of the lines would have to be half the measured depth, if explanation (*c*) were correct. It is very unlikely that such an effect would result from instrumental inaccuracy. Other possibilities are that the observed lines are deeper because the spectrum contains sky light, or because the K-corona still shows lines with a core of appreciable depth. The first possibility is ruled out as a major cause on account of the estimates made above. Computations by Mr VAN HOUTEN[1]) to check the second possibility have also given negative results. We have to conclude that it is not very likely that explanation (*c*) is correct.

The polar regions. The polarization in the inner regions resembles that near the equator. The deviation from the p_K curves sets in sooner than at the equator, because the F-component carries relatively more weight. Near $r = 1{\cdot}2$ the observed ratio p_{pole}/p_{eq} is about 0·75 (COHN), in agreement with the computed value 0·76 (minimum phase) although the p values themselves do not agree too well. About $r = 1{\cdot}5$, where the most reliable determinations have been made, the predicted p is about half the observed one. An obvious explanation would be that the measurements refer to an eclipse between the maximum and minimum phase. However, this suggestion does not work here, because both the computed p and the measured p depend heavily on data for the same eclipse of 1914.

We have to look again at the possibilities (*a*), (*b*) and (*c*). Suggestion (*a*) seems less plausible here than for the outer regions of the equatorial corona, because the brightness level is higher by a factor five. Suggestion (*b*) is rejected for the same reason as before. But explanation (*c*) has more of a chance in the polar regions. It seems possible that the main features of Figure 4 can be maintained, even if K_{pole} near $r = 1{\cdot}5$ is increased by a factor 1·5 to 2. There are no spectrographic observations by means of which we might check the corresponding increase of f from about 0·26 to 0·45. There is also no strong objection against the corresponding decrease of F by a factor 0·7. For the interplanetary dust might be concentrated to the plane of the ecliptic to such an extent that $F_{pole} = 0{\cdot}7\ F_{eq}$. All in all, the discrepancy in the polar regions is not very disconcerting and the f-values are still highly uncertain. Moreover, the brightness for the polar regions was inferred from data for two eclipses only.

Conclusion: All data mentioned in the beginning of this section agree with the model, except the polarization of the corona between $r = 1{\cdot}5$ and 3. We tend to believe that these are measuring errors caused by trouble with the elimination of sky light. Another discrepancy occurs in the polar regions, but there the model data are much less certain.

Desiderata: The first requirement for future observations is photoelectric photometry and polarimetry, primarily in the regions where discrepancies occur. Simultaneous spectrograms for measuring the f-ratios are desirable. It is very important to make a direct and accurate measure of sky light. Measures of the relative brightness in different parts of the corona seem more important than measures in different colors. Only extension to the infra-red would be of great interest in the present situation.

[1]) C. J. VAN HOUTEN, *B. A. N.* **11**, 160, 1950.

ON THE POLAR RAYS OF THE CORONA

BY H. C. VAN DE HULST

The first part of this paper contains a photometric study of the polar rays on some copies of a photograph taken by BARNARD at the eclipse of 1900. Accurate isophotes are determined for the north polar region and the calibration is effected by means of the known decrease of brightness outward. The surface brightness of the rays exceeds the brightness of the background by twenty per cent at most. When the F-corona is eliminated and the results converted to electron densities, N_e, we find N_e to be 5 times larger in the rays than between them. The density decreases outward along the ray in a very similar fashion as between the rays.

The second part contains a theoretical discussion of the equilibrium state of the entire corona (section 3) and of the polar rays in particular (section 4). Observational data contradict the hypothesis that the corona would be a collection of fast streams ejected by the sun. The concept of an isothermal atmosphere in static equilibrium is more nearly correct. The temperature is determined from the density gradient and a detailed discussion shows that magnetic acceleration does not change the equilibrium equations. Beyond $r = 3$, this picture may have to be modified for the escape of electrons and protons from the sun by their thermal velocities. It is shown that a loss of 10^{11} gram per second for the sun by means of this process does not contradict the observations.

The polar rays cannot be explained as ejected streams because their densities decrease too rapidly outward. It is suggested that they are in thermal equilibrium like their surroundings and that they coincide with lines of force of the general magnetic field of the sun. Thermal diffusion carries an initial disturbance along the lines of force in a few hours, thus forming a ray. Diffusion perpendicular to the lines of force is shown to be negligible, even if the field is very low. The forms of the rays may give information about the character of the magnetic field.

The polar rays are short brush-like plumes seen at the polar regions of the corona on any photograph taken near minimum phase. Good detail is shown only on large-scale photographs. Former speculations about the nature of these rays were based exclusively on their curved forms. This paper is a first attempt to determine also their brightness and electron densities. This may be done by means of uncalibrated plates, provided the general decrease of intensity outward is known. All data discussed below refer to the north polar region of the corona at the 1900 eclipse. The results have a profound bearing on the theoretical interpretation of these rays. This subject is discussed in the later sections of this paper.

1. *Photometry.*

Isophotes. The isophotes may be determined as lines of equal density on any well-exposed photograph, original or copy. They are roughly concentric circles but have a jagged character where they pass through the polar rays. The main problem is to determine the depth,

$$\Delta r = r_{high} - r_{low},$$

of the teeth in these isophotes. Here 'high' refers to a point in the ray, 'low' to a point between rays and r is the distance of a point on the isophote to the centre of the sun.

Visual estimates are no good because the eye has a strong tendency to overestimate the contrast. This tendency is well-known in spectral photometry; it is also beautifully illustrated by the artificial streamers reproduced in *Ap. J.* **89**[1]). Unbiased persons to whom I showed corona photographs overestimated Δr by a factor 5 or more; those warned for this tendency still were too high by a factor 2 or 3, usually. Another method is to judge printed raster photographs. I looked up some of those in books and publications and found with a magnifier the places where the white and black areas were equal squares. This method should be objective but most prints are fairly uneven and, somehow, there is still a systematic tendency to overestimate Δr by a factor $1\frac{1}{2}$ to 2. A very good method of finding isophotes is to make copies with increased contrast, e.g. by the Kodalith method, used by BRIAN O'BRIEN, STEWART and ARONSON. It was easiest for me, however, to obtain reliable isophotes by means of a photometer.

First I used the Yerkes Observatory microphotometer in connection with reduced-size copies of the Lick photographs[2]). It was rather difficult to retrace any except the most prominent streamers; the weaker ones vanish among the grain and irregularities of the tracing. It is also unpleasant that the tracing has to be made along a line instead of a circle. Further, I feared that fine detail might have been lost in these reduced copies. For all these reasons I distrusted the results obtained by this method, although they were not bad after all.

An independent and more satisfactory determination was made as follows. At the Yerkes Observatory, I had made some contact copies of a slightly enlarged positive of BARNARD's beautiful photograph of the 1900 eclipse. The diameter of the moon is 172 mm. The copies cover various parts and were made on Process 4″ × 5″ plates in order to step up contrast. The three copies used for the north polar region had medium density near r = 1·05, 1·18 and 1·32, respectively. These copies I measured with the Schilt photometer at Leiden. The light spot on the plate was a circle of about 0·3 mm diameter. By moving the plate in both co-ordinates, any number of points for which the galvanometer reading has a chosen value could be found; these points form one isophote. The co-ordinates of each point were read to 0·1 mm and plotted at once on a sheet of paper on a ten times enlarged scale. The advantages are that no reduction is needed, that doubtful parts are easily checked and that bad marks on the plate are noticed while measuring. Twelve isophotes based on about 50 points each were made. The composite picture is shown in Figure 1. Isophotes Nos. 7 and 9 from the bottom do not appear reliable; they came from a less well exposed part of the medium copy and have been omitted in the further reduction.

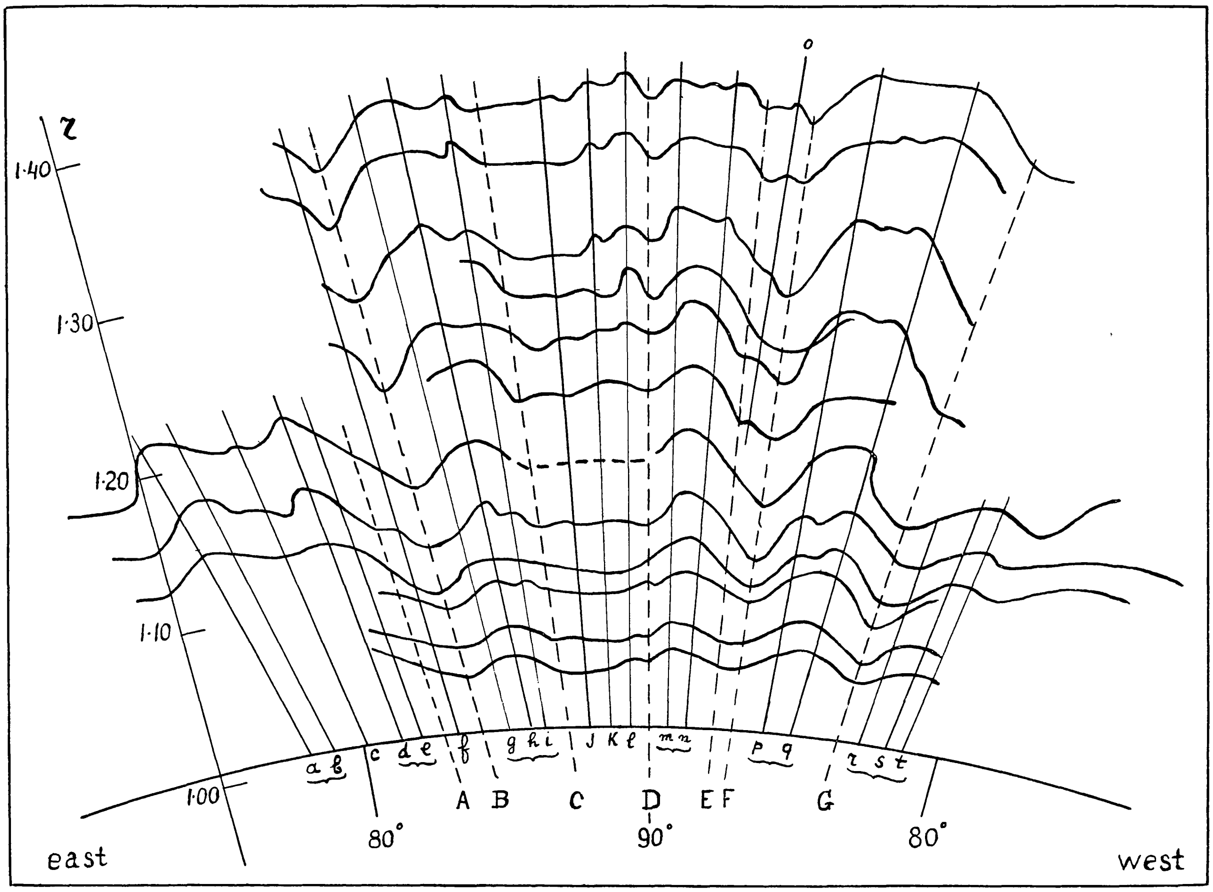
Description. The following description is based on direct inspection of the photographs. Small letters mark high points, or rays, and capitals mark low points, or spaces between rays. The isophotes show all except the very faint details. Positive contact copies (on film) of the Lick plates missed the sharp streamers like *o* and *r* but otherwise were comparable to BARNARD's plate. Also the prints of the Smithsonian Institution photographs[1]) have nearly the same quality. The reduced-size copies of the Lick plates have lost a little more detail but still show rays like *c*, *f*, and *l*, distinctly.

- *a, b:* first heavy ray from east, *b* slightly stronger than *a*, separation barely visible.
- *c:* individual ray, somewhat fuzzy, on even background.
- *d, e:* strong double, barely separated, *d* stronger than *e*.
- *A:* distinct low point.
- *f:* individual ray, stands out distinctly against sloping background.
- *B:* sharp and very deep low, conspicuous on short exposures.

1) BRIAN O'BRIEN, H. S. STEWART, and C. J. ARONSON, *Ap. J.* **89**, 26, 1939 (Plate VI).
2) H. C. VAN DE HULST, *B.A.N.* **11**, 135, 1950. This paper will further be referred to as paper I.

1) S. P. LANGLEY, 'The 1900 Solar eclipse expedition of the Astrophysical Observatory of the Smithsonian Institution', Washington 1904.

FIGURE 1
Isophotes of the north polar region of the 1900 corona.



g, h, i: one group: *g* is wide and fuzzy, bordered by *B* and by the sharp ray *i; h* is like *c* and *f* projected against *g* as background.

C, D: the lowest points of a fairly even plateau.

j, k: very faint and somewhat fuzzy rays, contrast growing with increasing *r*.

l: the sharpest ray present, conspicuous on long exposures.

m, n: a wide and heavy ray with a faint suggestion of separation.

E, F: form together a conspicuous separation, seen even on plates of poor definition.

o: a very faint but sharp ray barely visible in the hollow *E F*, somewhat slanting (// *p*).

p, q: a heavy and wide group; intensity fairly homogeneous across the group, no distinct separation.

G: a wide separation, like *E F*.

r: a very sharp but faint ray, west of the middle of *G*.

s, t: a double ray; here sideways sweeps start to come in, suggestive of streamers from low heliographic latitude. Just west of *t* is a prominence.

Calibration. The plates used had no calibration marks. But the known law by which the intensity decreases outward gives a sufficient calibration. In paper I the surface brightness of the 1900 corona along the polar axis was found from assumed data about the equatorial brightness. We now use the polar brightness distribution in order to find the photometric contrast of the streamers.

Let the smooth brightness distribution, previously assumed, be denoted by the function $H(r)$. We consider an isophote with a distance to the centre of the sun oscillating between r_h for the high points (rays) and r_l for the low points (background). Any point of this isophote has the same brightness, $H(\frac{1}{2}r_h + \frac{1}{2}r_l)$. Thus we find the brightness at one point of the ray and at one point in the background. By doing the same for other isophotes we can find many points and draw separate curves of the brightness as a function of r in the ray and between the rays. Since the gradients of these curves are not very different, the result simply is that the brightness at a fixed distance r of the centre is

$$\begin{aligned} &H(r) \text{ on the average,} \\ &H(r - \tfrac{1}{2}\Delta r) \text{ in the rays,} \\ &H(r + \tfrac{1}{2}\Delta r) \text{ between rays,} \end{aligned} \qquad (1)$$

where

$$\Delta r = r_h - r_l \quad (2)$$

is taken from the isophote that oscillates about the average value r.

I read from Figure 1 the differences, Δr, for the following combinations, *high-low: hi-B, nm-D, pq-EF*. Here *hi* means that the average of rays h and i was taken, etc. *High* and *low* were chosen close together in order to avoid effects from a possibly asymmetric plate veil. The values Δr are given in Table 1. The fourth column gives their average, where *nm-D* has been given half weight, because D does not seem to be a full low point. The three columns show a fairly similar run and the average seems to be significant to 2 or 3 thousandths of the solar radius. Some earlier measurements made on the Lick Floyd plate gave Δr-values of the same order, though not as accurate; they suggest that the apparent decrease of Δr in isophotes 11 and 12 may not be real.

TABLE 1

Depth of teeth of isophotes.

No	r	1000 Δr *hi-B*	*nm-D*	*pq-EF*	av.
1	1·054	7	9	11	9
2	1·071	8	9	14	11
3	1·099	11	9	17	13
4	1·114	14	12	21	16
5	1·140	18	18	25	21
6	1·178	20	14	28	22
8	1·254	33	13	37	31
10	1·312	39	19	40	35
11	1·365	37	11	32	30
12	1·402	38	10	30	29

TABLE 2

Surface brightness of rays and background.

r	Δr	$R+B+F$	$B+F$	F	B	R
1·04	·008	130·9	116·1	15·8	100·3	14·8
1·10	·014	60·5	51·1	11·6	39·5	9·4
1·16	·020	31·6	25·8	8·7	17·1	5·8
1·22	·028	18·6	14·8	6·7	8·1	3·8
1·28	·032	11·80	9·35	5·36	3·99	2·45
1·34	·032	7·87	6·58	4·33	2·25	1·29
1·40	·028	5·73	5·04	3·55	1·49	0·69

The further reduction (Table 2) is based on smoothed Δr-values. Since all photometric data include the F-corona, the calibration function $H(r)$ has been taken from the column $K_{pole} + F$ in Table 2 of paper I. Since the corona is transparent, the background is seen also where the rays are. If we write B = surface brightness of the real corona between rays and R = surface brightness of the rays seen without background, the observed brightness in and between the rays may be written as $R + B + F$ and $B + F$, respectively. These values have been computed from $H(r)$ and Δr by means of equation (1). The values of F, taken also from paper I, complete the data from which the single contributions B and R can be computed. All results are collected in Table 2. Since the photometry is fairly reliable, R may not be wrong by more than 20 per cent. However, the estimate of the zodiacal component, F, made in paper I, was uncertain. The polarization data, discussed in I, § 6 suggested that F might have half the adopted value; in that case B is somewhat stronger than given in Table 2. The brightness ratio R/B ranges from 0·14 to 0·60, according to Table 2. If F is halved, the range is from 0·14 to 0·37.

2. *Electron Densities.*

Densities in and between rays. Undoubtedly, the polar rays arise from electron scattering like the main light of the corona. The emission lines are far too weak to offer an explanation and the F-component cannot have a ray-like structure. So the electron densities must be locally higher in rays than between them.

The density ratio follows from the contrast of surface brightness, if due account is taken of the different depths of the rays and the background corona. Let us consider the data for $r = 1·20$. Table 2 gives $R/B = 0·40$ for the brightness ratio. But inspection of the isophotes that have enough measured points shows that single sharp rays like c, f, i, l, k, and o, have only 1/3 to 1/6 of this contrast. So we may take the ratio $R/B = 0·10$ for a typical single ray. The effective width of such a ray is of the order of $0°·5$ of heliographic latitude; this is, at $r = 1·20$, about $0·01\ R = 7000$ km, or the size of a small sunspot. The computations of I, § 5[1]) show the effective depth of the polar corona to be about $0·45\ R = 300000$ km. By this I mean that the correct surface brightness would be produced if the electron density were constant, and equal to the correct density at the polar axis, along a path of $0·45\ R$. So we may effectively reckon with an electron density that is constant with heliographic latitude within a cone around the polar axis having a half angle at its apex of $\frac{1}{2}\,\frac{0·45}{1·20}$ radians = 10°. Paper I, Figure 7B, suggests a similar value.

Now we have found that the single rays, for a moment considered as separate entities superposed on a homogeneous background, have the following ratios to this background:

ratio of surface brightness, $R/B = 0·10$,

ratio of effective depth, $7/300 = 0°·5/20° = 1/40$,

with the result that

[1]) In particular the graphs referred to below equation (35).

∴ ratio of electron densities = 40 × 0·10 = 4.

So the total density in the rays is 4 + 1 = 5 times larger than the background density for $\beta = 90°$. At $r = 1{\cdot}20$ we find (I, Figure 7B) $N_{ray} = 5 \times 4 \times 10^7 = 20 \times 10^7$.

The more conspicuous rays like *n m* are heavier complexes than those considered above. Their brightness ratio is the full amount of Table 2, $R/B = 0{\cdot}40$. But their width, and probably their depth, is of the order of 2°, so the resulting density is the same.

One may wonder whether the 'background' is possibly a superposition of unresolved streamers, while the rays only are chance agglomerations of these streamers. This possibility cannot definitely be excluded. But the picture of a rather even background is more attractive. For the sharp rays described above really give the impression of being single, each of them having its own peculiarity, like the increase of contrast in *l*, the different slope of *o*, etcetera. If we stick to this picture, some of the heavier rays still may be chance superpositions of two or three rays seen behind each other. Indeed, the distribution of the values of $r - r_{low}$ for a particular isophote somewhat resembles a Poisson distribution based on units of 1/3 Δr and an average number of 1·5 units in the line of sight.

The conclusion seems fair that all rays, single or double, have electron densities of about 5 times the background density. Seen from the earth we look through 0 to 4 of those single rays. The background, that has a 40 times larger depth than a single ray, contributes 70 to 100 per cent of the average surface brightness. Seen from above, the rays would cover at their base a fraction 1/20 of a small area around the pole. About 20 per cent of the total number of electrons would be found in rays and the remaining 80 per cent between them. All these numbers are estimated to be correct within a factor 2 or so.

Decrease of density along the rays. The density ratio of ray to background was found to be 5 at $r = 1{\cdot}20$, but may be different for other values of *r*. In other words, the density may decrease by a different law along the rays than in the background. There are individual differences; for instance, the brightness in rays *k* and *l* decreases less rapidly with increasing *r* than that in the average ray. Neglecting these differences we can base our conclusions directly on Table 2. An important fact is that all rays get wider with increasing *r*; the width, *w*, increases by about a factor 2 in the range considered. Changing from surface brightness to electron density we have to divide by *w* (the depth). Changing from surface brightness to the total number of electrons in a cross-section of the ray we have to multiply by *w* (the width). The results are given in Table 3. Here N_b has been taken from paper I, Table 5B. The next to last column refers to the data based on the adopted values for *F*; the last column shows the values obtained if the adopted *F* is halved, which may be more nearly correct.

TABLE 3

Factors by which various quantities change, if *r* increases from 1·04 to 1·34. Two different assumptions about the strength of the F-corona are made.

Quantity		*F* whole	*F* half
surface brightness of background,	B	1/45	1/25
surface brightness of ray,	R	1/11·5	1/11·5
width of ray,	w	2	2
ratio of surface brightness,	R/B	3·9	2·2
electron density of background,	N_b	1/25	1/14
ratio of electron densities,	$N_r/N_b = R/wB$	2·0	1·1
electron density in ray,	N_r	1/13	1/13
total number of electrons in an element of unit length along ray,	$w^2 N_r$	0·31	0·31
check:	$w^2 N_r/N_b = wR/B$	7·8	4·4

These data are crucial for the interpretation of the rays. The fact that N_r decreases with distance in nearly the same way, or possibly exactly the same way, as N_b, suggests at once some kind of equilibrium rather than a violent ejection of matter. But the argument may be sharpened: if the rays were streams of matter ejected with a large velocity, their densities would have to obey the equation of continuity. In this case,

$$w^2 N_r \propto 1/v,$$

which simply expresses the condition that the number of electrons passing through a cross-section per unit time is a constant along the ray. If the velocity of ejection were very large, $1/v$ would be a constant. If it were equal to the velocity of escape, *v* would change from 605 km/sec, at $r = 1{\cdot}04$, to 534 km/sec, at $r = 1{\cdot}34$, giving an increase of $1/v$ by a factor 1·13. If it were below the velocity of escape so that the ejected particles would reach $r = 4$ and fall back again, as was the conclusion of an investigation by MILLER [1]), the corresponding factor would be 1·21. So for this explanation to be correct, the value of $w^2 N_r$ would have to increase from $r = 1{\cdot}04$ to 1·34 by a factor 1·1 to 1·2. Actually (Table 3), it decreases by a factor 0·31. This discrepancy cannot be explained away. For the observed factor is independent of the assumed value of *F* and also insensitive to further errors of calibration. The conclusion must be that the polar rays must not be interpreted as ejected streams. This rules out a large class of suggested interpretations.

3. *A Review of Coronal Theories.*

The problem of the polar rays leads us to the wider question of the interpretation of the entire corona. Early theories dealt with interpretations of the light

[1]) J. A. MILLER, *Ap. J.* **27**, 286, 1908 and **33**, 303, 1911.

from the corona; this problem has been solved by the final distinction of three components: K (electron scattering), F (zodiacal light) and L (emission lines). The subsequent problem was to find in what state of equilibrium or non-equilibrium the free electrons and highly ionized atoms are maintained. Recent studies have emphasized two aspects of this problem:

1) How is energy transmitted from the sun to the corona, giving it the very high temperature of about 1000 000°?

2) What state of ionization and excitation is found at such temperatures and densities?

We shall not review the many answers given to these questions, for prior to them is a third question that we shall discuss in some detail[1]):

3) Is the coronal gas like a thermal gas (static equilibrium) or rather like a chaotic collection of streams and jets (dynamic equilibrium)?

In discussing questions 1) and 2) most authors use the specific concept of a thermal gas but are sceptic of it at the same time. For most of the observed features: large extension of the corona, strong Doppler broadening and high excitation, might be explained by strong stream motions as well as by a high temperature. Since the data described in the preceding sections have a direct bearing on this third question we shall critically review the suggested answers below.

A. *The corona as a collection of streamers.* Early investigators[2]) considered only explanations of this type, partly because the idea of a static corona of high extension seemed incredible and partly because the structure of the corona strongly suggested stream motions. In fact, the conventional term 'streamer' given to these structural details illustrates the strong bias in favor of this explanation. Turning to the observations we note first that the chromosphere shows streams and jets in great multitude, seen as prominences and spicules. But the chromosphere must not lightly be taken as an example for the corona. Although the inner corona certainly derives energy in some way from the lower layers, the chaotic motions of these layers do not seem to be propagated into the corona, as the following summary of observations suggests.

LYOT[3]) re-examined the transverse motions of structural details noticed during certain eclipses. He concludes that the alleged velocities of 10 or 20 km/sec are spurious. His own films, made in the red and green emission lines, show similar details but no motion at all, only changes in intensity. Radial velocities of the K-corona cannot be measured. The alleged effect of + 20 km/sec for the F-corona is probably spurious and at any rate unexplained[1]). WALDMEIER[2]) has extensive data on the emission lines. He finds that 10 km/sec must already be called a rather large speed. Higher velocities, up to 150 km/sec in one case are sometimes observed, but all cases where the velocity exceeded 20 km/sec were clearly connected with sunspot groups.

Since these data do not confirm the rapid ejection of matter, which is the common feature of explanations of type A, all variations need not be reviewed in detail. BREDICHIN and SCHAEBERLE advanced mechanical theories, in which the ejected matter described Kepler orbits. STÖRMER[3]) modified the theory for the influence of a general magnetic field. MILLER[4]) applied the mechanical theory to many observed streamers, while VON DER PAHLEN and KOHLSCHÜTTER[5]) tried to show by least-squares solutions that the observed forms are not exactly explained by those theories. The most recent theory in this class has been given by KIEPENHEUER[6]). He assumes that the sun has a general magnetic field that rapidly declines with increasing r and is virtually zero in the corona. He assumes that neutral bodies of gas are emitted by the sun and shows that they pick up induction currents when they pass into the corona. So they carry a magnetic moment with them. The further motion of these bodies is thought to be governed by their mutual magnetic fields, while gravitation is negligible. This attractive theory has to be rejected for the same reasons as the preceding ones, for the required velocities, of at least 600 km/sec, do not agree with observations. In addition, the author does not see any reason, theoretical or observational, why the sun's general magnetic field (if and when it exists) should be screened[7]).

B. *The corona as an atmosphere of high temperature.* The possibility that the high energy of the particles of the corona might be regarded as thermal energy was discussed systematically by ALFVÉN[8]). In the following considerations many of his suggestions were found useful. Let r be the distance from the sun's centre in terms of the sun's radius, R. Write G = gravitation constant, M = mass of the sun, m_H = mass of the proton, N_e = electron density (number per cm^3), k = Boltzmann constant, T = temperature, p = pressure and μ = molecular weight. If we assume a relative abundance of 5 H atoms and 1 He atom, all fully ionized, we have 7 electrons and 6 ions per 9

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1) Compare S. ROSSELAND, *Publ. Oslo* No. 5, 1933.
2) See S. A. MITCHELL, *Hdbuch d. Ap.* **4**, 343-353, 1929.
3) B. LYOT, *Ann. d'Astrophysique* **1**, 41-44, 1944.

1) Cf. *Ap. J.* **105**, 488, 1947.
2) M. WALDMEIER, *Astr. Mitt. Zürich* No. 151, 1947.
3) C. STÖRMER, *C.R.* **174**, 1447, 1922 and **152**, 425, 1911.
4) J. A. MILLER, *Ap. J.* **27**, 286, 1908 and **33**, 303, 1911.
5) E. VON DER PAHLEN and A. KOHLSCHÜTTER, *Veröff. Bonn*, No. 24, 1930.
6) K. O. KIEPENHEUER, *Z.f.Ap.* **10**, 260, 1935.
7) See section 4 of this paper.
8) H. ALFVÉN, *Arkiv f. Mat., Astr. och Fysik* **27A**, No. 25, 1941.

mass units; so $\mu = 9/13$. Hydrostatic equilibrium means that the sum of the gravitational force per unit volume and the pressure gradient is 0:

$$\frac{G M N_e}{(rR)^2} \cdot \tfrac{9}{7} m_H + \tfrac{13}{7} \frac{d(N_e k T)}{R\,dr} = 0.$$

The equation is easily reduced to

$$\frac{T_1 N_e}{r^2} = -\frac{d(N_e T)}{dr}, \qquad (1)$$

where $T_1 = GM\mu\, m_H / Rk = 16 \times 10^6\,{}^\circ\mathrm{K}.$

This equation may also be written in the form

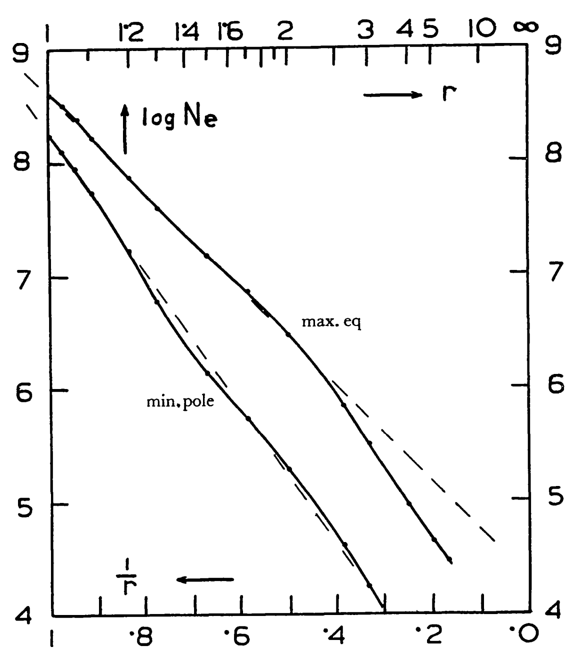
$$\frac{T_1}{T} = \frac{d \ln N_e}{d(1/r)} + \frac{d \ln T}{d(1/r)}. \qquad (2)$$

It is clear that from either equation (1) or equation (2) we must be able to find T as a function of r, if N_e as a function of r is given. The strict method is to find $N_e T$ by integration of equation (1); this method was used by ALFVÉN[1]), ALLEN[2]) and WALDMEIER[3]).

The integrations make it difficult to judge by this method to what extent errors of the original density curve influence the result. This drawback is overcome if we use equation (2). The last term of this equation is of minor importance because the temperature gradient is small. So we can plot at once the given values, log N_e, against $1/r$; then we expect a nearly

FIGURE 2

Graph for determining the temperature of the corona from its density gradient.



1) *L.c.*, section 3.
2) C. W. ALLEN, *M.N.* **107**, 426, 1947.
3) M. WALDMEIER, *Astr. Mitt. Zürich*, No. 154, 1948.

straight line, the slope of which is $0{\cdot}43\ T_1/T$. The last term of equation (2) may be taken into account in a second approximation. Figure 2 illustrates this method for the data of Paper I, Table 5. We see that straight-line solutions represent the data very well in the range $1 < r < 3$. The deviations of $\pm$ 20 per cent are not real, for the waves of the curves result from the analytic representation of the original brightness data by means of inverse powers. The solutions in this range are

maximum, equator: $\log N_e = 4{\cdot}28 + 4{\cdot}32/r$,
so $T = 1\,620\,000^\circ$,
minimum, pole: $\log N_e = 2{\cdot}17 + 6{\cdot}05/r$,
so $T = 1\,150\,000^\circ$.

Similar values are found by the authors cited. The resulting T is proportional to the adopted value of μ. Formulae of this type may also be written down directly on the basis of Boltzmann's equation for an isothermal gas. We conclude that an isothermal corona represents the data pretty well and that attempts to find the run of T with r from such data must be termed too optimistic. Only for $r > 3$ in the equatorial regions an indication is found that the temperature is lower by a factor 2, or so, than in the inner corona. Here, however, the observational data are rather uncertain, due to the F-corona, while also the simple temperature concept breaks down (see below).

The first modification of this theory involves the influence of electric fields. The electrons, which are light, have a tendency to form a more extensive atmosphere than the ions, that are heavier. But as soon as some separation of the charges is effected, an electric field is set up that counteracts the tendency. We have to find the equilibrium field. This is an old problem, solved by PANNEKOEK[1]) and ROSSELAND[2]), for it occurs in the equations of hydrostatic equilibrium throughout a star. COWLING[3]) has presented the solution in a very simple form by requiring that the total forces on the ions (protons) and on the electrons must be equal:

$$g\, m_H - e\, E = g\, m_e + e\, E,$$

where g is the local acceleration of gravity and E the local electric field. The field is found from this equation and hence the total charge and volume charge of the sun. The result is that the volume charge is positive throughout, and proportional to the density. As a result of this field we are permitted to solve the hydrostatic equation for the entire gas, with a mean molecular weight, just as if there were no electric field at all. In addition, there must be a slight repulsion between the sun and the corona, but I find the effect entirely negligible, at about 10^{-37} times the gravi-

1) A. PANNEKOEK, *B.A.N.* **1**, 110, 1922.
2) S. ROSSELAND, *M.N.* **84**, 720, 1924 (section 2).
3) T. G. COWLING, *M.N.* **90**, 140, 1929.

tational attraction. We may here remind of SCHWARZSCHILD's [1]) estimate that a corona consisting of only electrons around a sun with the corresponding positive charge could have an extension of only a few mm, because of electrostatic forces. A recent theory[2]), based on similar ideas, does not seem to be very promising. Conclusion: electrostatic forces are probably unimportant except in holding electrons and protons together.

ALFVÉN suggested yet another modification, based on the influence of an inhomogeneous magnetic field. If the field strength, H, changes along the lines of force, charged particles circling around the lines of force are accelerated along these lines in the direction of decreasing H. Accordingly, the electrons and protons in the polar regions of the corona should be pushed out. ALFVÉN[3]) computed that the magnetic force could account for about one third of the outward force, leaving two thirds to the pressure gradient, so that also the temperature had only two thirds of the value computed from the simple theory. However, this result was not correct, for the magnetic field does not influence hydrostatic equilibrium at all[4]). This would seem to follow directly from the fact that there is no energy connected with the Lorentz force, so that the Boltzmann factor is unaffected. Yet it is instructive to find the same result also from kinetic theory. In order to do so, we omit the gravitational acceleration; it is then to be shown that the equilibrium state is that of uniform density.

ALFVÉN derives the equations of motion of a charged particle in a very interesting paper[5]), to which the reader is referred for full details. The motion consists of a velocity v_z, with kinetic energy $E_{//} = \frac{1}{2} m v_z^2$, along the lines of force and superposed on it a circling motion around the lines of force. The radius of this circle is

$$\rho = \frac{v_\perp m c}{e H},$$

where c = velocity of light, m and e = mass and charge of the particle, while $v_\perp$ is its velocity component perpendicular to the lines of force, with the kinetic energy $E_\perp = \frac{1}{2} m v_\perp^2$.

ALFVÉN shows that the ratio

$$\mu = E_\perp / H$$

remains constant while the circle moves along the lines of force. So $v_\perp$ varies as $H^{\frac{1}{2}}$ and ρ as $H^{-\frac{1}{2}}$; this means that also the periphery of the circle moves exactly along a tube of force, that widens with decreasing H. Another consequence is that the particle in moving upward (Figure 3) loses transverse energy, $E_\perp$. Its total energy, however, is not changed by the Lorentz force, so the energy $E_{//}$ has to grow by the same amount: $dE_{//} = - dE_\perp$. This gives

$$m v_z \, d v_z = - \mu \, dH,$$

and, since the distance covered during the time dt is $dz = v_z \, dt$,

$$m \frac{d v_z}{dt} = - \frac{E_\perp}{H} \frac{dH}{dz}. \tag{3}$$

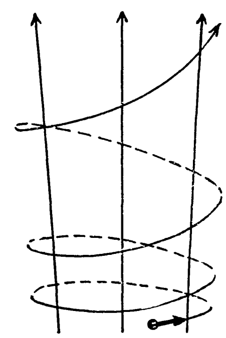


FIGURE 3

Path of a charged particle in an inhomogeneous field.

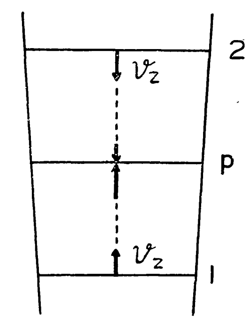


FIGURE 4

Kinetic explanation of the equilibrium state.

This result means that there is always an acceleration along the lines of force, towards smaller values of H; the acceleration is independent of $E_{//}$ and proportional to $E_\perp$.

This law of motion is all right, but we have to be cautious in its statistical application. Let P (Figure 4) be a test plane perpendicular to the field, and 1 and 2 planes at a small distance l at either side. Consider the circling particles passing up through 1 with energies $E_{//}$ and $E_\perp$ and passing down through 2 with the same energy components. If the temperatures at 1 and 2 are the same, these numbers will be proportional to the area, A, and the density, N. If l (comparable to the free path) is small enough, these particles need a time $1/v_z$ to travel from 1 and 2 to P. During this time the particles from 1 pick up speed by the amount

$$\Delta v = \frac{l}{v_z} \frac{d v_z}{dt},$$

while the particles from above slow down by the same amount. The criterion for equilibrium is that the momentum imparted to P from either side is the same, statistically. So

$$N_1 A_1 (v_z + \Delta v) = N_2 A_2 (v_z - \Delta v).$$

After the substitution $A_1/A_2 = H_2/H_1$, and repeated use of the assumption that l and Δv are small, this form is reduced to

1) See S. A. MITCHELL, *l.c.*
2) A. B. SEVERNY, *Astr. J. Soviet Un.* **16**, 16, 1939.
3) *L.c.*, sec. 8.
4) After this had been written, Dr ALFVÉN kindly informed me that he had found his argument defective for similar reasons as explained below.
5) H. ALFVÉN, *Arkiv f. Mat., Astr. och Fysik* **27A**, No. 22, 1940.

$$\frac{N_1}{N_2}\left(1 + 2\frac{l}{H}\frac{dH}{dz}\right)\left(1 - \frac{E_\perp}{E_{//}}\frac{l}{H}\frac{dH}{dz}\right) = 1. \qquad (4)$$

Since the velocity distribution of an isothermal gas is isotropic, we have also

$$E_\perp = 2\,E_{//} \quad , \qquad (5)$$

on the average. Equation (4) now shows that the equilibrium will be maintained if N_1 and N_2 are equal: the magnetic force does not result in a density gradient. ALFVÉN considered only particles with small $E_{//}$ and concluded that $N_2 > N_1$. On the other hand, particles with large $E_{//}$ and small $E_\perp$ have a tendency to make $N_1 > N_2$; these tendencies cancel on the average.

The preceding argument, although not a formal proof, may be sufficient indication that the magnetic field has no influence on the equilibrium state. The same holds true if there is a gravitational field, so the equations (1) and (2) do not need any modification.

C. *Influence of the long free path.* The ordinary properties of a thermal gas may safely be used if the changes of conditions are small over distances comparable with the free path of the molecules. The free path in an ionized gas is proportional to $N_e^{-1}\ T^2$. ALFVÉN estimates for the corona:

$r = 1{\cdot}2$, $\quad l = 5.10^8$ cm $= 0{\cdot}007\ R$;
$r = 2$, $\quad l = 10^{10}$ cm $= {\cdot}15\ R$;
$r = 4$, $\quad l = 7.10^{10}$ cm $= 1\ R$.

Here R is the sun's radius. We conclude that the preceding picture of the corona as a thermal gas in static equilibrium is fairly safe for the inner and medium corona. But in the outer corona, beyond $r = 3$, other effects may come in.

As a first effect, fast electrons may escape from the sun in the same way as light molecules are thought to escape from the upper atmosphere of a planet. The average velocity at $T = 10^6$ degrees is 6000 km/sec for electrons and 150 km/sec for protons, while the velocity of escape is 360 km/sec at $r = 3$. Further, electric fields will probably be set up, allowing electrons and protons to escape at equal rates. Finally, the magnetic force in the polar regions will tend to change the complete kinetic energy of a particle into energy of an outward motion, thus increasing its chance to escape before a collision takes place. The net result may be a general flow of matter out of the sun. Just for a guess, let us assume that near $r = 4$, ten per cent of the electrons and protons is moving outward with a velocity of 500 km/sec, while the remaining ninety per cent is at rest. The mean velocity of flow, u, is then 50 km/sec at $r = 4$. The number of electrons flowing per second through a sphere with radius r is

$$Z = 4\pi\, N_e\, r^2\, u; \qquad (6)$$

this must be a constant. By means of the observed densities[1]) N_e we find that

$u =$	·025	·2	1·5	6	50	100	km/sec
at $r =$	1	1·2	2	3	4	6	solar radii.

The total loss is $Z = 6.10^{34}$ electrons and protons per second. This corresponds to a loss of mass of 10^{11} g per second, which is 1/40 of the mass lost by the sun in the form of radiation.

If these ideas are correct, the inner corona is a gas in thermal equilibrium, with a very small outward drift needed to replace losses in the outer corona. In the outer corona this drift would gradually change to a general outward stream with rather high velocities. The magnitude of the effect is still unknown; it may well be 0·1 or 0·01 times the estimates made above. A complete theoretical discussion of this problem would be highly interesting.

4. *Suggested Interpretation of the Polar Rays.*

The interpretation of the polar rays as rapid streams of matter ejected by the sun is ruled out on account of the photometric data discussed in section 2[2]). The observed fact (Table 3) is that the electron densities in the rays decrease with r in a very similar way to the electron densities between the rays. This suggests also a very similar state of equilibrium, which in this range of r is hydrostatic equilibrium at a nearly constant temperature (section 3B).

The urgent question is: how can such regions of abnormally high density be so beautifully arranged along a specific ray? Here I can fully accord with the answer given by ALFVÉN[3]): the rays must coincide with magnetic lines of force. The explanation of the raylike structure is then straightforward. Let a lump of matter be placed somewhere in the field. The ions and electrons will diffuse in the normal manner along the lines of force. But the ions or electrons that would otherwise diffuse in the perpendicular direction are now kept close to the same line of force in spiral paths. The original lump of matter will, therefore, rapidly stretch out along the entire line of force and establish the ordinary density gradient along this line, determined by the gravitational force and the temperature of the medium. Very gradually, the ray will also widen itself by diffusion perpendicular to the lines of force. Eventually it will vanish in the background.

1) Paper I, Table 5A.
2) Local corpuscular emission from the sun is not denied by this argument. A proton with $v = 2 \times 10^8$ cm/sec can penetrate the entire corona without serious deflection; dense jets can do so a fortiori. But the issuing of slow streams from the sun with densities comparable to the corona and velocities of the order of the thermal velocities, as suggested by KIEPENHEUER (*Ap.J.* **105**, 408, 1947) would seem puzzling.
3) Arkiv 27A, No. 25, sec. 13.

The following estimates show that this picture is not in conflict with quantitative data. The free path, l, is of the order of 5000 km = 0·007 R in the regions where the rays are clearly seen. The time, τ, during two collisions is of the order of a second. The entire length of the rays is of the order of 0·5 R = 70 l. The establishment of equilibrium along this path requires about 70^2 = 5000 collisions, which takes a few hours' time. These data were taken from ALFVÉN's table for r = 1·2. The increase of τ with increasing r is offset by the decrease of l. So the order of a few hours for the formation of a ray is probably a correct estimate. The path perpendicular to the lines of force that is traveled between two collisions is of the order of the radius, ρ, of the spirals. This is 2·4 cm, or $0{\cdot}5.10^{-8}\ l$, for electrons and 1 m, or $2.10^{-7}\ l$, for protons. So the widening of the rays by diffusion is negligible, even during a week and even if the magnetic field would have only 1/100 the strength supposed by ALFVÉN. It has been found that the rays do not perceptibly change during the few hours in which an eclipse cone sweeps the earth's surface. This fact is in good agreement with the preceding estimates. Also the widening of the rays with increasing r is as expected from the divergence of the lines of force.

The sources of the increased densities are probably located at the sun's surface and should have life times of the order of hours, or days, and sizes comparable to small sun spots. Nothing much can be said about their nature but it seems quite reasonable to suppose that such disturbances exist. They cannot be ordinary prominences. For the polar rays are probably of the same nature as the general background density, which is fairly high just around the poles, unlike the prominences[1]).

Perhaps the most important consequence of the suggested explanation is that the polar rays really show us the general magnetic field of the sun. ALFVÉN assumed the conventional value of 25 gauss for the field strength at the poles. But the rays would still indicate the lines of force, even if the field strength would have to be scaled down by a factor 100 or 1000. So the strength of the field cannot be checked by means of these observations. That the field is really there, seems to agree with modern ideas that the field is not shielded by atmospheric currents. HALE's early evidence for a rapid outward decrease of H was very weak and CHAPMAN's[2]) theoretical explanation of the drift currents was shown to be incorrect a year later[3]). The grounds on which some authors[4]) still defend the idea of perfect shielding do not seem to be much safer. Moreover, the shielding would be imperfect close to the pole in any theory.

In order to try whether quantitative data about the form of the field could be obtained, I drew straight lines coinciding with conspicuous polar rays on some corona photographs.

These lines, when prolonged, converge to a point on the axis that has a distance q to the centre of the disk. The value q/R can be accurately measured and is a parameter characteristic of the form of the sun's field. The observed values, as given in Table 4, are

TABLE 4
Position of point of convergence of the polar rays.

Source	q/R	tangents drawn at
Corona 1900 north	0·70	r = 1·0 to 1·2, β = 85° to 75°
Corona 1900 south	0·71	
Corona 1901 north	0·64	
Corona 1901 south	0·65	
Observed mean	0·65 ± ·02	
Theory for dipole field	0·33	r = 1·0, β = 90°
	0·37	r = 1·1, β = 80°

definitely higher than the value for the field of an infinitesimal dipole at the sun's centre. This checks with the statement made by CAMPBELL, MOORE and BAKER[1]), that the rays 'more nearly resemble the lines of force obtained when the poles of the magnet are separated a little more than two thirds of the sun's diameter'.

Historically, the resemblance between the polar rays and magnetic lines of force has been noticed from the first time they were seen. SCHUSTER[2]), in his general discussion of the magnetic field of rotating bodies, mentioned the coronal rays as an interesting but not convincing evidence of the sun's general magnetic field. HALE[3]) started his measurements of the Zeeman effects due to this field as a conscious attempt to find a more convincing proof. Some six years later these efforts seemed to have been so successful indeed that the argument could be reverted. CAMPBELL, MOORE and BAKER[4]) measured the positions of the axes of the coronal rays at five different eclipses and found them to agree with the rotational axis of the sun. The difference with the magnetic axis computed from the Mount Wilson measurements was fairly large. So they cautiously concluded that: if the latter data are correct, the polar streamers are not influenced by the sun's general magnetic field. However, subsequent

1) Paper I, § 5.
2) S. CHAPMAN, *M.N.* **89**, 57, 1928.
3) T. G. COWLING, *M.N.* **90**, 140, 1929; also *M.N.* **105**, 166, 1945.
4) K. O. KIEPENHEUER, *l.c.*; C. WALÉN, *Arkiv f. Mat., Astr. och Fysik* **33A**, No. 18, 1946.

1) W. W. CAMPBELL, J. H. MOORE, R. H. BAKER, *P.A.S.P.* **35**, 163, 1923.
2) A. SCHUSTER, *Proc. Phys. Soc. London* **24**, 127, 1912.
3) G. E. HALE, *Ap. J.* **38**, 27, 1913.
4) *L.c.*