measurements of the Zeeman effect, notably by S. B. NICHOLSON, H. D. BABCOCK and G. THIESSEN, have shown that the earlier observers had been too optimistic about the accuracy of their results. The present situation is confused: it seems probable that the magnetic field is present but it may vary in strength[1]). At any rate, the idea of an accurately observed magnetic axis has to be abandoned. So it would seem just that the early order of problems be restored: The coronal rays indicate a magnetic field, now more convincingly than they did before, because no other interpretation seems to explain the observations. The strength of the field, and eventually, further data about its form and origin have to be provided by continued measurements of the Zeeman effect.

Added in proof. Recent studies by HOYLE and his associates[2]) emphasize another theoretical aspect of the corona: a stream of interstellar gas is thought to fall from all directions into the sun. This is just the reverse of the suggestion made above, section 3 C, p. 158. By HOYLE's estimates a cloud containing 45 hydrogen atoms per cm^3 and moving with a velocity of 10 km/sec with respect to the sun will lose all matter that would otherwise pass within 18 astronomical units from the sun's centre. When this material has fallen down to 1·2 solar radii from the centre, its velocity has become 570 km/sec and its density $2{\cdot}5.10^6$ protons and electrons per cm^3; in total $10^{37\cdot0}$ protons and electrons would fall into the sun per second. It may be noted, however, that more reliable data on the observed densities than the data of BAUMBACH, that are used by HOYLE, admit a stream of 1/10 or 1/20 times this value at the most. This is most evident if we consider the density in polar regions. As far as I am aware, there are no direct observational data that might prove or disprove the existence of such a reduced inward stream. It would still exceed the estimated outward stream by a factor 10. To what extent it would influence the heating of the corona remains a problem for further investigation.

1) H. D. BABCOCK, *P.A.S.P.* **60**, 244, 1948.
2) H. BONDI, F. HOYLE, and R. A. LYTTLETON, *M.N.* **107**, 184, 1947.
F. HOYLE, 'Some recent researches in solar physics', Cambridge University Press, 1949.

THE FORM OF FRAUNHOFER LINES IN THE INNER CORONA

BY C. J. VAN HOUTEN

The form of a Fraunhofer line in the spectrum of the solar corona resulting from scattering of sunlight by electrons has been computed. The difference in Doppler broadening for light scattered under different angles has been rigorously taken into account. Numerical results for a point somewhat outside the sun's limb show a line with a sharper core than corresponds to a Gaussian profile. The strongest Fraunhofer lines have central depressions of 7 per cent only, after scattering.

The main component of the light of the corona is sunlight scattered by free electrons. It has a continuous spectrum in which the absorption lines are almost obliterated by the Doppler effect arising from the electron velocities. Only a few strong Fraunhofer lines are still visible as slight depressions in the spectrum. The purpose of this paper is to determine the form of these depressions. This is not the familiar Gaussian form, for the broadening is a function of the scattering angle.

Let us consider an electron scattering light from one surface element of the sun in the direction of the earth. The shift of the Fraunhofer lines will be the algebraic sum of the Doppler shifts caused by the radial velocities of the electron relative to the sun and to the earth. We resolve the velocity of the electron in three components along the mutually perpendicular axes, ξ, η, ζ. The axes ξ and η are taken along the bisectors of the angle subtended at the electron by the directions from the surface element and to the earth. This angle will be denoted by θ. If the velocity components along the three axes are denoted by v_1, v_2, and v_3, respectively, the velocity of regression from the sun is

$$v_1 \sin \tfrac{1}{2}\theta + v_2 \cos \tfrac{1}{2}\theta$$

and the velocity of regression from the earth is

$$v_1 \sin \tfrac{1}{2}\theta - v_2 \cos \tfrac{1}{2}\theta.$$

If λ is the wave length of the line and c the velocity of light, the Doppler shift arising from the sum of both effects is

$$\Delta\lambda = \frac{2\lambda}{c} v_1 \sin \tfrac{1}{2}\theta,$$

independent of v_2 and v_3. The distribution function of one velocity component in a thermal gas is

$$\frac{1}{\alpha\sqrt{\pi}} e^{-v_1^2/\alpha^2} d v_1,$$

where $\alpha^2 = 2kT/m$ and m is the mass of the electron, k the Boltzmann constant and T the temperature. The corresponding distribution function of $\Delta\lambda$ thus becomes

$$w(\Delta\lambda, \theta)\, d(\Delta\lambda) = \frac{p}{\sqrt{\pi}} e^{-p^2 (\Delta\lambda)^2}\, d(\Delta\lambda), \qquad (1)$$

where

$$p = \frac{c}{2\, \lambda\, \alpha \sin \frac{1}{2}\theta}.$$

Equation (1) was derived for one surface element of the sun and one volume element of the corona, the combination of which defines one angle θ. There are many different combinations for which θ is the same. All of them will give the same distribution function, equation (1), provided T is constant through the corona. We now consider one point of the projected image of the corona at the distance x from the centre. Let $V(x, \theta)\, d\theta$ denote the fraction of the light seen at this point that arises from scattering under an angle between θ and $\theta + d\theta$. The shifts $\Delta\lambda$ in this light are then distributed according to the function

$$F(x, \Delta\lambda) = \int_0^{\pi} w(\Delta\lambda, \theta)\, V(x, \theta)\, d\theta \qquad (2)$$

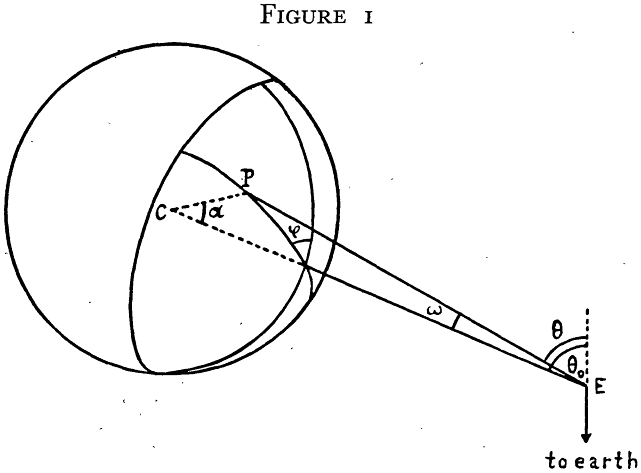
and the intensity distribution in a line with the equivalent width D is

$$I(x, \lambda) = I_0 \left\{ 1 - D . F(x, \lambda - \lambda_0) \right\}, \qquad (3)$$

or a somewhat different formula if the change of the intensity of the continuum with λ has to be taken into account; λ_0 corresponds to the centre of the line.

Computation of $V(x, \theta)$. The computation may be made in the following steps. (a) Determine the amount of light received by a volume element of the corona from the part of the solar surface that is seen under an angle between θ and $\theta + d\theta$ with the direction from the earth. (b) Find what fraction of this light is scattered in the direction of the earth. (c) Integrate along the line of sight.

FIGURE 1



Scattering of a light-ray from a point P of the sun's surface by an electron E of the corona in the direction of the earth. C is the sun's centre; CP = 1; CE = r; PE = l.

The line of sight passes at a distance x from the sun's centre (the radius of the sun is taken as the unit of length). Let y be the distance along this line from the point closest to the sun to the volume element we consider. Let the line of sight make the angle θ_0 at this point with the direction to the sun's centre and the angle θ with the direction to an arbitrary element of the sun's surface (see Figure 1). The corresponding distances will be denoted by r and l, respectively. We have

$$y = -x \cot \theta_0, \quad \text{so} \quad dy = \frac{x}{\sin^2 \theta_0}\, d\theta_0.$$

A point of the surface is specified by the polar coordinates, α and φ, where α is the angular distance from the axis of symmetry defined by the volume element and the sun's centre, and the angle φ is the angle between the planes sun's centre–earth–volume element of corona and surface element of sun–earth–volume element of corona, so defined that

$$\theta = \theta_0 - \omega \quad \text{for} \quad \varphi = 0 \;\; ; \;\; \theta = \theta_0 + \omega \quad \text{for} \quad \varphi = 180°.$$

In both cases the surface element lies in the plane defined by the line of sight and the sun's centre.

We now have the simple relations:

$$\operatorname{tg} \omega = \frac{\sin \alpha}{r - \cos \alpha} \;\; ; \quad l = \frac{\sin \alpha}{\sin \omega},$$

while it is seen from the relation

$$\sin(\alpha + \omega) = r \sin \omega$$

that the maximum values α_1 and ω_1 are defined by

$$\cos \alpha_1 = \sin \omega_1 = 1/r.$$

The angle of scattering follows from

$$\cos \theta = \cos \theta_0 \cos \omega + \sin \theta_0 \sin \omega \cos \varphi.$$

For a given volume element, θ is within the range between $\theta_0 + \omega_1$ and $\theta_0 - \omega_1$.

A surface element of the sun is represented by

$$dO = \sin \alpha\, d\alpha\, d\varphi.$$

It is transformed by means of $\sin \theta\, d\theta = \sin \theta_0 \sin \omega \sin \varphi\, d\varphi$ to

$$dO = \frac{l \sin \theta}{\sin \theta_0 \sin \varphi}\, d\alpha\, d\theta.$$

The solid angle under which it is seen from the volume element is

$$d\Omega = \frac{\cos(\alpha + \omega)\, dO}{l^2} = \frac{\sin \theta \cos(\alpha + \omega)}{l \sin \theta_0 \sin \varphi}\, d\alpha\, d\theta.$$

The sun's surface brightness in this solid angle may be denoted by

$$H_q = \frac{1 - q + q \cos(\alpha + \omega)}{1 - \frac{1}{3} q} H_0,$$

where H_o is the average brightness of the sun's disk and q is the coefficient of limb darkening.

The light received at the volume element consists of the contributions $H_q \, d\Omega$ from all surface elements for which $\alpha < \alpha_1$. The elements for which the scattering angle is in a particular interval $d\theta$ are singled out by integrating over α for a fixed θ. The result obtained from the preceding formulae is

$$H_q \, d\Omega = H_o \, S_q(\theta, \theta_o) \, d\theta,$$

where

$$S_q(\theta, \theta_o) = \frac{(1-q)\, S_o(\theta, \theta_o) + q\, S_1(\theta, \theta_o)}{1 - \frac{1}{3} q}$$

and

$$S_o(\theta, \theta_o) = 2 \frac{\sin\theta}{\sin\theta_o} \int_{\alpha_o}^{\alpha_1} \frac{\cos(\alpha + \omega)}{l \sin\varphi} \, d\alpha,$$

$$S_1(\theta, \theta_o) = 2 \frac{\sin\theta}{\sin\theta_o} \int_{\alpha_o}^{\alpha_1} \frac{\cos^2(\alpha + \omega)}{l \sin\varphi} \, d\alpha.$$

In these integrations ω, l and φ have to be considered as functions of α, θ_o and θ. The symmetry with respect to the zero meridian has been used. The limit of integration at this meridian, α_o, is found by means of the substitution $\omega = |\theta - \theta_o|$. The integrand is infinite at this limit, unless $\theta = \theta_o$.

The further calculation closely follows the pattern of the usual theory of scattering in the corona, the only difference being that the present formulae refer to the light scattered under angles between θ and $\theta + d\theta$. The volume element sends the amount

$$\frac{1}{4\pi} \cdot \tfrac{3}{4}(1 + \cos^2\theta)\, \sigma N H_o \, S_q(\theta, \theta_o)\, d\theta$$

per unit solid angle towards the earth. Here N is the electron density and σ the scattering cross-section of the electron. Integration over $dy = \frac{x \, d\theta_o}{\sin^2\theta_o}$ now gives for the contribution to the surface brightness at the point considered:

$$\frac{1}{4\pi} \cdot \tfrac{3}{4}(1 + \cos^2\theta)\, x \sigma H_o \, d\theta \int N(\theta_o)\, S_q(\theta, \theta_o) \frac{d\theta_o}{\sin^2\theta_o}.$$

The limits of integration are given by

$$\cot(\theta - \theta_o) = \frac{\cos\theta \pm x}{\sin\theta}.$$

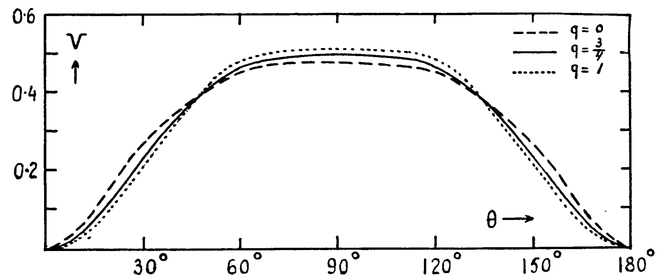
The plus sign gives the lower limit of θ_o, i.e. the point closest to the earth, from which rays from the sun may still be scattered under the angle θ. The minus sign defines the point farthest from the earth in a similar way. The surface brightness just found may be written as $PV(x, \theta)\, d\theta$, by definition. Writing R for the radius of the sun and $C = \frac{3}{4} \sigma R \, 10^{-8}$, we obtain

$$P \, . \, V(x, \theta) = \frac{10^8}{4\pi} x (1 + \cos^2\theta) \int C \, N(\theta_o)\, S_q(\theta, \theta_o) \frac{d\theta_o}{\sin^2\theta_o}.$$

The factor P is determined by the condition that $\int_0^\pi V(x, \theta)\, d\theta = 1$. This completes the scheme for computing the function $V(x, \theta)$.

Numerical results. The computations have been made with the values $x = 1{\cdot}03$, $q = 0{\cdot}75$ and the electron densities for the equatorial plane and minimum phase taken from the paper by VAN DE HULST. All integrations and interpolations were made from graphs. Table 1 shows the values of $S_{\frac{3}{4}}(\theta, \theta_o)$ that were the direct results of such integrations. Figure 2 gives the resulting distribution functions $V(x, \theta)$ for $q = 0$, $\frac{3}{4}$ and 1.

FIGURE 2



The distribution function $V(\theta)$ for a value $x = 1{\cdot}03$ and three values of q.

TABLE 1

Values of the function $S_{\frac{3}{4}}(\theta, \theta_o)$ for $x = 1{\cdot}03$.

θ_o \ θ	7°	15°	30°	45°	60°	90°	105°
90°	----	0·11	0·70	n.c.	2·01	2·65	n.c.
75°	0·11	n.c.	n.c.	n.c.	2·34	n.c.	n.c.
60°	0·18	0·54	1·49	n.c.	2·15	1·45	1·00
40°	0·25	n.c.	n.c.	n.c.	n.c.	----	----
30°	----	0·73	1·17	0·92	----	----	----

n.c. = not computed; ---- = outside limits of existence.

Various checks were made throughout the computations; comparisons have been made with results obtained by VAN DE HULST.

(a) The total illumination at the volume element is

$$\int S_q(\theta, \theta_o)\, d\theta = \pi(2A + B).$$

(b) The total radiation scattered by the volume element per unit solid angle to the earth is

$$\int (1 + \cos^2\theta)\, S_q(\theta, \theta_o)\, d\theta = \\ = 2\pi \left\{ A(1 + \cos^2\theta_o) + B \sin^2\theta_o \right\}.$$

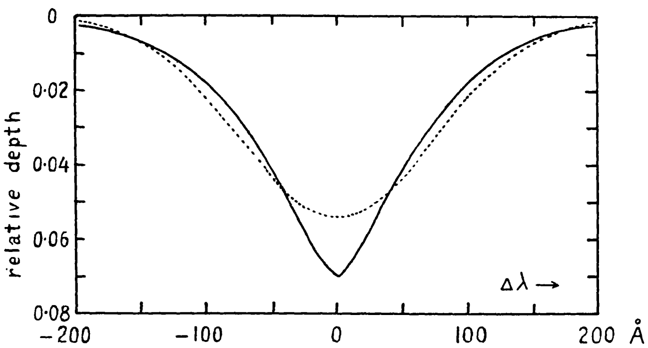
(c) The total surface brightness at the point considered, divided by the mean surface brightness of the sun, is

$$P = K \, . \, 10^{-8}.$$

The left-hand members of all equations were found by integrating our numerical results over θ; the right-hand members are the corresponding expressions from VAN DE HULST's paper. Deviations up to 5 per cent were found; they may be due to the rough graphical method.

Finally, the form of the spectral line was found from equations 2 and 3. It is shown in Figure 3 for a Fraunhofer line at $\lambda = 4000$ A that has an equivalent width of 10 A. An electron temperature of 10^6 °K was assumed, giving $\frac{1}{P} = 147 \sin \frac{1}{2} \theta$ in equation (1). The form that would be obtained if all scattering took place under the angle $\theta = 90°$ is shown for comparison.

FIGURE 3



Profile of the scattered Fraunhofer line, for $\lambda = 4000$ A and $x = 1{\cdot}03$. The dots refer to scattering under an angle of 90°.

Conclusions. The computation shows that the strongest Fraunhofer lines should leave depressions as deep as 7 per cent in the continuous spectrum of the corona. Measurements of these depressions, as made by GROTRIAN[1]), may be used for a temperature determination. Their contours do not have the familiar Gaussian form but have a sharper core. Values of the halfwidths for a standard temperature of 10^6 °K are as follows:

1) scattering contour at $x = 1{\cdot}03$ $h = 120$ A.
2) scattering under 90° angle only $h = 170$ A.
3) emission line contour $h = 120$ A.

The contours 1) and 2) were shown in Figure 3; contour 3 is obtained if we erroneously neglect the radial velocities of the electron relative to the sun, i.e. if we use the familiar theory for Doppler broadening of emission lines. Accidentally, this theory gives almost the correct halfwidth at $x = 1{\cdot}03$. It may be expected that the contours at points more distant from the sun's edge will more and more approach contour 2), with $h = 170$ A.

The usual determination of the brightness of the F-corona from the depth of the observed Fraunhofer lines is based on the assumption that the continuous corona has no traces of absorption lines at all. This is not strictly correct. One of the reasons for making the present computations was to see to what extent this determination is influenced by the widened Fraunhofer line of the continuous corona. Inspection of Figure 3 shows that the sharp core is still very shallow by ordinary standards; errors due to its neglection for moderately strong lines will probably stay below one per cent.

I wish to express my sincere thanks to Dr H. C. VAN DE HULST for his interest and help during the computations and the preparation of this paper.

1) *Z. f. Ap.* **3**, 199, 1931.

Addendum to page 137, *added in proof:* A point with ordinate 0·38, phase 0·3, may be added to Figure 1, according to photo-electric measurements of the eclipse of 1945[1]). The difference of brightness between maximum and minimum phase thus appears to be confirmed.

1) V. Nikonov and E. K. Nikonova, Bull. Crimean Astroph. Obs. **1**, 100, 1947.

CONTENTS