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An investigation of the constant of refraction from observations at Leiden and at the Cape

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error of $\pm .27$ instead of the assumed $.45$. Therefore a new computation of the definitive proper motions has been made, giving double weight to the computed μ 's.

I believe that the resulting proper motions will agree very closely with the system of the *P. G. C.* and that they thus will need the corrections given by RAYMOND of Albany *).

It is rather satisfactory that in general the new values show a strong tendency to be more nearly equal than the old proper motions. The average deviation of the proper motions of separate stars from the mean is found to be $\pm .002$ for μ_{α} and $\pm .003$ for μ_{δ} . From the mean values given at the foot of the columns it appears that there is no systematic difference between the new and old series of proper motions.

For the sake of completeness I have added the radial velocities, for which the mean has been taken of the determinations by the authorities given

*) *Astr. J.* Vol. 36, No. 857—858 (1926).

in the last column. The abbreviations used are:

Y = *Aph. J.* 64 (1926). E. B. FROST, S. B. BARRETT and O. STRUVE: Radial velocities of 368 Helium stars.

L = *Lick Public.* Vol. 16. Catalogue of Lick Observatory radial velocities,

W = *Mt Wilson Contr.* 258. W. S. ADAMS and A. H. JOY: Radial velocities of 1013 stars.

D = *Publ. Astr. Obs. Univ. of Michigan* Vol. I (1914). P. W. MERRILL: The radial velocity of Maia.

The interagreement of these velocities is not nearly so good as that of the proper motions. This must probably be ascribed to the uncertainty of the measures of these early type stars. The radial velocity given for Boss 851, 22.8 km/sec, is the mean of the Yerkes and Lick determinations of 44.5 and 1 km/sec respectively, of which the first is derived from four separate plates giving 46.8, 51.2, 22.9 and 56.9 km/sec. It may be that this discordance of the different determinations are due to variable velocity.

An investigation of the constant of refraction from observations at Leiden and at the Cape, by *J. H. Oort.*

The present investigation is based on fundamental observations of 460 bright stars from $+53^{\circ}$ to -34° declination which were made at Leiden during the years 1882—1897 *), and on the simultaneous observations of practically the same programme at the Cape Observatory **).

The Leiden observations were made with the 16 cm reversible meridian circle. Objective and eye-end are interchangeable in this instrument, and observations could thus be made in four different positions, viz: State I clamp East and West, and State II clamp East and West. An average number of 4 or 5 observations were obtained in each of these positions; two circles were read by four microscopes each. As a rule the nadir was determined twice each night, before and after the observations. No reflex observations

*) Catalogue of 460 stars observed at the Observatory of Leiden during the years 1882—'97, *Annalen Leiden*, XIII 4, in press.

***) Cape Catalogues for 1885 and 1890.

have been used in the formation of the mean Leiden declinations. Corrections for flexure were applied including a term proportional to $\sin z$ as well as one proportional to $\cos z$.

The refraction tables used in computing the refraction were constructed after RADAU's theory. The computed refractions agree exactly with those computed from RADAU's tables *) after the latter have been multiplied by the factor $(1 - 0.00027)$.

The latitude adopted for converting the zenith distances into declinations was derived from the observations of 84 close circumpolar stars **). With the refraction constant assumed by RADAU (see below) the latitude was found to be $+52^{\circ} 9' 19''.72$. We have however used the somewhat higher value $+52^{\circ} 9' 19''.81$, following from a provisional correction of RADAU's refraction by a factor $(1 - 0.002)$.

*) *Annales de Paris, Mémoires*, t. 19, 1889.

***) *Leiden Annals*, XIII 3, 1922.

The Cape declinations were measured with the non-reversible transit-circle, which instrument is equipped with one circle read by six microscopes. During the interval of the observations for the 1885 catalogue many reflex observations were made. These were, however, reduced to the system of the direct observations, the latter being corrected for flexure by means of the formula $-0''.46 \sin z$.

Before making the comparison with the Leiden positions those of the Cape 1885 catalogue were

corrected in accordance with the table of systematic errors printed on the page preceding the catalogue, extended to declinations above $+45^\circ$ by means of the constants given in the introduction. The latitude adopted for the declinations thus corrected is $-33^\circ 56' 3''.54$; the refractions used are those of BESSEL's Tabulae Regiomontanae diminished by 0.00218 times their amount. Both the latitude and the correction to the refraction were derived from a comparison with Greenwich and from observations of circumpolar stars.

TABLE I.

(1) Limits of declination	(2) $z_{Leid.}$	(3) z_{Cape}	(4) Leid.-Cape ^{'85}		(5) Leid.-Cape ^{'90}		(8) $L-C'$ ^{'85} corr. I	(9) $L-C'$ ^{'90} corr. I	(10) $L-C'$ ^{'85} corr. II	(11) $L-C'$ ^{'90} corr. II	(12) L -Auw. corrected		(13) L -Boss corrected		(14) L -Newc. corrected		(15) L -Pulk. corrected					
			$\Delta\delta$	n	$\Delta\delta$	n					$\Delta\delta$	n	$\Delta\delta$	n	$\Delta\delta$	n	$\Delta\delta$	n	$\Delta\delta$	n	$\Delta\delta$	n
+ 52 ^o 0' to + 53 ^o 0'	0.3	86.4	"	—	"	—	"	"	"	"	"	"	"	"	"	"	"	"				
+ 51 + 52	0.7	85.4	-3.48	1	—	—	-2.14	—	-3.48	—	—	+0.12	8	+0.24	9	-0.02	8	+0.23	6			
+ 50 + 51	1.7	84.4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—			
+ 49 + 50	2.7	83.4	-1.70	3	-0.69	3	-0.84	+0.17	-1.69	-0.68	—	—	—	—	—	—	—	—	—			
+ 48 + 49	3.7	82.4	-1.10	3	-.54	6	-.40	+ .16	-1.09	-.53	—	—	—	—	—	—	—	—	—			
+ 47 + 48	4.7	81.4	-1.06	1	-.52	2	-.50	+ .04	-1.05	-.51	—	—	—	—	—	—	—	—	—			
+ 46 + 47	5.7	80.4	-0.70	4	-.30	4	-.24	+ .16	-0.69	-.29	—	—	—	—	—	—	—	—	—			
+ 43 + 46	7.7	78.4	-.65	7	-.06	8	-.35	+ .24	-.63	-.04	—	—	—	—	—	—	—	—	—			
+ 40 + 43	10.7	75.4	-.35	5	+ .05	5	-.21	+ .19	-.33	+ .07	—	—	—	—	—	—	—	—	—			
+ 35 + 40	14.7	71.4	+ .17	9	-.01	9	+ .17	-.01	+ .20	+ .02	—	—	—	—	—	—	—	—	—			
+ 30 + 35	19.7	66.4	-.01	7	+ .16	8	-.10	+ .07	+ .03	+ .20	—	—	—	—	—	—	—	—	—			
+ 25 + 30	24.7	61.4	+ .05	12	+ .35	12	-.09	+ .21	+ .10	+ .40	—	—	—	—	—	—	—	—	—			
+ 20 + 25	29.7	56.4	+ .21	9	+ .17	9	+ .04	.00	+ .27	+ .23	—	—	—	—	—	—	—	—	—			
+ 15 + 20	34.7	51.4	+ .32	7	+ .29	6	+ .14	+ .11	+ .40	+ .37	—	—	—	—	—	—	—	—	—			
+ 10 + 15	39.7	46.4	+ .28	13	+ .25	13	+ .11	+ .08	+ .37	+ .34	—	—	—	—	—	—	—	—	—			
+ 5 + 10	44.7	41.4	+ .18	23	+ .25	23	+ .03	+ .10	+ .29	+ .36	—	—	—	—	—	—	—	—	—			
+ 0 + 5	49.7	36.4	+ .32	26	+ .34	22	+ .20	+ .22	+ .45	+ .47	—	—	—	—	—	—	—	—	—			
- 5 + 0	54.7	31.4	-.01	59	-.04	27	-.08	-.11	+ .15	+ .12	—	—	—	—	—	—	—	—	—			
- 10 - 5	59.7	26.4	-.05	55	-.05	33	-.05	-.05	+ .14	+ .14	—	—	—	—	—	—	—	—	—			
- 15 - 10	64.7	21.4	-.18	49	-.22	25	-.08	-.12	+ .05	+ .01	—	—	—	—	—	—	—	—	—			
- 20 - 15	69.7	16.4	-.22	60	-.22	28	+ .04	+ .04	+ .08	+ .08	—	—	—	—	—	—	—	—	—			
- 23 - 20	73.7	12.4	-.52	35	-.63	29	-.07	-.18	-.15	-.26	—	—	—	—	—	—	—	—	—			
- 26 - 23	76.7	9.4	-.68	15	-.69	15	+ .02	+ .01	-.20	-.21	—	—	—	—	—	—	—	—	—			
- 29 - 26	79.7	6.4	-.91	9	-.85	9	+ .16	+ .22	-.28	-.22	—	—	—	—	—	—	—	—	—			
- 30 - 29	81.7	4.4	-.82	4	-.75	3	+ .65	+ .72	-.02	+ .05	—	—	—	—	—	—	—	—	—			
- 31 - 30	82.7	3.4	-1.48	6	-1.41	5	+ .31	+ .38	-.51	-.44	—	—	—	—	—	—	—	—	—			
- 32 - 31	83.7	2.4	-1.07	1	-1.10	1	+ 1.13	+ 1.10	+ .13	+ .10	—	—	—	—	—	—	—	—	—			
- 33 - 32	84.7	1.4	-2.75	1	-1.68	4	+ .06	+ 1.13	-1.19	-.12	—	—	—	—	—	—	—	—	—			
- 34 - 33	85.7	0.4	—	—	-2.49	1	—	+ 1.26	—	-.30	—	—	—	—	—	—	—	—	—			
- 35 - 34	86.7	0.6	-3.20	4	-2.48	6	+ 2.18	+ 2.90	+ 0.18	+ .90	—	—	—	—	—	—	—	—	—			

The latter observations were extended and again thoroughly discussed in the introduction to the 1890 catalogue. The resulting latitude, $-33^\circ 56' 3''.45$, was adopted in the formation of the final declinations in that catalogue*), and the refractions were corrected in the same way as for the 1885 catalogue.

The detailed comparison of the Cape declinations with those of Leiden is shown in table I, columns 4 and 6. The numbers of stars compared are given under n . The second and third columns indicate the average zenith distance at Leiden and at Cape respectively.

It is well known that the various refraction tables which are used at the present time show very nearly the same run with the zenith distance; the refractions

*) In the following comparisons we have used the declinations as given in the columns headed "seconds of final declination."

for normal conditions as computed from various tables differ only by factors which are independent of the zenith distance, at least up to about 80°. This may be illustrated by the following little table copied partly from ALBRECHT*), giving the refractions as computed from several well known tables for the following conditions:

External temp. = temp. of barometer = + 10° C.,
 barom. 760 mm, latitude = + 45°, altitude = 0,
 $p_{H_2O} = 6$ mm..... (1)

The last two columns were computed from the third through multiplication by the factors (1 - 0.0041) and (1 - 0.0047) respectively. The last column but one is seen to agree rather accurately with the Pulkowa column, whereas the last column is nearly identical with DE BALL and ALBRECHT. The refractions from the Tabulae Regiomontanae agree with those of RADAU except for a difference of nearly 0".3 at 80° zenith distance.

TABLE 2

Mean refractions.

Z_{app}	BESSEL Tab. Regiom.	RADAU	Pulkowa	DE BALL	ALBRECHT	(1-0.0041) × RADAU	(1-0.0047) × RADAU
40°	48".77	48".77	48".57	48".54	48".54	48".57	48".54
45	58.10	58.10	57.86	57.82	57.82	57.86	57.83
50	69.20	69.20	68.92	68.88	68.88	68.92	68.88
60	100.39	100.39	99.99	99.91	99.92	99.98	99.92
70	158.40	158.44	157.78	157.69	157.67	157.79	157.70
80	318.42	318.71	317.36	317.1	317.15	317.40	317.21

In the following we shall start by trying to determine a constant correction-factor to the mean refraction.

Supposing that the average refractions adopted in Leiden require corrections of x times their amount and those adopted at Cape corrections of y times their amount and that the difference of latitude Leiden minus Cape requires a correction $\Delta\phi$, we get the following equations of condition:

$$\delta_L - \delta_C = xr_L + yr_C - \Delta\phi$$

in which r_L represents the mean refraction at Leiden and r_C the analogous quantity for Cape.

Three different solutions were made because it was feared that anomalies in the refractions at large zenith distances might have a preponderating influence. Solution I includes all stars up to zenith distances of 83°, solution II those up to 80° and solution III only the stars with zenith distances smaller than 75°.

*) *Hilfstafeln f. Geogr. Ortsbestimmungen*, 4^{te} Auflage, p. 341.

The numbers of stars in each interval were used as weights, so that the unit of weight in tables 3 and 4 corresponds to a mean error of one difference (about $\pm 0".30$). The limits of the three solutions have been marked by spaces in table 1.

TABLE 3

Solution for refraction constants and for correction to latitude.

Catalogues	Solution	x (weight)	y (weight)	$\Delta\phi$ (weight)
L-C '85	I $z < 83^\circ$	-.0051 (172)	-.0041 (115)	-.63 (59)
»	II $z < 80^\circ$	56 (89)	40 (51)	-.67 (34)
»	III $z < 75^\circ$	62 (30)	36 (13)	-.72 (12)
L-C '90	I $z < 83^\circ$	50 (152)	25 (139)	-.58 (50)
»	II $z < 80^\circ$	54 (81)	22 (50)	-.60 (29)
»	III $z < 75^\circ$	30 (25)	+ 11 (12)	-.20 (10)

The $(L-C)$'s corrected according to the above solution [adopting $x = -.0053$, $y = -.0032$, $\Delta\phi('85) = -.65$, $\Delta\phi('90) = -.59$], are shown under the headings $L-C$, corr. I, in the eighth and ninth columns of table 1. A small additional correction was applied to the Leiden declinations below -23° (see below).

Except for the values with lowest weights, the various solutions are rather consistent in giving a correcting factor of (1 - 0.0053) to the Leiden refractions and one of (1 - 0.0032) to the Cape refractions. The first correction would make the mean refraction at 45° zenith distance and at the normal conditions (1) 57".74, whereas the second correction would lead to the value 57".84. Both values are nearly equal to the mean refraction assumed by DE BALL and ALBRECHT (compare table 2).

The value found for the third unknown, $\Delta\phi$, is less consistent with the results of other observations. It is extremely difficult to see how the adopted Leiden latitude could be in error by as much as + 0".6. The latitude derived from the circumpolar stars would be increased by 0".16 if we used the refraction constant as derived from the above solution, so that the disagreement would become still worse; a similar remark applies to the latitude of the Cape. It is probably still more difficult to understand how the latitude of the Cape instrument could be considerably in error, as this latitude has been very accurately confirmed from observations with the new Cape transit circle*). Yet we may perhaps not be justified in

*) See HOUGH, Introduction to Cape fund. cat. for 1900, p. XLV, and Introduction to second Cape fund. cat. for 1900, p. XXII.

altogether dismissing from our thoughts the possibility of such corrections. In this connection it is of interest to call to mind the discussion concerning the latitude of the Pulkowa Observatory recently made by BONSDORFF *). In this discussion (page 94) the conclusion is reached that the latitude as determined with the vertical circle by ERTEL differs by $0''.50$ from the latitude determined by a similar vertical circle by REPSOLD (which had been temporarily erected at Pulkowa) and that it is, as yet, impossible to decide which is the right latitude. **)

But I think we *are* justified in provisionally dismissing the above solution for the practical purpose of determining the systematic correction to the Leiden places, be it only because we cannot separate $\Delta\varphi_{\text{Leid.}}$ and $\Delta\varphi_{\text{Cape}}$. A second set of solutions has therefore been made on the assumption that $\Delta\varphi = 0$. These are shown in table 4.

The mean refractions at the normal conditions (I) following from these solutions are $57''.94$ for Leiden and $58''.06$ for the Cape.

The differences Leiden—Cape corrected according to this solution (y being neglected and the adopted value of x being -0.0019), are shown in columns 10 and 11 of table I. In these columns as well as

TABLE 4.

Solution for refraction constants on the assumption $\Delta\varphi = 0$.

Catalogues	Solution	x (weight)	y (weight)
Leiden-Cape '85	I $z < 83^\circ$	$- .0019$ (706)	$- .0006$ (259)
»	II $z < 80^\circ$	$- 18$ (572)	$+ 05$ (151)
»	III $z < 75^\circ$	$- 18$ (360)	$+ 27$ (72)
Leiden-Cape '90	I $z < 83^\circ$	$- 22$ (574)	$+ 02$ (327)
»	II $z < 80^\circ$	$- 22$ (427)	$+ 16$ (157)
»	III $z < 75^\circ$	$- 18$ (235)	$+ 28$ (70)

in columns 8 and 9 and in all following columns of the table additional corrections have been applied to the Leiden declinations below -23° , as suggested by investigations on the distribution of temperature in the upper atmosphere by the late professor H. G. VAN DE SANDE BAKHUIJZEN. The corrections are quite small up to 83° zenith distance and they could not have influenced the above solutions in any appreciable manner.

If we now compare the residuals obtained from

*) Resultate der absoluten Deklinationsbestimmungen des Pulkowaer Katalogs, 1915.

**) I am indebted to Mr. J. J. RAIMOND for drawing my attention to this passage.

the first solution with those obtained from the second it is seen that, excepting the declination-intervals below -29° and those above $+49^\circ$, the residuals from the first solution are satisfactorily small and unsystematic (especially if we combine the results of Cape '85 with those of Cape '90). On the other hand the second solution leaves residuals which are systematically negative and rather large above $+40^\circ$ and which are systematically positive between -10° and $+40^\circ$ declination, with a maximum of about $+0''.40$ at $+15^\circ$, that is in the region most conveniently observed from both observatories.

In order to try to find indications as to which solution would be right, the Leiden positions were also compared with the fundamental catalogues of AUWERS (system of Neuer Fundamental Katalog, as outlined in *A. N.*, Bd. 164, 246), BOSS (Preliminary General Catalogue), NEWCOMB*) and with a simultaneous fundamental catalogue of Pulkowa **). The systematic differences and the numbers of stars used in the formation of each difference will be found in columns 12 to 19 of table I. The Leiden declinations have always been corrected with the results of solution II.

The result of the comparisons seems indecisive. Both AUWERS and BOSS are slightly lower than Leiden, so that the Leiden latitude would have to be diminished by about $0''.15$ to make it agree with these catalogues. After elimination of this difference the BOSS declinations would still be somewhat smaller than Leiden in the region from 0° to $+25^\circ$ declination, in the same sense but less than the Cape positions. NEWCOMB's places appear to show a very good agreement with Leiden over the entire range from -20° to $+53^\circ$. The comparison with Pulkowa shows a considerable constant difference; the differences run up to a maximum near $+15^\circ$ just as Cape and BOSS. However, it must be noticed that if we should adopt the latitude of Pulkowa as derived by BONSDORFF from observations with the REPSOLD vertical circle ***) a correction of $-0''.59$ should be applied to the differences in column 18, whereby the systematic residuals between 0° and $+30^\circ$ would be very nearly eliminated.

With regard to the possibility of an instrumental error affecting the Leiden declinations it may be remarked here that the values obtained in the four different instrumental positions agree excellently with each other. The differences between the declinations derived in various positions are small and show no measurable variation with zenith distance ****). Thus, if the systematic differ-

*) *Astronomical Papers Am. Eph.*, Vol. 8, Pt 2.

**) NYRÉN, Déclinaisons Moyennes des ét. princ. pour 1885.0 *Publ. de l'Obs. central Nicolas*, Série II, vol. 1.

***) Compare the first footnote on the first column of this page

****) Compare the introduction, *Leiden Annals*, XIII 4, in press

ences with the Cape would be ascribed to such an error, it would have to be one which is accurately the same function of the zenith distance for the four instrumental positions and for both circles. In one point all the catalogues seem to agree, namely that the negative differences between $+40^\circ$ and $+49^\circ$ in columns 10 and 11 are not due to anomalies in the Leiden declinations; it looks as if these have to be ascribed to an error in the Cape refraction. But if we apply a corresponding correction to the refraction constant all the positive residuals will be considerably increased and we shall approach our first solution, namely that of a large positive error in the difference between the latitudes of Leiden and Cape.

It may be of interest briefly to compare our results to those obtained by some other investigators of the same problem.

In his introduction to the Cape catalogue for 1885 GILL has made comparisons of this catalogue with the Greenwich Ten-Year catalogue for 1880. Rejecting the reflex observations at the Cape he finds the following values for the mean refractions at 45° zenith distance ("mean refraction" corresponding to the conditions (1) as defined above):

for Greenwich $57''.94$, for Cape $58''.03$
(both reduced to 45° latitude)

from a solution including the stars common to both catalogues as well as circumpolar stars. The Cape latitude found from this solution is $-33^\circ 56' 3''.54$.

A very similar investigation has been made more recently by DYSON and CULLEN from the declinations of the Greenwich Observations for 1915–21 and the second Cape fundamental catalogue for 1900*). The results were that neither the Cape refractions nor the Greenwich refractions for northern stars (both from the Pulkova tables) required any important correction, but that the southern stars observed at Greenwich required a correction of $-0''.10 \tan z$ to the declinations. The mean refractions at 45° are thus deduced to be $57''.90$ for Greenwich north, $57''.99$ for Cape, and $58''.00$ for Greenwich south (reduced to 45° latitude).

I want to mention one other comparison between northern and southern observatories, namely that between the extensive catalogues of Albany and San Luis, as recently discussed by BOSS and JENKINS**). The differences between Albany and San Luis are equated to formulas of the following form: $A \sin z + B \tan z + C \tan^3 z + D \tan^5 z$. Practically the

latitudes were assumed to be those derived from fairly close circumpolars after correction with provisional values of the coefficients A, B, C and D. It seems rather doubtful, on account of the introduction of the unknown sine-terms, whether the values of B derived in this way may be taken to represent the true correction to the Pulkova refractions used; if we assume this to be so, the mean refraction as defined above is found to be $58''.13$ for southern stars at Albany, and $58''.39$ for northern stars at San Luis. In the same paper a similar solution of the coefficient B has been made from Albany circumpolar stars and the corresponding mean refraction for north stars is found to be $58''.09$, in rather good agreement with the value for southern stars. For San Luis a much more uncertain similar solution yields $58''.22$ for the southern circumpolars.

Conclusion.

Two types of solutions of the refraction constant have been made from differences between Leiden and Cape declinations. In the first type a correction, $\Delta\varphi$, to the assumed difference of latitude between Leiden and Cape was introduced as an unknown. The solutions give the very large correction $\Delta\varphi = -0''.6$ and for the refractions at 45° zenith distance and at the normal conditions (1) (page 138) $57''.74$ and $57''.84$ for Leiden and Cape respectively. These values agree approximately with the corresponding Pulkova refraction. In the second type $\Delta\varphi$ was assumed to be zero; the values for the mean refractions come out very different in that case, viz. $57''.94$ for Leiden and $58''.06$ for Cape. These are about half way the refractions computed from BESSEL's *Tabulae Regiomontanae* (RADAU) and those of Pulkova. Considerable systematic residuals remain in this case and we are led to the conclusion that if we do not wish to ascribe the differences to a considerable error in the latitudes and to uncertainties of the refractions we must admit the existence of systematic instrumental errors varying from about $-''.7$ to $+''.4$. In view of the very good interagreement of the zenith distances derived in the four positions of the Leiden instrument such errors in the Leiden positions are considered improbable.

In general the discussions in this paper show again that it is eminently important to determine declinations fundamentally at *many different* observatories and, if possible, by different methods.

A provisional investigation of the constant of refraction, based upon nearly the same material as that used in this note, was made many years ago by the

*) *Monthly Notices R.A.S.*, **84**, 470, 1924.

) *Astronomical Journal*, **37, 173, 1927.

late Dr. E. F. VAN DE SANDE BAKHUYZEN. As appears from the "Verslag van den staat der Sterrewacht te Leiden, 1906—1908, page 18", his conclusions were that RADAU's refraction constant needed a correction of 0.0020 of its amount. This is in close agreement with the result of my solutions II, and apparently

he has not considered the possibility of a correction to the difference of latitude Leiden—Cape.

No details, nor any discussions concerning this investigation, could be found in the papers left by Dr. E. F. VAN DE SANDE BAKHUYZEN, so that the above discussion had to be carried out entirely anew.