BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS.

1927 November 23 Volume IV. No. 138.

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Note on the excitation of "forbidden" lines in the nebular spectrum,

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The identification by I. S. BOWEN of lines of until recently unknown origin in the nebular spectrum has given rise to the question (A. FOWLER, *Nature* Oct. 29 1927, p. 617) how it is possible that an atom only radiates those lines of its spectrum that under ordinary laboratory conditions do not occur, and does not show those already known from terrestrial experiments.

The following analysis has taken its origin in a discussion with J. H. OORT, that has suggested the influence of different optical thickness of the nebula in different frequencies.

1. S. ROSSELAND *) has pointed out the influence of metastability on the relative concentration of different energy levels in nebular matter. Taking for the sake of simplicity an atom with three stationary states (1, 2, 3) and supposing the transition probability $1 \rightleftarrows 2$ zero, it appeared that the ratio of n_2 and n_1, the numbers of atoms per unit volume in states 2 and 1, is to a large extent independent of the dilution of the radiation and closely approximates to its value in thermodynamic equilibrium. The application to the case that the transition probability is not exactly zero but exceedingly small requires special attention to ascertain the precise meaning of "exceedingly small". Let $A_{i\to k}\,\rho_{\nu_{ik}}$ and $A_{k\to i}$ $(i<k)$ be the probabilities of the transitions $i\to k$ and $k\to i$ if the density of radiation is $\rho_{\nu_{ik}}$.

Neglecting the stimulated transitions we have the well known equations expressing the constancy of the numbers n_1, n_2, n_3:

$$\begin{aligned} n_1\,(A_{1\to2}\,\rho_{\nu_{12}} + A_{1\to3}\,\rho_{\nu_{13}}) &= n_2\,A_{2\to1} + n_3\,A_{3\to1} \\ n_2\,(A_{2\to1} + A_{2\to3}\,\rho_{\nu_{23}}) &= n_1\,A_{1\to2}\,\rho_{\nu_{12}} + n_3\,A_{3\to2} \\ n_3\,(A_{3\to1} + A_{3\to2}) &= n_1\,A_{1\to3}\,\rho_{\nu_{13}} + n_2\,A_{2\to3}\,\rho_{\nu_{23}} \end{aligned}$$

The third equation is equivalent to the sum of the first two equations. Solving for n_2/n_1 and n_3/n_1 we have:

$$\frac{n_2}{n_1} = \frac{A_{1\to2}(A_{3\to1}+A_{3\to2})\,\rho_{\nu_{12}} + A_{3\to2}\,A_{1\to3}\,\rho_{\nu_{13}}}{A_{2\to1}\,(A_{3\to1}+A_{3\to2}) + A_{3\to1}\,A_{2\to3}\,\rho_{\nu_{23}}} \qquad \frac{n_3}{n_1} = \frac{A_{1\to2}\,A_{2\to3}\,\rho_{\nu_{12}}\,\rho_{\nu_{23}} + A_{1\to3}\,A_{2\to3}\,\rho_{\nu_{13}}\,\rho_{\nu_{23}} + A_{1\to3}\,A_{2\to1}\,\rho_{\nu_{13}}}{A_{2\to1}\,(A_{3\to1}+A_{3\to2}) + A_{3\to1}\,A_{2\to3}\,\rho_{\nu_{23}}}$$

Supposing $A_{3\to1}$ and $A_{3\to2}$ of the same order of magnitude it is seen that, as n_2/n_1 is relatively large if the denominator has no term without the factor ρ, the expression "exceedingly small" means

$$(A) \qquad A_{2\to1} << A_{2\to3}\,\rho_{\nu_{23}}.$$

This I shall suppose to be the case **); hence the line ν_{12} is "forbidden", the line ν_{13} is a usual line.

As

$$\frac{n_3}{n_2} = \frac{A_{2\to3}\,\rho_{\nu_{23}}\,(A_{1\to2}\,\rho_{\nu_{12}} + A_{1\to3}\,\rho_{\nu_{13}})}{A_{3\to2}\left\{A_{1\to2}\left(\frac{A_{3\to1}}{A_{3\to2}}+1\right)\rho_{\nu_{12}} + A_{1\to3}\,\rho_{\nu_{13}}\right\}},$$

if we neglect $A_{2\to1}$ with respect to $A_{2\to3}\,\rho_{\nu_{23}}$ according to condition (A) we have

$$\frac{n_3}{n_2} \backsimeq \frac{A_{2\to3}\,\rho_{\nu_{23}}}{A_{3\to2}},$$

the value that we should obtain if 2 and 3 were the only possible stationary states. Hence condition (A) leads to the relation

$$\frac{n_3}{n_2} >> \frac{A_{2\to1}}{A_{3\to1}}$$

or

$$(B) \qquad n_2\,A_{2\to1} << n_3\,A_{3\to1}.$$

2. If I_ν is the intensity of radiation emerging from

*) *Astroph. J.* 63, p. 232.

**) This is the limiting case; gradually weakening of the inequality, succeeded by an equality as regards order of magnitude, does not alter the conclusions of this note.

the nebula, E_ν is the emission coefficient per unit volume, t_ν is the optical depth, T_ν the optical thickness we have:

$$I_\nu = \int_0^{T_\nu} \frac{E_\nu}{x_\nu \rho} e^{-t_\nu} dt_\nu ;$$

x_ν is the mass-absorption coefficient, ρ the density of matter.

Case I. Suppose the optical thickness to be very small for both frequencies ν_{12} and ν_{13}; then if l is the linear depth of the nebula and we neglect variation of E_ν within the nebula we have:

$$I_\nu = \frac{E_\nu}{x_\nu \rho} T_\nu = E_\nu l$$

Hence: $I_{\nu 12} : I_{\nu 13} = n_2 A_{2\to 1} : n_3 A_{3\to 1}$, thus according to (B) the "forbidden" line is much weaker than the usual line.

Case II. Suppose the optical thickness to be very large for both frequencies ν_{12} and ν_{13} then:

$$I_\nu = \frac{E_\nu}{x_\nu \rho}$$

and

$$I_{\nu 12} : I_{\nu 13} = \frac{n_2 A_{2\to 1}}{n_1 A_{1\to 2}} : \frac{n_3 A_{3\to 1}}{n_1 A_{1\to 3}}.$$

As $\frac{A_{2\to 1}}{A_{1\to 2}} = \frac{8\pi h}{c^3} \nu_{12}{}^3$ (h constant of PLANCK, c velocity of light) we have:

$$I_{\nu 12} : I_{\nu 13} \backsim n_2 : n_3$$

hence the "forbidden line" is by far the most intense.

Case III. Suppose the optical thickness in ν_{13} is large, in ν_{12} is small, then:

$$I_{\nu 12} : I_{\nu 13} = \frac{n_2 A_{2\to 1}}{n_1 A_{1\to 2}} T_{\nu 12} : \frac{n_3 A_{3\to 1}}{n_1 A_{1\to 3}}.$$

The right hand member is of the order of magnitude of

$$\frac{n_2}{n_3} T_{\nu 12}.$$

As n_2/n_3 is large, the small value of $T_{\nu 12}$ may be compensated so as to make the "forbidden" line the stronger one. This is more clearly seen if we write:

$$\frac{n_2}{n_3} T_{\nu 12} = \frac{n_2}{n_3} \frac{A_{1\to 2}}{A_{1\to 3}} T_{\nu 13} \backsim \frac{n_2}{n_3} \frac{A_{2\to 1}}{A_{3\to 1}} T_{\nu 13}$$

If the inequality (A) is not too strong, the large value of $T_{\nu 13}$ may easily compensate the small value $\frac{n_2 A_{2\to 1}}{n_3 A_{3\to 1}}$ and the "forbidden" line may be the only one visible.

One might ask whether there is no contradiction in the two suppositions $T_{\nu 12}$ small and inequality (A) not too strong. Suppose $\rho_{\nu 23}$ to be equal to the value of black-body radiation density multiplied by a dilution factor W. Then inequality (A) leads to the relation

$$(C) \qquad A_{1\to 2} << A_{1\to 3} \frac{W}{e^{\frac{h\nu_{23}}{kT}} - 1}$$

hence $T_{\nu 12} << T_{\nu 13} W$ generally. Hence if W is small enough $T_{\nu 13}$ may be large and $T_{\nu 12}$ small without interfering with the condition that inequality (A) is not too strong.

As $T_{\nu 13}$ probably is exceedingly large, case I is excluded; then the foregoing analysis shows that it is easy to conceive a value for the transition probability of the "forbidden line" which makes the intensity of this line large compared with that of the usual line; and thus provides an explanation of the empirical fact that only "forbidden lines" of O II, O III and N II are visible in the nebular spectrum. *)

*) In a recent note (*Publ. of the Astr. Soc. of the Pac.*, Oct. 1927) BOWEN identifies the lines λ 3313, 3342, 3445, 3759 with 4 of the five brightest lines of O III. The intensities of these lines as given by WRIGHT (*Lick* XIII, table 11) are small compared with that of the "forbidden" O III line λ 5007; however, as the connection of WRIGHT's intensities with an energy scale is not clear, it is difficult to be definite about the matter.