In Figure 3 the observations are shown as arranged according to six intervals of phase ψ, which are indicated in each section of the figure. It should be realized that these curves are not actual light-curves. During the time of one primary period of 2.568 days, the phase ψ changes by .37 and one has to move in Figure 3 through 2 or even 3 consecutive sections. The number of observations is far too small to construct real ψ-curves for very small intervals in ψ. The fact that the intervals used in Figure 2 are rather wide no doubt causes a part of the dispersion in the observations. Another part of this dispersion may be due to the fact that we have tried to represent the observations by two periods only, while in the case of AI Vel for example more periods are involved. But it is quite clear from Figure 2 that the pronounced changes in the height of maximum and in the form of the light-curve are reasonably well represented by this beat period. Personally I am convinced that the value of the beat period derived here is essentially correct. The more so as in the following article on TU Cas it will be shown that the many photometric observations and radial velocity determinations of this Cepheid can be accurately represented by its primary period P_0 and a beat period equal to 2.445 P_0.

The number of observations of U TrA is far too small to determine the effects of phase- and magnitude distortions, and the present analysis is all one can do with this material.

It seems of great importance that the variations in the light-curves of these two classical Cepheids obey exactly the same laws as the light-variation of stars like AI Vel and SX Phe. It is interesting to notice that these last two variables have extremely short periods as compared with most RR Lyrae-type stars, whereas TU Cas and U TrA have very short periods as compared with other classical Cepheids. There can practically be no doubt that the irregularities in the light-curves of the Cepheids mentioned in the beginning of this paper are due to the same phenomenon, but the number of observations is by far too small to derive the value of their beat periods.

THE LIGHT-VARIATION AND THE RADIAL-VELOCITY CURVE OF TU CAS EXPLAINED IN TERMS OF A PRIMARY PERIOD AND A BEAT PERIOD

BY P. TH. OOSTERHOFF

Two series of photo-electric observations are given, made at the Leiden Observatory by the author and by Mr BAKOŠ. The phase of the ascending branch and the height of maximum vary greatly, as was noticed by all other observers. The variations in the light-curve and also in the radial-velocity curve can be accurately described by the combination of the fundamental period of 2.139 days with a beat period of 5.230 days. The photo-electric observations by KRON and by WORLEY and EGGEN, as well as the visual observations by RYBKA and by CASTEELS are in complete agreement with the formulae derived from my own observations.

In 1947 and 1948 the writer made a number of photo-electric measures in white light of the well known Cepheid TU Cas with the Zunderman reflector of the Leiden Observatory. As comparison star BD $+ 49°72 \equiv$ HD 2055 of spectral type A0 was used. The observations have been listed in Table 1. The first column gives the heliocentric Julian Day, the second the magnitude difference between variable and comparison star and the third column the phase φ, which was computed according to the formula:

$$\text{phase } \varphi = .4674437 \text{ (J.D.} - 2420000).$$

This value of the reciprocal period corresponds with a period of $2^d.13930$, which gives a fair representation of the existing observations. My own observations have been graphically represented in Figure 1. The observations of a single night are given in the form of a line segment. The numbers in this figure and the phases ψ in Table 1 will be explained below. Figure 1 clearly shows that the height and phase of maximum vary enormously. This confirms the conclusions from nearly all other observers who made visual, photographic and photo-electric observations or who observed the radial-velocity curve of this variable.

In 1948 I spent much time on my own observations and on the visual estimates by RYBKA [1]) in order to see whether any other periodicities than the fundamental period of $2^d.139$ could be found in the light-variation of this Cepheid. Although I could convince myself that some features in the light-variation seemed to repeat themselves with some regularity, I did not succeed in deriving any other period but the fundamental period mentioned above.

When I had been successful in deriving a beat period for U TrA, which has been discussed in the

[1]) *Acta Astronomica b*, Vol. **1**, 1929.

TABLE I

J. D. hel. —2430000	Δm	phase φ	phase ψ
d	m		
2424.399	—.227	.707	.30
.401	—.198	.708	
.404	—.211	.709	
.407	—.214	.711	
.410	—.231	.712	
.413	—.255	.714	.30
.454	—.496	.733	.31
.458	—.524	.735	
.460	—.518	.736	
.463	—.541	.737	
.465	—.562	.738	
.468	—.560	.739	
.470	—.588	.740	
.473	—.617	.742	
.475	—.643	.743	
.479	—.654	.744	
.482	—.661	.746	
.484	—.703	.747	
.487	—.696	.748	
.488	—.712	.749	
.489	—.756	.749	
.491	—.735	.750	
.493	—.762	.751	
.495	—.760	.752	.31
.496	—.767	.752	.32
.498	—.773	.753	
.499	—.776	.754	
.501	—.803	.755	
.502	—.809	.755	
.504	—.814	.756	
.506	—.807	.757	
.507	—.824	.758	
.509	—.852	.758	
.510	—.867	.759	
.512	—.871	.760	
.513	—.868	.760	.32
2425.589	.056	.263	.52
.592	.060	.265	.52
.594	.096	.266	.53
.598	.082	.268	
.601	.114	.269	
.603	.104	.270	.53
2426.386	—.325	.636	.68
.389	—.309	.637	
.392	—.334	.639	
.395	—.359	.640	
.397	—.370	.641	
.400	—.355	.642	
.402	—.386	.643	
.405	—.400	.645	
.408	—.408	.646	.68
.431	—.507	.657	.69
.434	—.503	.658	
.436	—.489	.659	
.440	—.525	.661	
.443	—.539	.662	
.445	—.539	.663	
.468	—.614	.674	
.470	—.590	.675	
.473	—.626	.677	
.478	—.623	.679	
.480	—.638	.680	.69
.484	—.658	.682	.70
.508	—.672	.693	
.510	—.691	.694	
.513	—.670	.695	
.517	—.684	.697	
.519	—.690	.698	
.521	—.693	.699	
.524	—.672	.700	
.526	—.696	.701	
.528	—.689	.702	.70
.554	—.682	.714	.71
.558	—.727	.716	
.560	—.744	.717	
.563	—.742	.719	
.565	—.659	.720	
.568	—.643	.721	
.573	—.708	.723	
.576	—.689	.725	
.578	—.711	.726	.71
2430.349	.205	.488	.43
.351	.224	.489	.43
.354	.190	.491	.44
.357	.203	.492	
.359	.219	.493	
.361	.208	.494	
.403	.196	.514	.44
.442	.254	.532	.45
.444	.228	.533	
.446	.252	.534	

J. D. hel. —2430000	Δm	phase φ	phase ψ
d	m		
2430.449	.246	.535	
.451	.218	.536	.45
.491	.221	.555	.46
.494	.204	.556	
.496	.222	.557	
.499	.202	.558	
.501	.229	.559	
.503	.225	.560	.46
.542	.207	.579	.47
.545	.196	.580	
.547	.204	.581	
.550	.230	.582	
.552	.232	.583	
.554	.205	.584	.47
2434.472	.062	.416	.22
.475	.076	.417	
.478	.072	.418	
.481	.092	.420	
.483	.053	.421	
.485	.046	.422	.22
.523	.102	.439	.23
2439.404	—.232	.721	.17
.406	—.151	.722	
.407	—.189	.722	
.409	—.195	.723	
.410	—.188	.724	
.412	—.192	.725	.17
.472	—.319	.753	.18
.474	—.379	.754	
.475	—.414	.754	
.477	—.386	.755	
.478	—.388	.756	
.480	—.378	.757	.18
.548	—.602	.788	.19
.550	—.634	.789	
.551	—.647	.790	
.552	—.659	.790	
.554	—.688	.791	
.555	—.690	.792	.19
2441.473	—.713	.688	.56
.475	—.734	.689	
.476	—.739	.690	
.478	—.746	.691	
.479	—.758	.691	
.480	—.768	.691	
.482	—.794	.692	
.484	—.796	.693	
.485	—.793	.694	
.486	—.797	.694	
.488	—.808	.695	
.489	—.795	.696	
.492	—.812	.697	
.494	—.832	.698	.56
.495	—.836	.698	.57
.499	—.863	.700	
.501	—.879	.701	
.502	—.858	.702	
.504	—.868	.703	
.506	—.868	.704	
.507	—.860	.704	
.509	—.880	.705	
.510	—.887	.705	
.512	—.892	.706	
.516	—.911	.708	
.518	—.924	.709	
.520	—.923	.710	
.522	—.902	.711	
.524	—.911	.712	
.526	—.917	.713	
.531	—.938	.715	
.533	—.948	.716	
.535	—.948	.717	
.542	—.926	.720	
.544	—.924	.721	
.546	—.934	.722	.57
.550	—.942	.724	.58
.551	—.942	.725	
.553	—.966	.726	
.554	—.946	.726	
.556	—.956	.727	
.557	—.953	.727	
.587	—.933	.741	
.589	—.918	.742	
.590	—.911	.743	
.592	—.900	.744	
.593	—.905	.744	
.595	—.914	.745	
.598	—.884	.747	
.599	—.929	.747	.58
.600	—.922	.748	.59

J. D. hel. —2430000	Δm	phase φ	phase ψ
d	m		
2444.492	—.330	.099	.14
.493	—.340	.100	
.496	—.318	.101	
.500	—.314	.103	
.503	—.317	.105	
.505	—.306	.105	
.512	—.298	.109	
.516	—.316	.111	
.518	—.279	.112	.14
.549	—.296	.126	.15
.551	—.288	.127	
.552	—.267	.127	
.556	—.280	.129	
.558	—.291	.130	
.561	—.287	.132	.15
2454.454	—.318	.756	.04
.455	—.325	.757	
.456	—.336	.757	.04
.474	—.345	.765	.05
.475	—.354	.766	
.477	—.366	.767	
.490	—.389	.773	
.491	—.382	.773	
.492	—.364	.774	
.508	—.394	.781	
.510	—.402	.782	
.511	—.404	.783	.05
.525	—.448	.789	.06
.527	—.432	.790	
.528	—.438	.791	
.545	—.474	.799	
.547	—.451	.800	
.548	—.466	.800	
.564	—.488	.807	
.565	—.515	.808	
.567	—.490	.809	.06
.587	—.504	.818	.07
.589	—.494	.819	
.590	—.483	.820	
.592	—.523	.821	
.593	—.521	.821	
.595	—.518	.822	
.609	—.496	.829	
.611	—.518	.829	
.612	—.526	.830	
.615	—.550	.831	
.617	—.542	.832	
.618	—.545	.833	.07
.629	—.527	.838	.08
.631	—.524	.839	
.633	—.568	.840	
.634	—.492	.840	
.635	—.495	.841	
.637	—.541	.842	.08
2471.300	.076	.631	.26
.304	.096	.632	
.306	.078	.633	.26
.311	.094	.636	.27
.314	.105	.637	
.317	.091	.639	
.322	.053	.641	
.324	.059	.642	
.327	.033	.643	
.331	.088	.645	
.333	.076	.646	
.335	.074	.647	
.360	.015	.659	
.362	.002	.660	.27
.366	.002	.661	.28
.376	—.005	.666	
.380	.018	.668	
.382	—.008	.669	
.394	—.021	.675	
.397	—.018	.676	.28
.425	—.133	.689	.29
.428	—.126	.690	
.431	—.176	.692	
.433	—.128	.693	
.436	—.151	.694	
.438	—.125	.695	
.441	—.106	.697	
.449	—.174	.700	
.451	—.209	.701	
.453	—.115	.702	.29
2606.282	—.226	.727	.07
.284	—.240	.728	
.286	—.292	.729	
.288	—.232	.730	
.291	—.177	.731	
.294	—.218	.733	
.297	—.258	.734	

TABLE I (*continued*)

J. D. hel. —2430000	Δm	phase φ	phase ψ	J. D. hel. —2430000	Δm	phase φ	phase ψ	J. D. hel. —2430000	Δm	phase φ	phase ψ
d	m			d	m			d	m		
2606.299	—.259	.735		2608.290	—.196	.666		2608.349	—.584	.693	
.302	—.313	.736		.294	—.219	.668		.351	—.576	.694	.46
.312	—.265	.741	.07	.297	—.250	.669		.356	—.608	.697	.47
.314	—.247	.742	.08	.300	—.237	.670	.45	.360	—.656	.698	
.316	—.264	.743		.303	—.257	.672	.46	.363	—.690	.700	.47
.323	—.304	.746		.322	—.402	.681					
.325	—.312	.747		.336	—.463	.687					
.327	—.299	.748	.08	.339	—.456	.689		2615.331	—.262	.957	.80
				.342	—.521	.690		.335	—.266	.959	
2608.273	—.096	.658	.45	.344	—.529	.691		.338	—.260	.960	
.286	—.152	.664		.347	—.536	.692		.341	—.248	.962	.80

preceding article, I decided to apply the same method to my own observations of TU Cas. With the aid of Figure 2 of the article on U TrA it proved very easy to assign provisional phases in fractions of a beat period to the line segments in Figure 1 and without any difficulty these phases could be accurately represented by a beat period of $5^{d}.2306$ or 2.445 times the fundamental period $P_{\circ}$ of $2^{d}.139$. For the reciprocal value of this beat period P_b I adopted $d^{-1}.19118$.

FIGURE 1

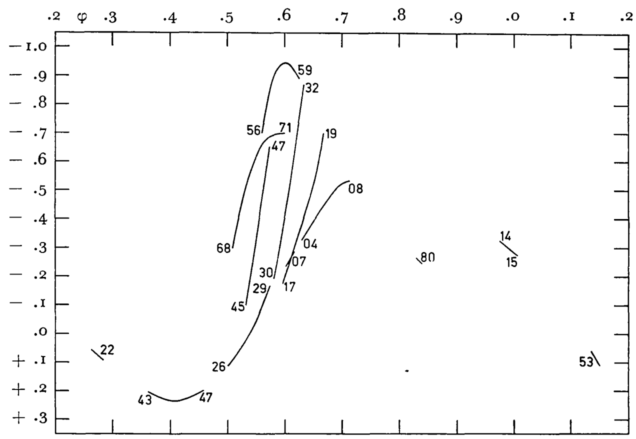


FIGURE 2

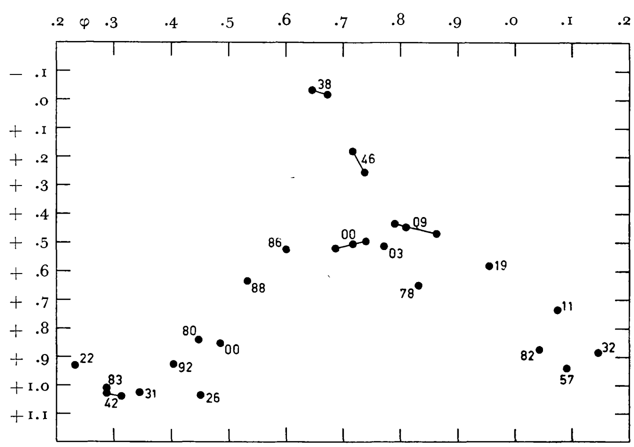
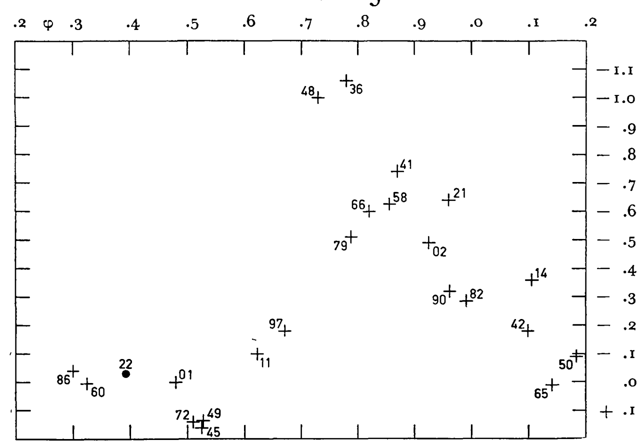


FIGURE 3



The phases ψ, expressed in fractions of this beat period, and computed with the formula:

$$\text{phase } \psi = .19118 \text{ (J.D.} - 2420000)$$

have been given in the fourth column of Table 1 and they are also shown in units of one hundredth on both ends of each part of the light-curve in Figure 1. It is

TABLE 2

Observations by KRON

J. D. hel. —2430000	Δm	phase φ	phase ψ
d	m		
2039.862	.652	.958	.78
46.802	.735	.202	.11
55.813	1.011	.414	.83
56.849	.513	.898	.03
57.836	.928	.359	.22
68.769	1.024	.470	.31
81.824	.842	.573	.80
89.733	.885	.270	.32
2100.736	1.032	.413	.42
.793	1.041	.439	.43
03.732	.521	.813	.99
.797	.505	.844	.00
.845	.494	.866	.01
05.782	—.035	.772	.38
.838	—.019	.798	.39
06.734	.940	.217	.57
13.820	.927	.529	.92
14.650	.434	.917	.08
.694	.447	.937	.09
.806	.468	.990	.11
18.732	.524	.825	.86
23.746	.875	.169	.82
24.694	.853	.612	.00
25.696	.582	.080	.19
39.771	.634	.659	.88
41.734	1.037	.577	.26
63.698	.182	.844	.46
.741	.253	.864	.46

TABLE 3

J. D. hel. —2430000	Δ*m*	phase φ	phase ψ
d	m		
3272.369	—.17	.085	.41
.378	—.22	.089	
.385	—.20	.093	.41
.394	—.13	.097	.42
.399	—.13	.099	
.408	—.14	.104	
.416	—.22	.107	
.423	—.21	.111	.42
3301.416	—.13	.663	.96
.423	—.18	.666	.97
.432	—.16	.671	
.443	—.22	.676	
.449	—.24	.679	.97
3302.318	—.34	.085	.14
.329	—.38	.090	
.336	—.40	.093	
.343	—.40	.096	.14
.379	—.32	.113	.15
.387	—.34	.117	
.393	—.36	.120	.15
3305.342	.10	.498	.72
.352	.14	.503	
.360	.20	.507	
.366	.09	.510	
.373	.15	.513	
.383	.09	.517	
.390	.10	.521	.72
3306.299	—.36	.946	.90
.306	—.34	.949	
.316	—.37	.954	
.322	—.31	.956	
.325	—.33	.958	.90
.336	—.33	.963	.91
.344	—.30	.967	
.353	—.28	.971	
.362	—.30	.975	
.371	—.34	.979	.91
3310.293	—.61	.813	.66
.299	—.61	.815	
.307	—.59	.819	.66
.316	—.60	.823	.67
.322	—.59	.826	.67
3311.283	.00	.275	.85
.290	—.03	.279	
.297	—.03	.282	.85
.304	—.04	.285	.86
.312	—.02	.289	
.318	—.01	.292	
.325	—.04	.295	
.336	—.06	.300	
.343	—.03	.303	
.349	—.03	.306	
.354	—.05	.309	.86
.360	—.08	.311	.87
.368	.02	.315	
.374	—.01	.318	
.381	—.01	.321	.87
3330.323	—.09	.176	.49
.333	—.04	.180	.49
.344	—.14	.185	.50
.352	—.06	.189	
.360	—.14	.193	
.370	.01	.197	.50
3335.310	.15	.507	.44
.319	.16	.511	.45
.326	.15	.514	
.333	.22	.517	
.344	.18	.523	
.352	.15	.526	
.364	.16	.532	.45
.372	.12	.536	.46
.381	.16	.540	
.392	.13	.545	.46
3339.348	—.03	.394	.22
3340.356	—.78	.865	.41
.372	—.73	.873	
.379	—.72	.876	.41
3341.321	—.01	.316	.59
.328	.02	.320	.60
.335	—.01	.323	
.341	.03	.326	
.350	.02	.330	.60

J. D. hel. —2430000	Δ*m*	phase φ	phase ψ
d	m		
3342.307	—.52	.777	.78
.319	—.50	.783	.78
.327	—.48	.787	.79
.336	—.54	.791	
.348	—.53	.797	.79
3479.497	—.48	.906	.01
.522	—.46	.918	.02
.527	—.46	.920	
.535	—.48	.924	
.542	—.50	.927	
.549	—.51	.930	
.558	—.51	.934	
.566	—.48	.938	
.572	—.51	.941	.02
3560.394	— .94	.721	.48
.398	— .96	.723	
.402	— .96	.724	
.405	—1.04	.726	
.411	—1.00	.729	
.414	— .96	.730	
.418	—1.02	.732	
.422	—1.08	.734	
.426	— .99	.736	
.429	—1.03	.737	
.432	—1.04	.739	
.436	—1.00	.740	.48
3567.328	— .34	.962	.80
.332	— .30	.964	
.334	— .29	.965	
.338	— .28	.967	
.341	— .31	.968	
.344	— .31	.969	.80
.348	— .26	.971	.81
.351	— .27	.973	
.354	— .26	.974	
.357	— .31	.976	
.360	— .24	.977	
.363	— .33	.978	
.367	— .30	.980	
.370	— .28	.982	
.373	— .27	.983	
.377	— .27	.985	
.382	— .24	.987	
.386	— .23	.989	
.394	— .30	.992	.81
.414	— .25	.002	.82
.417	— .22	.004	
.421	— .15	.005	
.428	— .27	.009	
.430	— .21	.010	
.433	— .18	.011	
.437	— .20	.013	
.440	— .24	.014	
.444	— .18	.016	
.448	— .48?	.018	.82
.451	— .13	.019	.83
.454	— .20	.021	
.457	— .32	.022	.83
3568.365	— .21?	.447	.00
.371	— .03	.450	
.377	.03	.452	
.380	.05	.454	
.385	.04	.456	
.387	.02	.457	
.390	— .01	.458	.00
.394	.03	.460	.01
.398	.01	.462	
.403	— .05	.465	
.406	.06	.466	
.409	— .03	.467	
.412	.12	.469	
.415	.04	.470	
.421	.03	.473	
.424	.04	.474	
.428	.03	.476	
.431	.03	.478	
.434	.02	.479	
.439	.06	.481	
.442	.10	.483	.01
.444	.01	.484	.02
.448	.02	.486	
.451	.04	.487	
.475	.01	.498	
.478	.10	.500	
.481	— .02	.501	
.484	— .11	.502	
.488	.04	.504	
.492	— .07	.506	
.495	— .02	.508	.02
.498	— .12	.509	.03

J. D. hel. —2430000	Δ*m*	phase φ	phase ψ
d	m		
3368.501	— .03	.510	
.504	.02	.512	
.508	.12	.514	
.511	.00	.515	
.514	.06	.516	
.516	.02	.517	
.519	— .08	.519	.03
3569.419	— .68	.939	.20
.423	— .69	.941	
.426	— .68	.943	
.429	— .69	.944	
.432	— .66	.946	
.436	— .64	.947	.20
.438	— .64	.948	.21
.443	— .64	.951	
.445	— .66	.952	
.448	— .68	.953	
.467	— .66	.962	
.470	— .61	.963	
.474	— .62	.965	
.478	— .75	.967	
.480	— .60	.968	
.485	— .58	.970	
.487	— .56	.971	.21
.491	— .64	.973	.22
.494	— .67	.974	
.497	— .62	.976	
.501	— .59	.978	
.504	— .66	.979	
.507	— .48	.981	.22
3571.348	— .75	.841	.57
.351	— .66	.843	
.357	— .58	.845	
.360	— .64	.847	
.363	— .68	.848	
.370	— .66	.851	.57
.374	— .64	.853	.58
.378	— .70	.855	
.380	— .53	.856	
.384	— .54	.858	
.389	— .60	.860	
.392	— .63	.862	
.396	— .59	.864	
.399	— .48	.865	
.403	— .59	.867	
.409	— .59	.870	
.412	— .38?	.871	
.415	— .54?	.872	
.421	— .73?	.875	.58
.425	— .52?	.877	.59
.430	.10?	.879	
.464	— .36?	.895	
.466	— .48?	.896	
.472	— .46?	.899	
.476	— .23?	.901	.59
3575.461	— .83	.764	.36
.465	—1.06	.766	
.469	— .94	.767	
.473	— .83	.769	
.476	— .92	.771	
.480	—1.06	.773	
.484	—1.24	.774	
.488	—1.06	.776	
.492	—1.10	.778	
.494	—1.13	.779	
.498	—1.06	.781	
.502	—1.05	.783	.36
.505	— .98	.784	.37
.508	—1.19	.786	
.511	—1.13	.787	
.516	—1.28	.789	
.519	—1.03	.791	
.521	—1.08	.792	
.524	—1.12	.793	
.527	—1.00	.795	
.532	— .95	.797	.37
3579.333	.03	.574	.10
.338	.02	.576	
.342	— .08	.578	
.347	.04	.580	
.350	— .02	.582	
.356	.00	.584	
.359	— .04	.586	
.362	— .03	.587	
.364	.00	.588	
.367	.08	.590	
.371	— .17	.591	
.375	— .24	.593	.10
.378	.02	.595	.11
.381	— .03	.596	

TABLE 3 (*continued*)

J. D. hel. —2430000	Δm	phase φ	phase ψ	J. D. hel. —2430000	Δm	phase φ	phase ψ	J. D. hel. —2430000	Δm	phase φ	phase ψ
d	m			d	m			d	m		
3579.384	— .01	.598		3579.544	— .14	.672		3581.392	.14	.536	
.391	— .13	.601		.548	— .42?	.674		.394	.16	.537	
.394	— .05	.602		.551	— .12	.676		.396	.19	.538	
.396	— .17	.603		.554	— .26	.677		.400	.18	.540	
.401	— .03	.605		.557	— .47?	.678		.402	.22	.541	
.403	.14	.606		.560	— .11	.680		.404	.20	.542	
.407	.07	.608		.564	— .25	.682		.407	.15	.543	
.410	— .08	.610		.568	— .16	.684		.410	.18	.545	.49
.414	.00	.612		.571	— .19	.685		.418	.15	.548	.50
.416	.10	.612		.573	— .24	.686		.421	.22	.550	
.419	— .04	.614	.11	.576	— .31	.687	.14	.423	.14	.551	
.460	— .01	.633	.12					.426	.16	.552	
.463	— .41?	.634						.428	.20	.553	.50
.466	— .06	.636		3581.328	.20	.506	.48				
.471	— .03	.638		.330	.13	.507					
.473	— .10	.639		.333	.14	.509		3608.327	— .05	.127	.64
.478	— .01	.641	.12	.337	.15	.510		.330	— .07	.128	
.482	— .05	.643	.13	.339	.06	.511		.333	.14	.130	
.487	— .27	.646		.344	.18	.514		.336	— .02	.131	
.489	— .31	.647		.346	.13	.515		.339	.02	.132	
.493	— .17	.648		.348	.16	.516		.343	— .07	.134	
.498	.12	.651		.351	.13	.517		.347	.02	.136	
.501	— .08	.652		.355	.10	.519		.350	— .08	.137	
.503	.09	.653		.358	.13	.520		.353	.02	.139	.64
.506	— .21	.655		.361	.06	.522	.48	.355	.08	.140	.65
.509	— .31	.656		.364	.11	.523	.49	.360	.05	.142	
.514	— .04	.658		.366	.03	.524		.362	.03	.143	
.516	— .07	.659		.368	.22	.525		.365	.20	.144	
.519	— .05	.661		.373	.14	.527		.368	.04	.146	
.522	.06	.662		.376	.16	.529		.371	.15	.147	
.526	— .01	.664		.378	— .06	.530		.376	— .02	.150	
.532	.05	.667	.13	.380	.09	.531		.378	.07	.151	
.535	— .14	.668	.14	.383	.20	.532		.381	— .09	.152	
.537	— .22	.669		.387	.07	.534		.383	— .10	.153	
.541	— .05	.671		.389	.16	.535		.386	.08	.154	.65

seen from this figure that the maxima for values of ψ near .3, .4 and .5 are high and that those for ψ-values near .8 and .0 are low. But the number of observations is too small to consider the proposed value of the beat period as definitely determined.

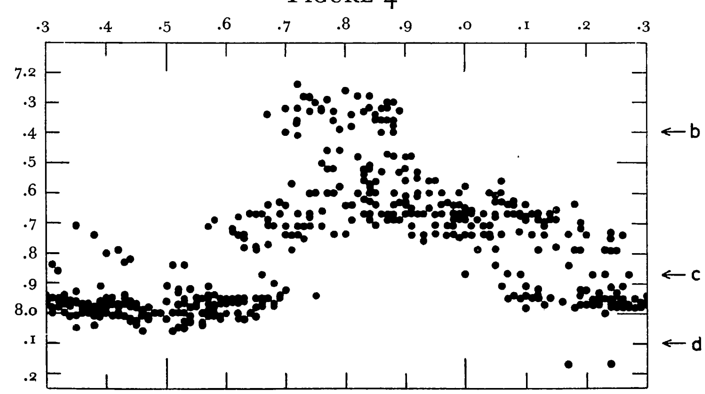
In Table 2 phases φ and ψ, computed with the same formulae, are given for the observations published by KRON[1]). These photo-electric observations are shown in Figure 2, in which the ψ-values have been indicated in the same way as in Figure 1. A comparison of Figures 1 and 2 shows that they are in complete agreement, although KRON's observations were made one year before my own.

By the courtesy of Mr BAKOŠ I had also at my disposal a series of photo-electric observations made by him in 1950 at the Leiden Observatory with the 10″ visual refractor combined with photomultiplier and pulse counting device. The individual observations have been listed in Table 3 together with the phases φ and ψ. The accuracy of these observations is low, mostly due probably to poor observing conditions. In Figure 3 all observations of a single night have been represented by one cross, next to which the value of the ψ-phase has been indicated. This figure also confirms the results of Figures 1 and 2, although there seems to be an indication that Figure 3 needs a small shift along the abscissa towards the left in order to coincide well with Figures 1 and 2. This could be achieved by a slight change in the fundamental period P_o. On the other hand these three figures indicate that the value of the beat period derived above is sufficiently accurate to cover the years from 1946 to 1950.

The photo-electric observations discussed so far are few in number and as there are some old extensive series of visual observations, we shall analyse two of these series here.

RYBKA made visual estimates in the years 1926, 1927 and 1928. A plot of all his observations, given in the article quoted above, against phase φ, computed with the same formula, is shown in Figure 4. The large dispersion of the observations at all phases is evident. Further it is clear that his estimates show a pronounced systematic tendency not to be equal to the brightness of the comparison stars, which have

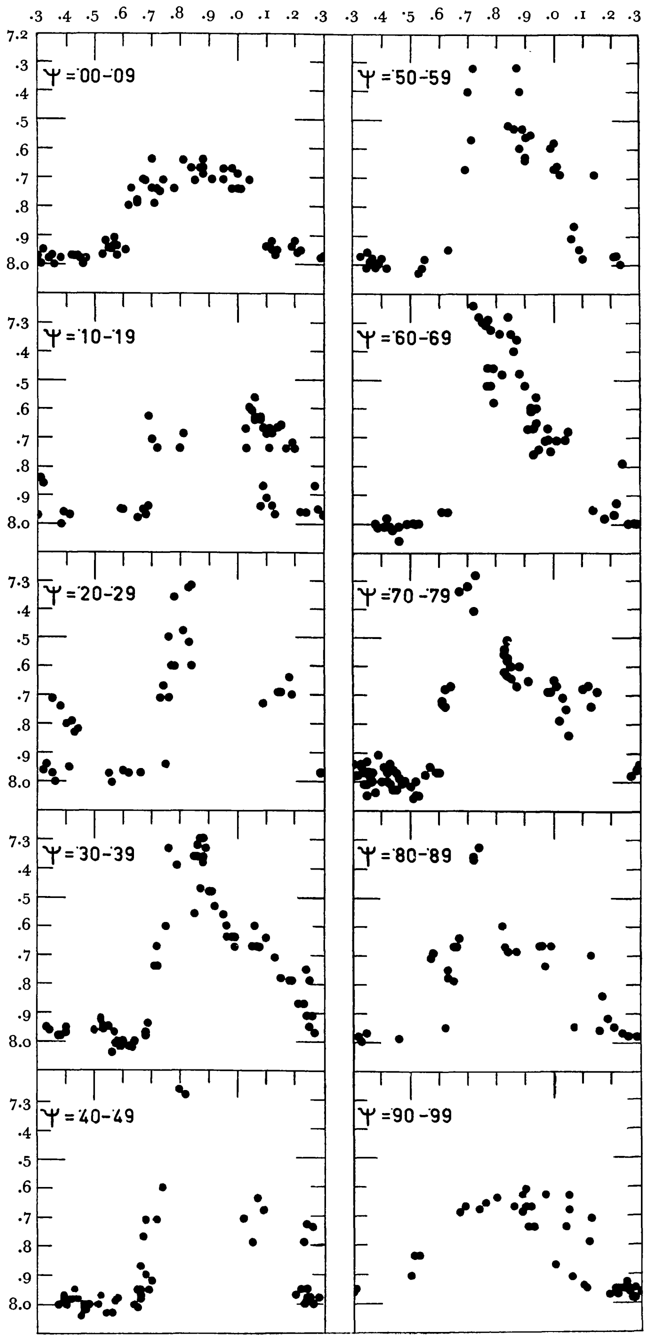
FIGURE 4



[1]) *Ap. J.* **106**, 321, 1948.

been indicated on the right-hand side of the figure. This means that all parts of the light-curve must be considerably distorted. Then for all observations phases ψ were computed with the same formula as used before, and the observations were divided in ten groups according to ten equal intervals of the phase ψ. The resulting light-curves are shown in Figure 5. It should be emphasized that these curves do not correspond with the light-curves actually observed, as during the time of one fundamental period of 2.139 days, the phase ψ changes as much as .41. In

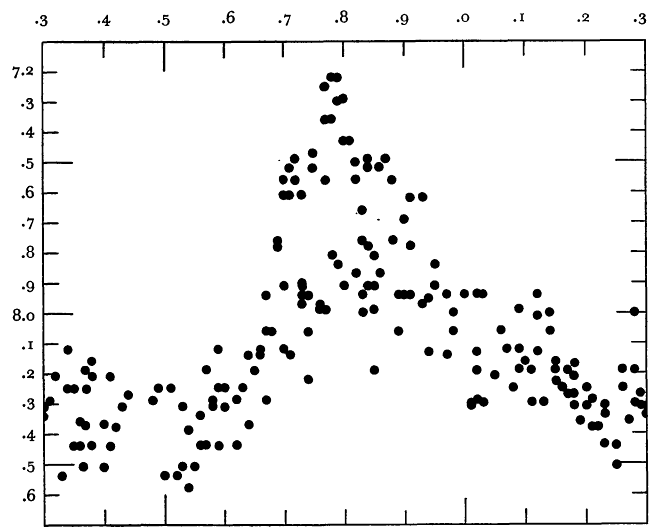
FIGURE 5



constructing a light-curve, as one could actually observe, it would be necessary to use four successive curves of Figure 5. For this reason I shall refer to these curves as ψ-curves. Although the ψ-curves still show a considerable dispersion, which is partly due to the distortion mentioned above and partly to the fact that the observations of one curve correspond with a rather large interval in ψ, it is clear from Figure 5 that the ψ-curves for ψ from .90 to .09 are very flat. For ψ from .10 to .19 the curve becomes somewhat irregular in changing into a curve with a steep ascending branch and a sharp maximum. The curve keeps this shape for ψ between .20 and .79. In this interval the ascending branch moves gradually towards the left to smaller values of phase ψ. For ψ between .80 and .89 the curve changes very rapidly into the flat shape.

The next series of observations we shall discuss here are the visual observations by CASTEELS, made in the years 1911 and 1912[1]). These observations were treated in exactly the same way as those made by RYBKA. The individual observations have been plotted in Figure 6, the resulting ψ-curves in Figure 7.

FIGURE 6



The general picture of these ψ-curves is very much the same as that shown in Figure 5. The flat curves now occur for values of ψ between .00 and .29. A very small correction to the beat period P_b would make these flat curves occur at the same ψ-values as found for RYBKA's observations.

Finally we have analysed the radial-velocity measures by SANFORD[2]). These observations were made in the years from 1917 to 1947. The phases φ

[1]) *Ann. Obs. Royal Belgique, Nouvelle Série*, **13**, 184, 1914.
[2]) *Mt Wilson Contr.* No. 367; *Ap. J.* **68**, 169, 1928 and *Ap. J.* **114**, 331, 1951.

FIGURE 7

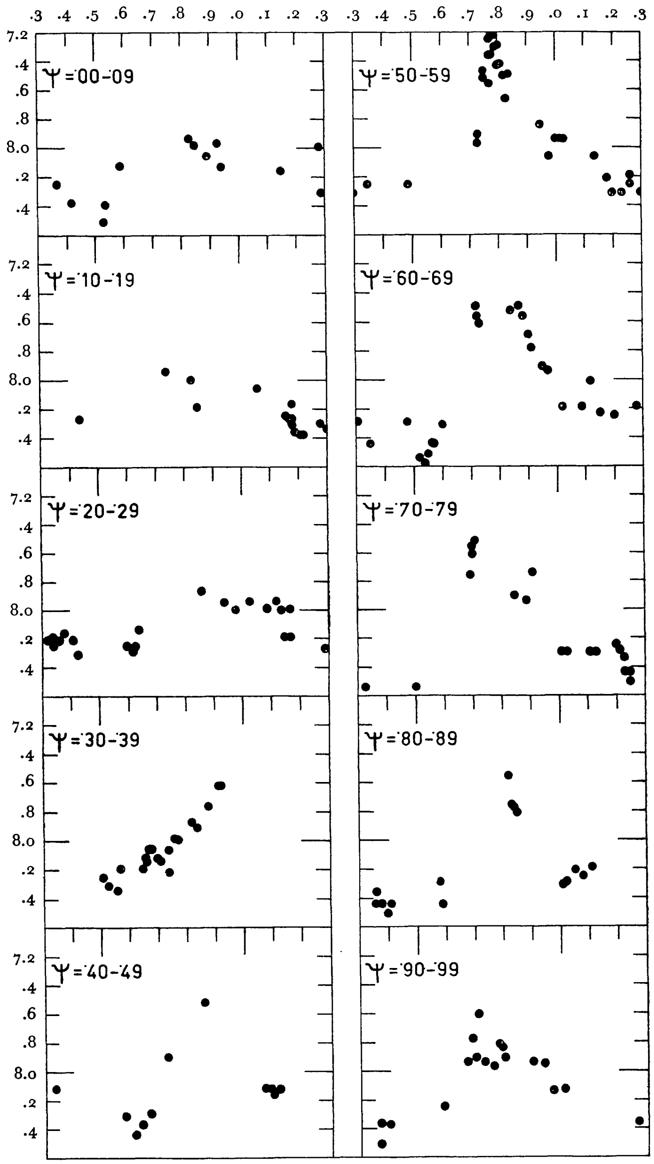


FIGURE 8

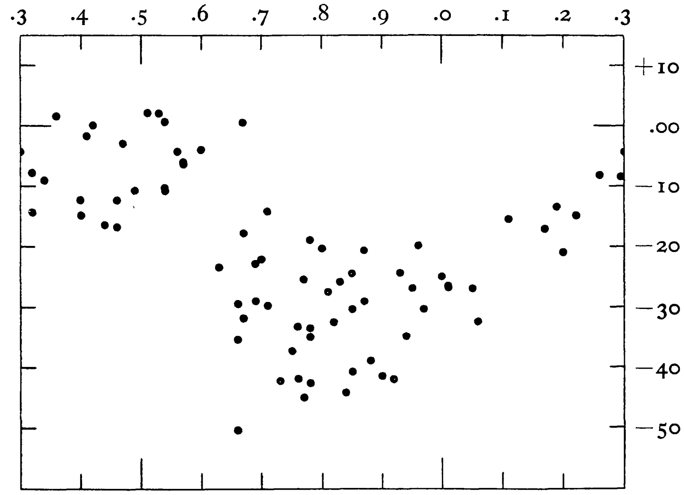
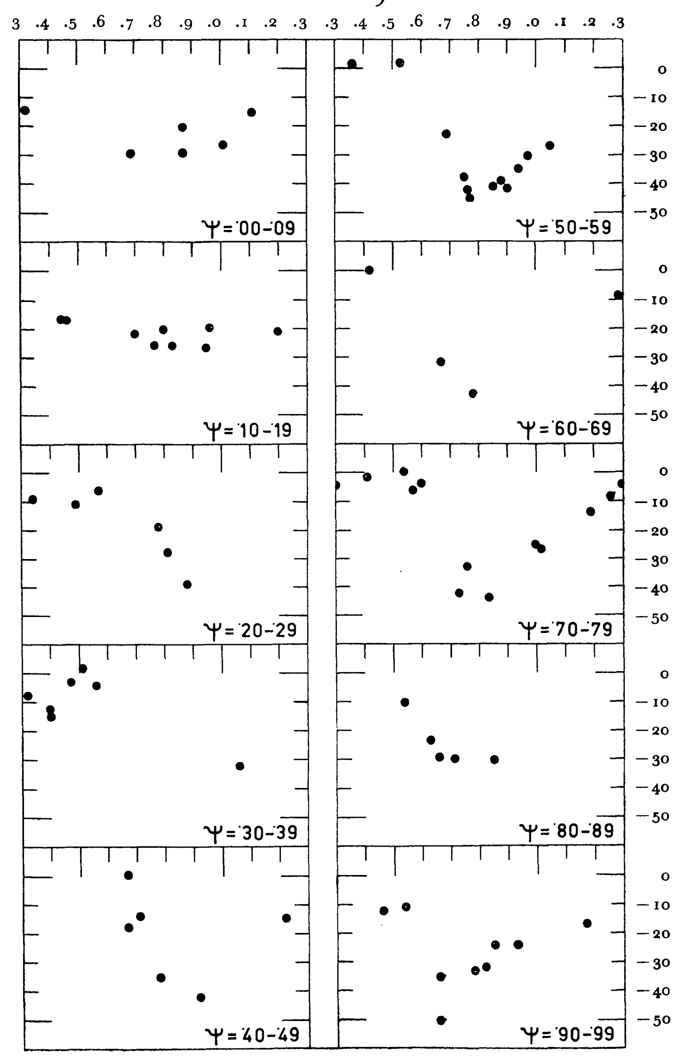


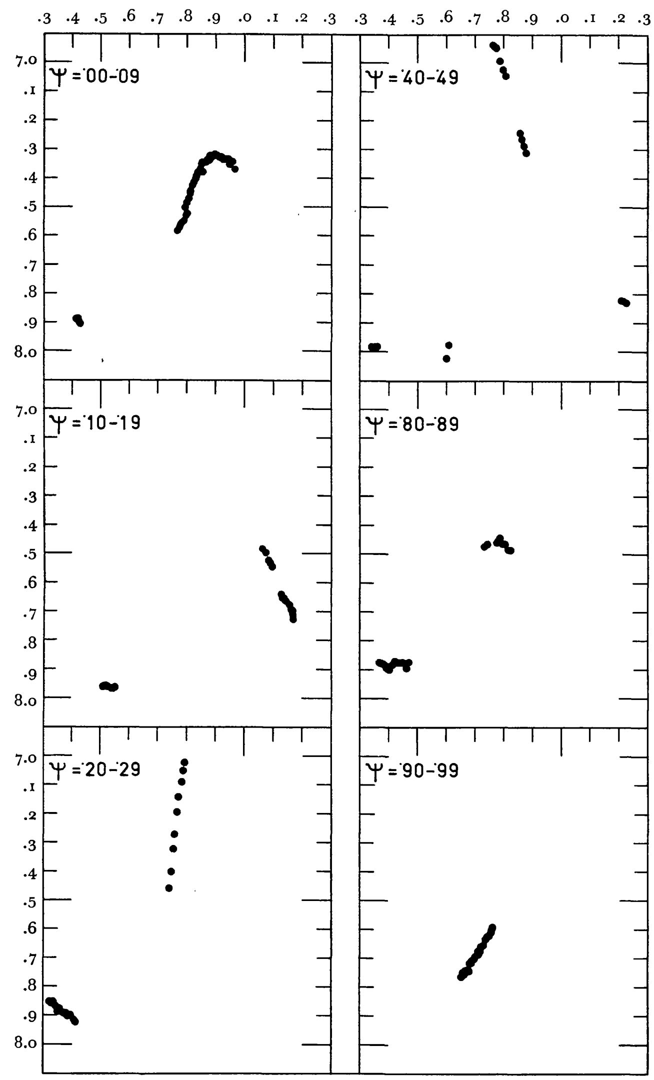
FIGURE 9



and ψ were computed again with the same formulae as before. The individual radial velocities have been plotted against phase φ in Figure 8 and the resulting ψ-curves are shown in Figure 9. It is interesting to notice that the difference between the flat and the steep ψ-curves is as pronounced in the radial velocities, as in the photometric measures. The flat curves occur here for ψ values between .00 and .19. As these observations cover a time interval of about 30 years it is to be expected that the representation of the observations can probably be slightly improved by a small adjustment of the beat period.

When the manuscript of this article was nearly ready for print, I received from Dr EGGEN a typescript of an article for the *Astronomical Journal* on TU Cas by CHARLES E. WORLEY and OLIN J. EGGEN with a number of new photo-electric observations made

FIGURE 10



mostly in the years 1954 and 1955 at the Lick Observatory. A plot of the individual observations of this series can be found in the article in the *Astronomical Journal* [1]). The resulting ψ-curves are shown here in Figure 10. For four intervals in ψ of .10 the number of observations is so small that the ψ-curves have been omitted. The remaining six curves show clearly that the observations by WORLEY and EGGEN obey strictly to the rules derived from my own observations. If the fundamental period of $2^d.139$ is considered as primary period P_0, the beat period P_b of $5^d.2306$ originates from the combined effect of the primary period P_0 and a secondary period P_1 of $1^d.5183$. None of the series of observations discussed here is accurate enough or extensive enough to derive the amplitudes of the harmonic curves connected with the primary and secondary periods and the phase- and magnitude distortions. For such an investigation a more or less continuous series of photo-electric observations over a period of two or three months in a good observing climate would be needed.

In *B.A.N.* **13**, 65 (No 470), 1956, Woltjer concluded that in the case of SX Phe and AI Vel the light-curve after removal of the effect of the magnitude-distortion resembles closely the radial-velocity curve. For TU Cas we have found here that the ψ-curves for the light-variation are very similar to those for the radial-velocity curve, which may well mean that the magnitude-distortion is much less important for TU Cas than for the two short-period variables mentioned above. For these reasons further observations of TU Cas will be of great interest.

From the ψ-curves for the observations by CASTEELS, RYBKA and WORLEY and EGGEN we can derive the value of phase φ at which the highest maxima occur. In all three cases we find a value close to .75, which proves that the value of the primary period used in this article has been sufficiently accurate. The approximate values of phase ψ at which the flattest ψ-curves occur are found to be:

CASTEELS	.15
RYBKA	.00
WORLEY-EGGEN	.90

These values show that the beat period used in this article needs a small correction. If we assume that the counting of the number of beat periods elapsed between these three sets of observations has been correct, the best value of the beat period becomes $5^d.23026$.

1) *A. J.* **62**, 104, 1957.