radial velocity of a Cepheid variable, as observed, may not be the simple phenomenon it used to be thought, and though it is not certain whether the velocities observed at different phases of the light-variation refer to the same layer in the atmosphere, yet it is interesting to look for the secondary period in the radial velocities too, all the more so, because prediction of the shape of the radial-velocity curve according to the pulsation theory is simpler than that of the light-curve.

Of course, the observed values of the radial velocity are far from sufficing for the actual determination of the two arguments when these are not known, but once these have been found from the light-variation, as in the case of RR Lyrae, it should be possible to derive the amplitudes of the various periodic terms in the radial velocity. Moreover, RR Lyrae is the most suitable object for such an investigation, because it is the brightest of the cluster-type variables known.

Accordingly I have repeated my earlier attempts at analysis of the radial velocities determined for this star at the Mount Wilson Observatory in the years 1916 to 1930[1]).

Though there are some slight indications of the influence of the secondary period, the observations proved to be not sufficiently accurate for the present purpose, so that no definitive results can be given yet.

It is to be regretted that on the whole the determinations of radial velocity of Cepheid variables fall so far behind the observations of brightness, in number as well as in accuracy, though, of course, this is partly due to the greater amount of work involved in the determination of radial velocities. Also the remark may be allowed that it is often difficult for astronomers, not working in stellar spectroscopy themselves, to judge about the accuracy of the published radial velocities.

It would certainly be very desirable to have series of accurate velocities for a few selected variable stars. For the reasons given above it is especially to be hoped that such measures will be made for RR Lyrae, if possible with simultaneous accurate determinations of brightness. A hundred observations with an accuracy of 2 to 3 km/sec, evenly distributed over all phases in the 41-days period, would probably suffice for the intended analysis.

I wish to mention that the above remarks have, for the greater part, resulted from discussions with the late Dr Woltjer.

Discussion of Stebbins's recent[2]) photo-electric measures of δ Cephei,

by *A. J. Wesselink.*

A discussion of Stebbins's photo-electric measures at six different effective wave-lengths, is given. It is found that the complete relative distribution of energy from $^{\mu}$·42 — 1$^{\mu}$·03 depends uniquely on the colour-index. The same relative distribution of energy occurs therefore twice during a period since the colour-index reaches the same value twice. It is shown by means of the measures at the ultra-violet wave-length that this property of the relative distribution of energy does not hold in the ultra-violet. An accurate parameter *c*, which defines the relative distribution of energy in the interval of wave-lengths just mentioned, has been determined at each of the 25 phases. *c* is a colour-index on the wave-lengths $^{\mu}$·42 and 1$^{\mu}$·03. It is more accurate than the difference between the magnitudes at these wave-lengths as the brightnesses measured at the three wave-lengths inside the interval have been used as well in its determination.

A mean light-curve $\overline{m'}$ has been determined; $\overline{m'}$ is the mean of the five nearly simultaneously observed magnitudes at different wave-lengths (*U* being excluded). The effective wave-length of this light-curve of extreme accuracy is $^{\mu}$·560. A simple formula is given by which a light-curve at any wave-length between $^{\mu}$·42 and 1$^{\mu}$·03 may be calculated from $\overline{m'}$ and *c*. In the next article the property of the relative energy distribution mentioned above is shown to be a necessary condition for the determination of the mean radius along the lines set forth in *B.A.N.* No. 368.

The observational material consists of 26 sets. Each set comprises six differences in magnitude *m* (δ Cep)— — *m* (ε Cep) at the six effective wave-lengths given in Table 1. The observations were made with a single photo-electric cell attached to the sixty-inch reflector at Mt Wilson. The various effective wave-lengths were realized by means of suitable filters. One set is printed between parentheses and is discordant. It has been omitted in the following. For each set a single date is given. The corresponding 25 phases have been computed with the formula: phase $= ^{d-1}$·18634702 × (J. D. Hel. M.A.T. Grw. — 2420000), and are well distributed over a complete period. The reciprocal period used corresponds to the value of the period derived on page 89, *B.A.N.* **10**.

Table 1

magnitude	$\lambda_{eff.}$	$f(\lambda)$
U	$^{\mu}$·353	
V	·422	1·000
B	·488	·658
G	·570	·424
R	·719	·175
I	1·030	·000

The 6^{m}·6 companion of δ Cep was excluded from the measures.

The spectrum of the comparison star ε Cephei is A6.

[1]) R. F. Sanford, *Ap. J.* **67**, 319, 1928, *Mt Wilson Contr.* No. 351; *Ap. J.* **81**, 149, 1935, *Mt Wilson Contr.* No. 510.

[2]) J. Stebbins, *Ap. J.* **101**, 47, 1945.

We shall denote magnitude differences between δ Cep and ε Cep by capital letters:

U(ltra-violet), *V*(iolet), *B*(lue), *G*(reen), *R*(ed), *I*(nfra-red) and call them briefly: magnitudes.

Consider the difference between two magnitudes belonging to the same set, as a function of the phase. In total 15 types of such differences are possible, although they are not all independent of one another. Each difference is a difference in colour-index between δ Cep and ε Cep at the corresponding pair of wave-lengths. We denote such differences briefly with "colour".

Each colour varies periodically with the time. Hence each value is reached at least twice during a cycle, with the exception of the extremes which are reached only once. The most general figure to be expected therefore when two simultaneous colours of different types are plotted is obviously a closed curve, which is described once during a complete cycle. This is in fact what is found if at least one of the magnitudes involved in the colours is *U* (compare Figure 2). This ultra-violet magnitude is however exceptional as it is found that the colours that may be derived from the five magnitudes (*U* excluded), are single-valued functions of one another, the closed curves reducing to lines that are described twice in opposite directions during a cycle. This is a remarkable observational fact, the more so as the observations are very accurate and cover the considerable range of wave-lengths from μ·42 — 1μ·03.

It follows that the relative distributions of energy through which the radiation of the variable passes during a cycle form a one-dimensional series, each relative distribution of energy occurring twice during a cycle.

In the following we shall leave out the magnitude *U* and proceed to determine from the remaining five magnitudes *V*, *B*, *G*, *R* and *I* a parameter, which in a simple and accurate manner determines the relative distribution of energy in the interval μ·42 — 1μ·03. This parameter assumes the same value twice during a cycle.

From a plot against $V-I$ of respectively $V-B$, $G-I$ and $R-I$ it was found that the three relations may be represented satisfactorily by straight lines. Therefore three least-squares solutions were made, in which the 25 equations of condition, each corresponding to one set, were assumed to have equal weight.

The results with their mean errors are:

$$V-B = +\cdot3420\,(V-I) - {}^{\mathrm{m}}\cdot087 \qquad (1)$$
$$\pm\cdot0057 \qquad \pm\cdot007$$
$$G-I = +\cdot4245\,(V-I) + {}^{\mathrm{m}}\cdot100 \qquad (2)$$
$$\pm\cdot0057 \qquad \pm\cdot007$$
$$R-I = +\cdot1752\,(V-I) + {}^{\mathrm{m}}\cdot083 \qquad (3)$$
$$\pm\cdot0057 \qquad \pm\cdot007$$

Some justification should be given of the form of these equations of condition. If the magnitudes *V*, *B* etc. are of the same accuracy, then the six quantities $V-B$, $V-I$ etc. are also equally accurate. The solution of the equations in the form given above, by means of least squares assumes the quantities to the left of the equality sign to be subject to accidental error only, whereas the right-hand members are treated as exact. Actually the right-hand members are uncertain too. The adopted form is however a good approximation, the right-hand members being more accurate than the left-hand ones. That the errors to the right and to the left of the equality sign are dependent is of negligible importance. In Table 2 the frequencies of the absolute values of the residuals are shown.

TABLE 2

Frequency of absolute values of residuals from equations (1), (2) and (3)

m	(1) n	(2) n	(3) n
·00	10	10	10
·01	11	11	11
·02	4	4	3
·03	0	0	1

Equations (1), (2) and (3) may be written:

$$V-(B-\cdot087) = +\cdot3420\,(V-I) \qquad (1')$$
$$(G-\cdot100)-I = +\cdot4245\,(V-I) \qquad (2')$$
$$(R-\cdot083)-I = +\cdot1752\,(V-I) \qquad (3')$$

We now introduce five new quantities:
$V' = V$, $B' = B - {}^{\mathrm{m}}\cdot087$, $G' = G - {}^{\mathrm{m}}\cdot100$,
$R' = R - {}^{\mathrm{m}}\cdot083$, $I' = I$.

The accented quantities are differences in magnitude m' between δ Cep and a fictitious star (ε′ Cep) which at the violet and the infra-red wave-length has the same brightness, but which at the blue, green and red effective wave-lengths respectively is ${}^{\mathrm{m}}\cdot087$, ${}^{\mathrm{m}}\cdot100$ and ${}^{\mathrm{m}}\cdot083$ fainter than ε Cep.

It follows from (1′), (2′) and (3′) that the differences in magnitude m' between δ Cep and ε′ Cep in the interval of wave-lengths μ·42 — 1μ·03 may be represented by the following formula:

$$m' = c\,f(\lambda) + d \qquad (4)$$

where $f(\lambda)$ is a universal function, which depends on the coefficients in (1′), (2′) and (3′). We have in fact:

$$f_V - f_B = +\cdot3420\,(f_V - f_I)$$
$$f_G - f_I = +\cdot4245\,(f_V - f_I)$$
$$f_R - f_I = +\cdot1752\,(f_V - f_I)$$

We take $f_V = 1\cdot000$ and $f_I = \cdot000$ and obtain the values at the other wave-lengths as given in Table 1.

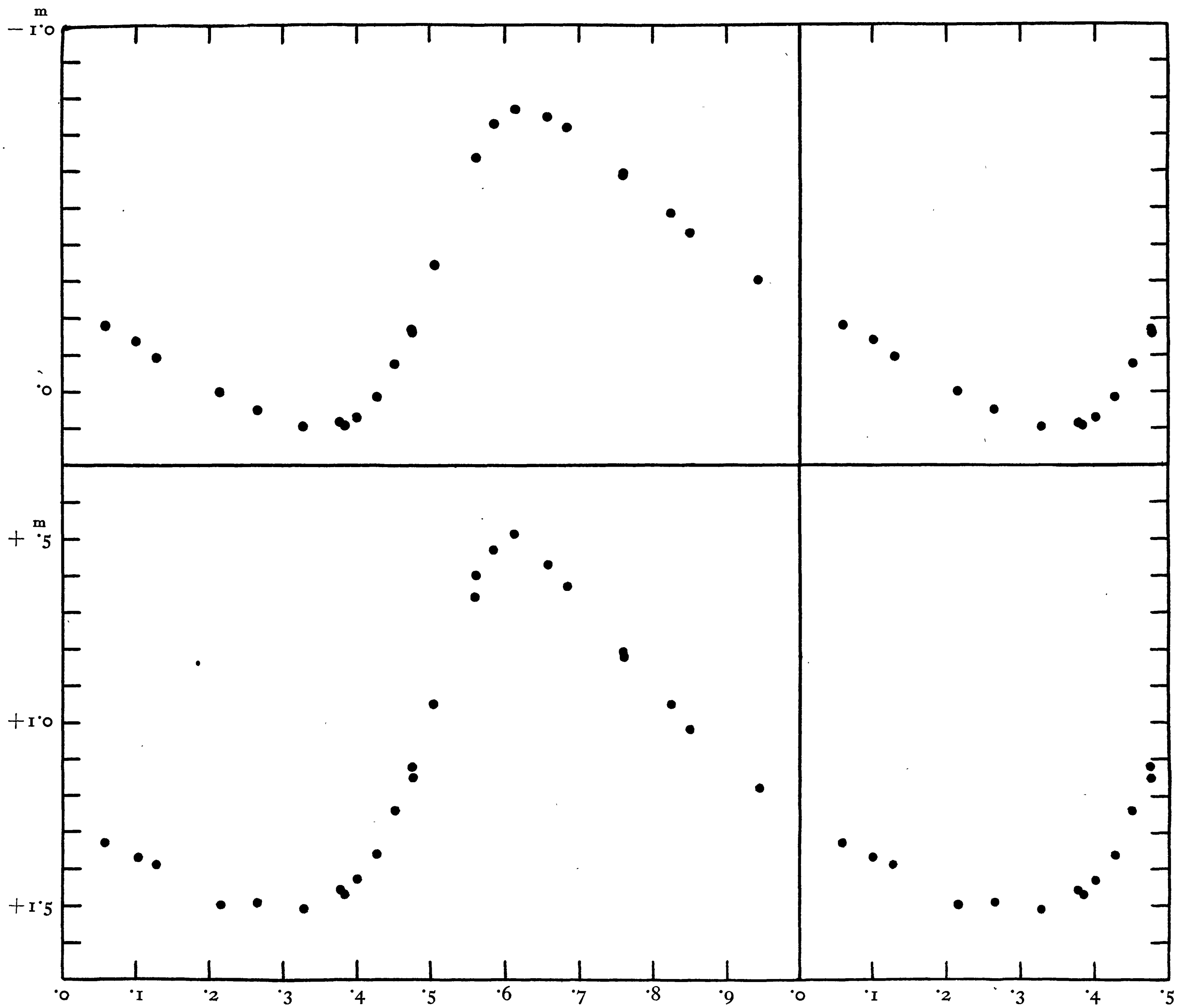
It is clear that any linear function of the function $f(\lambda)$ we have defined in Table 1 would serve our purpose as well.

We made 25 least-squares solutions. In each solution the quantities c and d in formula (4) were determined from the five values V', B', G', R' and I' occurring in a set. The quantity c is the parameter which defines the relative distribution of energy in the interval of wave-lengths μ·42 — 1μ·03 of δ Cep. Since we choose f in such a way that $f_V - f_I = 1$, it follows that c is just the difference in colour-index between δ Cep and ε' Cep or ε Cep on the wave-lengths μ·42 and 1μ·03, somewhat improved by the measures B, G and R. Colour-indices on any pair of wave-lengths may be obtained from c by multiplying with an appropriate factor.

c is given as a function of the Julian date and of the phase in Table 3. It is shown graphically in Figure 1.

It follows from formula (4) that $\overline{m'} = 1/5\ (V' + B' + G' + R' + I')$ is the difference in magnitude between δ Cep and ε' Cep at an effective wave-length, which corresponds to the mean value of $f(\lambda)$, which is $\bar{f} = $ ·4514. The corresponding wave-length is μ·560. $\overline{m'}$ is given as a function of the Julian date and of the phase in Table 3. It is shown graphically in Figure 1. $\overline{m'}$ is of course more accurate than any of the five light-curves separately. The accidental errors in $\overline{m'}$ and in c are independent. Any light-curve

FIGURE 1



In both diagrams abscissae are phases: d^{-1}·18634702 (JD—2420000). The ordinate in the upper diagram is $\overline{m'}$, in the lower diagram the ordinate is the colour aequivalent c.

whose effective wave-length is between μ·42 and 1μ·03 may be calculated from the curves $\overline{m}'$ and c according to the formula:

$$m = \overline{m}' + (f - \overline{f})\, c \qquad (5)$$

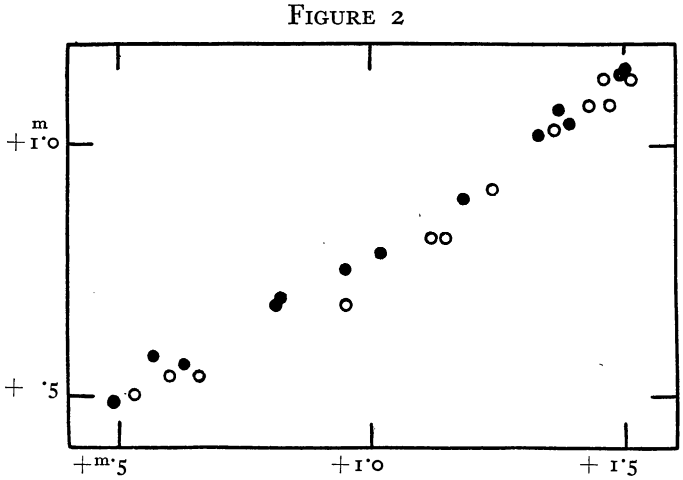
TABLE 3

$\overline{m}'$ and c as functions of the Julian date and of the phase.

J.D. −2430000	$^{d-1}$·18634702 × (J.D. −2420000)	$\overline{m}'$= 1/5($V'+B'+G'+$ $+R'+I'$)	c
d	P	m	
233·908	·058	− ·182	+ 1·33
904·933	·102	− ·138	1·37
550·897	·128	− ·094	1·39
234·756	·216	+ ·002	1·50
610·667	·266	+ ·048	1·49
610·997	·328	+ ·094	1·51
911·779	·377	+ ·084	1·46
911·808	·383	+ ·092	1·47
911·912	·402	+ ·072	1·43
584·696	·427	+ ·018	1·36
611·663	·452	− ·074	1·24
584·958	·475	− ·170	1·12
230·785	·476	− ·162	1·15
611·951	·505	− ·346	·95
912·758	·560	− ·638	·66
912·767	·562	− ·640	·60
912·894	·585	− ·728	·53
585·699	·613	− ·768	·49
585·931	·657	− ·750	·57
231·896	·683	− ·720	·63
584·912	·758	− ·594	·81
913·825	·759	− ·592	·82
205·819	·824	− ·484	·95
232·788	·850	− ·432	1·02
549·901	·943	− ·306	1·18

We shall now return to the ultra-violet magnitude U. The exceptional behaviour of U is illustrated by Figure 2, where $U - G$ has been plotted against c.

FIGURE 2



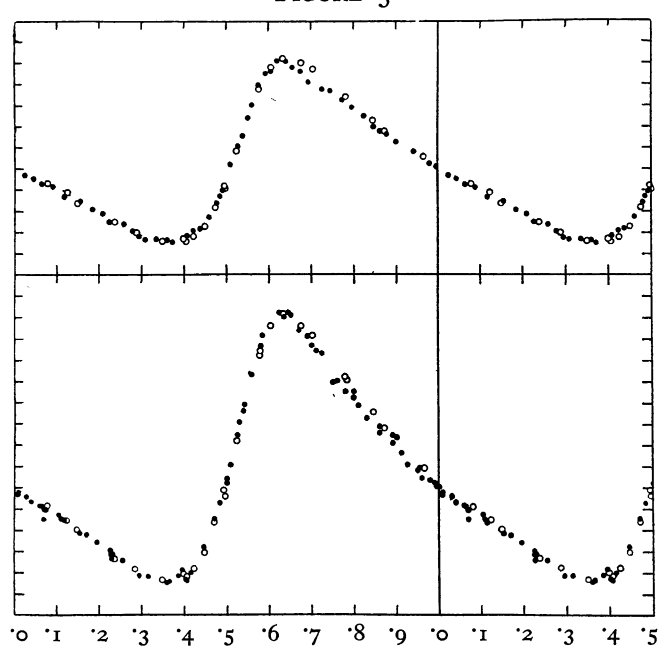
Abscissa: c.
Ordinate: $U - G$.

The errors in the two co-ordinates are practically independent. Points having phases between ·300 and ·600 are represented by open circles (rising branch of c-curve) whereas points with phases outside this interval (descending branch of c-curve) are represented by dots. The position of the open circles is seen to be systematically different from that of the dots, a fact which is not shown if a colour is plotted against c, which is independent of U.

From the residuals of the various least-squares solutions I find a mean error of one difference m (δ Cep)— — m (ε Cep) of ± m·009. Systematic errors, which are the same for the differences belonging to the same set, are negligible. The total weight of each of the six light-curves is consequently 25 × ·009^{-2} = 310000 m^{-2}. The total weight of $\overline{m}'$ = 1500000 m^{-2}. The mean error of a single determination of c equals ± m·011. The corresponding weight of the c-curve is 200000. If c is reduced to the ordinary scale of colour-indices on the wave-lengths μ·43 and μ·55 we have to multiply c with ·48 and the total weight of the resulting colour-index curve becomes 800000 m^{-2}.

The effective wave-length of $\overline{m}'$ is so nearly equal to that of the writer's photovisual light-curve that a direct comparison of the two light-curves is possible. In Figure 3 the writer's photovisual light-curve is represented by dots, whereas the light-curve from STEBBINS's photo-electric observations $\overline{m}'$ is represented by open circles. The phases are the same as in *B.A.N.* **10**, 86. The ascending branches are made to coincide by an appropriate shift in phase of $\overline{m}'$. The amplitudes are practically the same; a shift in the

FIGURE 3



Upper diagram: Open circles $\overline{m}'$; dots the photovisual curve of *B.A.N.* **10**, 86.
Lower diagram: Open circles ½ ($V + B$); dots the photo-electric light-curve (GUTHNICK and SMART, *B.A.N.* **10**, 89, 94). Each division in the ordinates corresponds to m·1.