1968, *Bull. Astr. Inst. Netherlands* **20** 69–72; *Communication from the Observatory at Leiden*

A TWO-COMPONENT LOG $N-$LOG S RELATIONSHIP

EDWARD B. FOMALONT

Received 5 April 1968

It is shown that the slope of the log $N-$log S relationship for radio sources of high brightness temperature is larger than the slope for sources of low brightness temperature. Using the optical identification of these radio sources the slope for radio galaxies is found to be ~ 1.5 and the slope for quasars ≥ 2.5.

1. Introduction

One of the easiest methods of using radio observations as a test of cosmological models is the use of the log $N-$log S relationship, where N is the number of radio sources in a complete survey with a flux density greater than S. The relationship is a manifestation of the density distribution and the luminosity distribution of radio sources, and the space-time structure, combined in a rather complicated manner. For a spatially homogeneous distribution of sources in a static Euclidean universe, N is proportional to the volume of space probed and S is inversely proportional to the square of the distance so that $N \propto S^{-1.5}$*. The redshift-distance relationship decreases the value of the slope below 1.5 for the fainter, more distant sources and in this way Olber's paradox of a very large sky brightness for an unbounded universe is avoided.

To quickly review the observational data, the slope of the log $N-$log S relationship for sources above one flux unit (10^{-26} W · m^{-2} · Hz^{-1}) is certainly larger than 1.5. There is excellent agreement among all of the surveys for a slope of 1.75. The confusion observations (HEWISH, 1962) and deep surveys of faint sources (LONGAIR, 1966) at 178 MHz do show the expected drop in the slope at a flux density level corresponding to about 0.2 flux units at 1425 MHz. A possible explanation of the relationship is that the density and/or luminosity of radio sources increase with distance: it is only for the very distant sources for which a large redshift sufficiently attenuates the radiation to cause a decrease in the slope. This explanation is substantiated by SCHMIDT (1968) who has found an increase in the density of quasars with distance.

It is still unknown, however, whether all radio sources partake in a density increase with distance. VÉRON (1966) has shown that the log $N-$log S relationship for identified sources in the 3C Revised Catalogue (BENNETT, 1962) has a slope of 1.55 for radio galaxies (homogeneous distribution) and 2.2 for quasars (density increase with distance). But the limited number of identifications, some of which are not well-established, and the incompleteness of the identifications to a reasonably low flux density level may seriously affect the result. JAUNCEY (1967) has reanalysed the data and shows that the doubtful identifications and the empty fields (which account for 45 per cent of the sample) are responsible for the steepening of the slope. Until the uncertain identifications are confirmed and more information is known about the optical type represented by the empty fields, the results of Véron are in doubt.

2. Observations

In 1965 the one-dimensional brightness distribution of over 500 sources was obtained at a frequency of 1425 MHz using the Caltech twin-element interferometer to synthesize an east-west fan beam of 45″. The general information concerning the observations and the results are given in two previous publications (FOMALONT, 1967, 1968). The source list included all of the then known sources with a diameter less than 10′, a

* In the relation $N \propto S^{-x}$ usually considered for number-source counts, x is the slope of the log $N-$log S relation.

flux density greater than 2 flux units, and a declination north of $-50°$. The completeness varies considerably and depends on the frequency and depth of the survey used in covering the different portions of the sky. See FOMALONT (1968) for a discussion of the source list. Overall the list is 80 per cent complete above a flux density of 2.0 flux units at 1425 MHz.

In order to obtain a valid $\log N - \log S$ relationship for a group of sources, this group must be distinguishable by a radio source property which is independent of distance. Selection of a subset of all radio sources by some distance-dependent criterion would distort the $\log N - \log S$ relation. Examples of acceptable selection properties are: radio sources identified with galaxies, radio sources identified with quasars, radio sources with a core-halo structure, and so forth. Another quantity which is available from one-dimensional observations is an average brightness temperature T_b defined by

$$T_b \propto S \cdot d^{-2}, \quad (1)$$

where S is the flux density of the source and d is the east-west angular extent of the source.

The observed sources were divided into two groups, "high-brightness" sources with $T_b > T_0$ and "low-brightness" sources with $T_b < T_0$. The value of T_0 was chosen to be $2.0/(0.25)^2$ flux units $\cdot$ (arc minutes)$^{-2}$ which corresponds to 7000 °K. This value was chosen for several reasons: it is the highest brightness temperature for which each source above 2 flux units can be placed unambiguously into either group, approximately half of the sources are in each group, and it is known that very few quasars have a brightness temperature <7000 °K at 1425 MHz.

3. Results

In figure 1 the $\log N - \log S$ curves for the two groups of sources have been plotted using all of the sources in the observing program. There is only a slight difference in slope for the high- and low-brightness sources and this difference is not statistically significant. Because the source list was obtained from surveys at a finding frequency lower than 1425 MHz, there is no doubt that a majority of the missing sources have flat spectra and thus a high brightness temperature (KELLERMANN, 1964). The detailed effect of the missing sources on the slopes is unclear for it depends critically on their distribution with flux density.

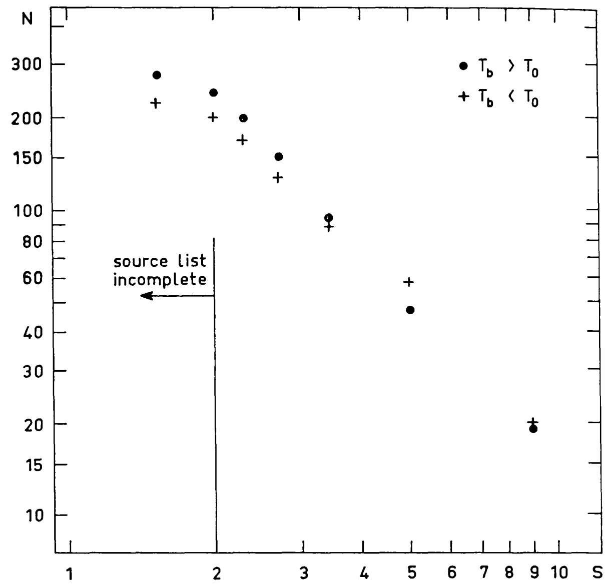

Figure 1. The $\log N - \log S$ relationship for high- and low-brightness sources using the entire source list.

In order to consider a sample which is as complete as possible, only sources within the declination range $0° < \delta < 20°$ and $24° < \delta < 30°$ were considered. For the first range the Parkes Catalogue (DAY *et al.*, 1966) was used. It is complete at 408 MHz to 2.5 flux units, thus at 1425 MHz it will be deficient only for sources with a spectral index greater than -0.2. For the second range of declinations the CTD catalogue (KELLERMANN and READ, 1965) was used to compile the source list. Since it was a finding survey at the same frequency as these observations to a flux density of 1 flux unit, this range of declinations should be complete. The area within 15° of the galactic plane was excluded.

The resulting $\log N - \log S$ curves for this sample of sources is given in figure 2. There is a significant difference of slope for the two groups. A least-squares solution gives

$$N = 300\, S^{-2.16}, \qquad T_b > T_0 \quad (2)$$

$$N = 140\, S^{-1.44}, \qquad T_b < T_0 \quad (3)$$

for $S > 2$ flux units in 2.2 steradians. The standard error of the slope is 0.16 in each case. The slope difference is statistically significant at the one per cent level (excluding systematic effects).

The splitting of the two groups has, to a large extent, separated the quasars and the radio galaxies. This is shown in table 1. The sources comprising the low-

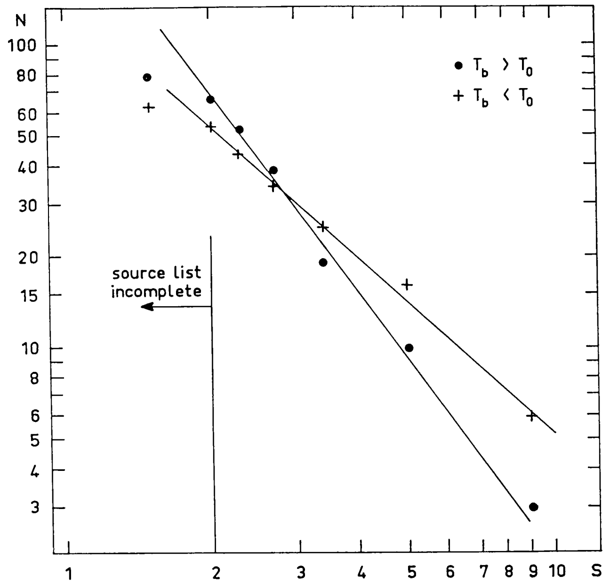

Figure 2. The log $N-$log S relationship for high- and low-brightness sources in the declination range 0° to 20° and 24° to 30°.The lines give the best power law fit.

brightness group are nearly all radio galaxies. Only one identified quasar is in this group and most of the unidentified sources probably are radio galaxies below the Palomar Sky Survey limiting magnitude of 19 or 20. Thus the low-brightness log $N-$log S relationship is that of radio galaxies. The slope of 1.44 ± 0.16 suggests that their distribution in space is homogeneous. The average redshift of the identified galaxies is 0.15 with a few redshifts of 0.6 expected for some of the unidentified sources.

TABLE 1

Identification of high- and low-brightness radio sources

Optical type	Number $T_b > T_0$	Number $T_b < T_0$
Quasars	18	1
Galaxies	16	39
Unidentified	31	13
Total	65	53

For the 65 high-brightness sources there are approximately equal numbers of quasars and radio galaxies. However, nearly half of the sources are unidentified. This is expected since high-brightness sources tend to be intrinsically more luminous than low-brightness sources (HEESCHEN, 1966) and are therefore farther away, making their optical identification more difficult. At the present time the predominant optical type of the unidentified sources is not known. Even in the extreme cases the ratio of quasars to galaxies will not be greater than three nor less than one-third for the high-brightness group. Thus the high-brightness sources contain an approximately equal number of quasars and radio galaxies.

It is likely that the high-brightness galaxies, taken alone, will give a slope no larger than 1.5. While they are somewhat more distant than the low-brightness galaxies, the volume of space surveyed with the high-brightness galaxies is not much larger than that for the low-brightness galaxies. Thus their slopes should not differ appreciably. In order to obtain the large slope of 2.16 for the high-brightness sources, the slope of the log $N-$log S relationship for the quasar component must be in excess of 2.5.

4. Discussion

There are several selection effects which should be discussed in order to make more certain that the slope splitting is real—not an observational effect, such as an incompleteness of the source list or the effect of redshift.

A comparison of figures 1 and 2 shows that even an incompleteness of 20 per cent can severely alter the log $N-$log S relation of the two groups because the incompleteness is related to the spectral index of the source which is correlated with the source brightness. In the restricted range of declinations 0° to 20° and 24° to 30° the completeness is nearly 95 per cent. At the present time only seven sources greater than 2.0 flux units have been discovered which were not in the original observing list. Since their radio structures are unknown at this time they cannot be included in the data. Their inclusion would hardly affect the results.

A more important effect is that due to the redshift of the radio sources. The value T_b defined in eq. (1) is no longer independent of distance if the redshift is related to the distance of radio sources, as is usually assumed. With redshift effects included the proper relation should be (MCVITTIE, 1965)

$$T_b \propto S \cdot d^{-2}/(1+z)^{3-\alpha}, \tag{4}$$

where z is the redshift and α is the spectral index of the source (usually negative). Thus the defined value of T_0 separating the two groups corresponds to a larger intrinsic brightness temperature for sources of large redshift in their rest-frame, so the larger the redshift the more likely it is that a source will be put in the low-brightness group.

There is good evidence that the fainter the source, the higher its redshift. This is clear for radio galaxies which follow a well-defined redshift-distance relation, and a similar trend is seen for the identified quasars (LONGAIR and SCHEUER, 1967). In this case, the redshift should produce a *deficiency* of high-brightness sources at low flux density levels. However, the slope difference suggests an *excess* of high-brightness sources for weak sources—in the opposite sense to that expected from the redshift alone.

5. Conclusion

These results suggest that radio galaxies are distributed homogeneously in space to a distance corresponding to a redshift of about 0.4. Quasars, which are nearly all of high brightness and consist of one-quarter to one-third of the radio sources above 2.0 flux units at 1425 MHz, are not distributed homogeneously. The slope defined by their $\log N - \log S$ relationship is in excess of 2.5. Redshift and incompleteness effects are not responsible for the slope difference of the low- and high-brightness sources. Since the sampled volume for galaxies is much smaller than for quasars, no comparison can be made between their space distribution.

There is good agreement with the results of VÉRON (1966) who used only the identification of radio sources. The two methods of analysis are independent since the sources here were divided by radio brightness temperature—the identifications were used to show only that the quasars and radio galaxies were separated well between the two brightness groups.

The derived slope of ≥ 2.5 for quasars is in good agreement, considering the statistical uncertainties, with the slope for quasars derived by SCHMIDT (1968) of $3/2 + A/(1+A)$, where A is the luminosity distance of a "typical" source in the sample. The value of A for identified sources is about 1 although it may be larger if many of the unidentified quasars have a redshift greater than 2. Thus the predicted slope for quasars would be in the range 2.0 to 2.5.

The method of separating sources into two brightness groups can be easily done experimentally by observations of a complete sample of radio sources with a large interferometer spacing. Sources which are not resolved (or only partially resolved if they are strong) are then of high brightness; the others low in brightness. Since the identification of radio sources is time consuming, ambiguous and difficult, especially for the weaker sources, the grouping by radio brightness temperature may be more useful in the investigation of the $\log N - \log S$ relationship of galaxies and quasars at low flux density levels.

Acknowledgements

I am grateful to Dr. H. van der Laan for helpful comments and criticisms. The observational program at Caltech is supported by the United States Office of Naval Research under Contract N00014-67-A-0094-0008, and the research was supported by a National Science Foundation Postdoctoral Fellowship.

References

A. S. BENNETT, 1962, *Mem. Roy. Astr. Soc.* **68** 163
G. A. DAY, A. J. SHIMMINS, R. D. EKERS and D. J. COLE, 1966, *Australian J. Phys.* **19** 35
E. B. FOMALONT, 1967, *Pub. Owens Valley Radio Obs.* **1** No. 3
E. B. FOMALONT, 1968, *Ap. J. Suppl.* **15** 203
D. S. HEESCHEN, 1966, *Ap. J.* **146** 517
A. HEWISH, 1962, *Mon. Not. Roy. Astr. Soc.* **123** 167
D. L. JAUNCEY, 1967, *Nature* **216** 877
K. I. KELLERMANN, 1964, *Ap. J.* **140** 969
K. I. KELLERMANN and R. B. READ, 1965, *Pub. Owens Valley Radio Obs.* **1** No. 2
M. S. LONGAIR, 1966, *Mon. Not. Roy. Astr. Soc.* **133** 421
M. S. LONGAIR and P. A. G. SCHEUER, 1967, *Nature* **215** 919
G. C. MCVITTIE, 1965, *General Relativity and Cosmology* (Univ. of Illinois Press) 181
M. SCHMIDT, 1968, *Ap. J.* **151** 393
P. VÉRON, 1966, *Nature* **211** 724