5 gives the unweighted as well as the weighted solution in this way.

TABLE 5. *Solution without weights, all declinations.*

Mag.	n	X	Y	A	$P\cos D$	$Z=P\sin D$	D	P
$\leq 8^m.0$	455	$+0''.0048$	$-0''.0114$	$293^\circ \pm 5^\circ.7$	$+0''.0124$	$+0''.0198$	$58^\circ \pm 3^\circ.6$	$0''.0234 \pm 19$
$>8·0$	909	+0·0026	—0·0061	293 ± 4·8	+0·0066	+0·0178	70 ± 5·4	0·0190 ± 23

Solution with weights, all declinations.

Mag.	n	X	Y	A	$P\cos D$	$Z=P\sin D$	D	P
$\leq 8^m.0$	455	$+0''.0055$	$-0''.0086$	$302^\circ \pm 5^\circ.0$	$+0''.0102$	$+0''.0170$	$59^\circ \pm 2^\circ.4$	$0''.0198 \pm 17$
$>8·0$	909	+0·0027	—0·0052	297 ± 7·2	+0·0059	+0·0172	71 ± 2·3	0·0182 ± 22

That the mean errors of the results of the second group containing 909 stars are not smaller than those of the first group numbering 455 stars, must be explained by the fact, that the proper motions of the fainter stars are determined with an appreciably smaller accuracy.

From Table 5 we may conclude:

1° that there is again no essential difference between the results of the weighted and unweighted solutions.

2° that there is no difference between the R. A.'s of the sun's apex, as derived from both groups.

3° that there is a rather marked difference in the declination of the sun's apex for both groups.

4° that the resulting values of the parallactic motion of both groups do not agree with the data in *Groningen Publications* **29**, where the ratio 1·5 is given for the parallaxes of stars of this type with the magnitudes $7^m.5$ and $8^m.5$.

However, the most important features of the resulting values of the co-ordinates of the apex are the very large value of D, well known from other investigations, and the wide deviation of A from the usually adopted 270°.

Before an extensive material of radial velocities concerning the same stars is at hand, it is difficult to decide, especially about the second fact, whether these results are real or influenced by systematic errors in the proper motions; perhaps a mixture of both causes is present.

Thus *e. g.* the position of the apex could be explained by assuming that nearly all the stars belong to the second drift of KAPTEYN or that a very large percentage of the stars belong to the stars of great velocity. Dr. J. H. OORT in *Groningen Publications* **40** has found an increasing value of A and D of the sun's apex with an increasing percentage of these stars in a group.

On the other hand, it is nearly a certainty that the proper motions in declination depending on the BOSS system need a systematic correction, which will have a greater influence on the co-ordinates of the apex as the mean parallactic motions of the stars are decreasing. For that reason it is an advantage that in the solutions of this section the X and Y are wholly independent from the μ_δ's; now it is possible to derive the correction to the μ_δ's, keeping the values of X and Y unchanged.

4. *Derivation of the μ_δ correction.*

I have computed a correction to the μ_δ's on either of the following assumptions:

a. The resulting D computed from all the stars must be 30°.

b. The resulting D computed from all the stars must be 45°.

c. The resulting values of P, computed from the groups $\leqq 8^{m}\cdot 0$ and $> 8^{m}\cdot 0$ must have the ratio 1·5.

In the solution without weights from all the stars (Table 3)

$X = +0''\cdot 0038$, $Y = -0''\cdot 0089$, $P \cos D = +0''\cdot 0097$.

Assuming $D = 30°$, $Z = P \sin D$ must be $+0''\cdot 0097 \tan 30° = +0''\cdot 0056$.

Assuming $D = 45°$, $Z = P \sin D$ must be $+0''.0097 \tan 45° = +0''\cdot 0097$.

The third normal equation of the set of μ_δ equations without a correction to the μ_δ's is:

$$-7\cdot 4\, X + 2\cdot 1\, Y + 105\cdot 1\, Z = +1''\cdot 9427.$$

Substituting, however, the values of X, Y and Z given above, the right-hand member of this normal equation becomes:

$+0''\cdot 5418$ and $+0''\cdot 9727$ for the 1st and 2nd assumption respectively.

This means that the right-hand member of the original third normal equation must be corrected by the amounts:

$-1''\cdot 4009$ and $-0''\cdot 9700$ for the 1st and 2nd assumption respectively.

The right hand member of the original third normal equation can be written in the form:

$$-\Sigma_0^{23}\, \Sigma_1^7 \cos \delta_i\, \overline{\mu_\delta}$$

in which δ_i is the mean declination of each of the seven consecutive zones, $\overline{\mu_\delta}$ are the hourly means of the proper motions in declination.

A constant correction to the μ_δ's, k, will change this member by the amount:

$$-k\, \Sigma_1^7\, a_i \cos \delta_i$$

in which δ_i is the same as above, a_i is the number of hourly means in the zone with index i.

The value of $\Sigma_1^7\, a_i \cos \delta_i$ is

$(24.0\cdot 993 + 21.0\cdot 934 + 19.0\cdot 887 + 19.0\cdot 843 + 22.0\cdot 793 + 21\cdot 0\cdot 737 + 21.0\cdot 669) = 123\cdot 3.$

We thus find:

for the assumption $D = 30°$, $k = +0''\cdot 0113$
" " " $D = 45°$, $k = +0''\cdot 0079$.

I have made a similar computation starting from the results of the weighted solution.

According to Table 4 $X = +0''\cdot 0037$, $Y = -0''\cdot 0064$, $P \cos D = +0''\cdot 0074$.

Then Z must be:

$+0''\cdot 0043$ and $+0''\cdot 0074$ for the 1st and 2nd assumption respectively.

Substituting these values of X, Y and Z in the third normal equation of the weighted solution, $-222\cdot 3\, X + 99\cdot 6\, Y + 941\cdot 9\, Z = +15''\cdot 0601$, it is found, that the right-hand member must be corrected by the amounts:

$-12''\cdot 4703$ and $-9''\cdot 5507$ for each of the assumptions respectively.

A constant correction to the μ_δ's changes the right hand member by the amount:

$$-k\, \Sigma_1^7\, n_i \cos \delta_i\ ,$$

where δ_i is the same as above, n_i is the total number of stars in the zone i.

Then, $\Sigma_1^7\, n_i \cos \delta_i = 1124\cdot 6$ and:

for the assumption $D = 30°$, $k = +0''\cdot 0111$
" " " $D = 45°$, $k = +0''\cdot 0085$.

To derive the value of the μ_δ correction on the third assumption viz: a ratio of 1·5 between the resulting P's from the groups $\leqq 8^{m}\cdot 0$ and $> 8^{m}\cdot 0$, I have made three tentative solutions, using as corrections to the μ_δ the amounts $+0''\cdot 007$, $+0''\cdot 009$ and $+0''\cdot 011$.

In the solution without weights the corrections to the right-hand member of the third normal equation are then:

for the group $\leqq 8^{m}\cdot 0$ $-0''\cdot 742$, $-0''\cdot 954$ resp. $-1''\cdot 166$
for the group $> 8^{m}\cdot 0$ $-0''\cdot 748$, $-0''\cdot 961$ resp. $-1''\cdot 175$

and the third normal equation becomes:

$-10\cdot 1\, X + 1\cdot 2\, Y + 92\cdot 5\, Z = 1''\cdot 0246$ resp. $0''\cdot 8126$ resp. $0''\cdot 6006$ for the stars $\leqq 8^{m}\cdot 0$
$-9\cdot 4\, X - 2\cdot 0\, Y + 89\cdot 8\, Z = 0''\cdot 8165$ resp. $0''\cdot 6035$ resp. $0''\cdot 3895$ for the stars $> 8^{m}\cdot 0$