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Mixed shocks: spectral selection of the class of solutions

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Abstract. We investigate the influence of the Mach number on the acceleration of relativistic particles by stationary adiabatic mixed shocks assuming non-relativistic hydrodynamics. We solved the structure equations corresponding to adiabatic mixed shocks and we found a critical value $\Delta_c \simeq 50$ for the pressure jump through the shock, such as

1. when $\Delta > \Delta_c$: if the pressure of the relativistic particles is a very small fraction, i.e. greater or equal to 10^{-5} times the total pressure in the upstream flow, there are three solutions to the structure equations. Two solutions correspond to a downstream pressure dominated by a non-relativistic component and have an efficiency of about 10^{-3} and 10^{-2}, their radio spectral indices are $\alpha_r(1) \geq 0.5$ and $\alpha_r(2) \leq 0.5$ respectively. The third solution corresponds to a downstream pressure dominated by a relativistic component and has an efficiency greater than 0.5 and a radio spectral index $\alpha_r(3) \leq 0.5$. In the strong shock limit, observations of bow shocks in strong extragalactic radio sources and of supernova remnant shocks indicate that they correspond to the solutions with a downstream pressure dominated by a non-relativistic gas. The third solution, whose efficiency is unity, has never been observed. Moreover, the radio spectral index of radio supernovae which do not contain a pulsar corresponds to the radio spectral index of the solution (1), i.e. $\alpha_r \geq 0.5$.
2. when $\Delta < \Delta_c$: there are from one to three solutions depending on both the fraction of the relativistic gas in the upstream flow and the pressure jump.

Key words: acceleration mechanism – galaxies: radio

1. Introduction

The acceleration of cosmic rays by repeated first order Fermi process through a shock in a diffusive medium has been proposed by several authors (Axford et al. 1977; Krimsky 1977; Bell 1978; Blandford & Ostriker 1978). To take into account the back reaction of the cosmic rays on the flow, a nonlinear theory of the mixed shock structure has been performed by Drury & Völk (1981), Mc Kenzie & Völk (1982). Further developments of mixed shocks have been done by Axford et al. (1982), Lagage & Cesarsky (1983a,b), Völk et al. (1984), Achterberg et al. (1984), Eichler (1984), Webb et al. (1986), Pelletier & Roland (1986), Achterberg (1987), Pelletier & Roland (1988), Kang & Jones (1990), Fraix-Burnet & Pelletier (1991).

In a previous article, Pelletier & Roland (1986) developed an analytical determination of the structure function ϕ of mixed shocks and a spectral theory of mixed shocks parametrized by the ratio θ of the non-relativistic pressure p_c to the relativistic pressure p_r, namely $\theta = p_c/p_r$. In the following, we will denote upstream and downstream quantities with indices 1 and 2 respectively. In Pelletier & Roland (1986), the shock was supposed to be strong which means that the ratio Δ between the downstream pressure and the upstream pressure is infinite, i.e. $\Delta = (p_{c2} + p_{r2})/(p_{c1} + p_{r1}) \rightarrow \infty$. In other words, the Mach number M was taken to be infinite.

In this article we are studying the influence of a finite Mach number on the acceleration of relativistic particles by mixed shocks (i.e. we will suppose that $\Delta \geq 10$), and on the spectral index of the power law distribution of the relativistic particles. This is a key step toward an accurate determination of the structure and spectral index of magnetized mixed shocks. An important consequence of the theory is the tentative explanation of the Liu-Laing-Garrington effect (Roland et al. 1992).

Moreover, we will assume that $p_{r1} \neq 0$, namely we will suppose that $\theta_1 \leq 10^5$. So, we do not take into account the possibility of an injection of relativistic particles at the shock front. In fact, some thermal electrons can be accelerated by whistler waves and reach the threshold for resonant interaction with Alfven waves. Apart from the usual threshold for resonant scattering, there is an additional condition for particle acceleration by the first order Fermi process through the shock: the incident velocity of the particle v_p must be much greater than the bulk velocity v of the plasma flow. Precisely, it must satisfy:

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$v_p \cos\phi \gg v$, where ϕ is the angle between the magnetic field lines and the shock normal (obliquity effects have been analysed in detail by Drury 1983). That kind of acceleration clearly does not work if the shock is nearly perpendicular, with $\cos\phi < v/c$. In quasi-perpendicular shocks, with $v/c < \cos\phi < \sqrt{2}/2$, acceleration occurs but injection of thermal particles is quite impossible. In quasi-parallel shocks, some particles in the tail of the thermal distribution could be accelerated if the Mach number is not too high. When the previous strong inequality is fulfilled, a transmitted particle has a probability close to one of coming back and cross the shock upstream. Concerning the question of relativistic particles acceleration assuming $p_{r1} = 0$, see Kang & Jones (1990) and references therein.

We will neglect the second order Fermi process which has no influence in the subshock and does not modify the distribution function just after the shock. However, inside the hot spots in situ reacceleration cannot be neglected (Roland et al. 1988), i.e. it will change the distribution function in the downstream flow.

In the next section we will remind the basic assumptions and the equations of mixed shocks. In Sect. 3., we will study the influence of a finite Mach number for the acceleration of relativistic particles by mixed shocks and we will discuss the general properties of the structure function corresponding to stationary adiabatic mixed shocks. In Sect. 4., we will calculate the spectral index of the distribution of the relativistic particles behind the shock with the help of the previously computed structure function and we will select the different classes of solutions.

2. Basic assumptions and equations of mixed shocks

2.1. Basic assumptions

Throughout the article, we will suppose that,

1. the shock is a mixed shock, i.e. the ratios θ_1 and θ_2 are arbitrary.
2. the shock is adiabatic.
3. the relativistic particles are accelerated in the precursor with width δ_r and the thermal particles are heated in the subshock with width $\delta_c \ll \delta_r$. We define a ratio δ by $\delta \equiv \delta_c/\delta_r = \kappa_c/\kappa_r$, where κ_c and κ_r are the diffusivity of the thermal and relativistic particles respectively. The parameter δ is much less than unity but cannot be made equal to zero in order to calculate the spectral index of the relativistic particles distribution behind the shock.
4. when losses are disregarded and when the flow is unidirectional the energy distribution $\rho_r(E, x, t)$ of the relativistic particles is governed by the following equation (Skilling 1975; Ginzburg & Ptuskin 1985)
$$\frac{\partial \rho_r}{\partial t} + u\frac{\partial \rho_r}{\partial x} = \frac{1}{3}\frac{\partial u}{\partial x}E^3\frac{\partial}{\partial E}(\frac{\rho_r}{E^2}) + \frac{\partial}{\partial x}(D_r\frac{\partial \rho_r}{\partial x}) \quad (1)$$
where u is the velocity of the scattering centers and E is the energy of the relativistic particles. We will disregard the energy dependence of the diffusion coefficient D_r and thus the diffusivity κ_r is $\kappa_r = \left(\int \rho_r(E) D_r E dE\right)/\int \rho_r(E) E dE \simeq D_r$.
5. the oscillations of the shock are small enough to consider the quantities as averaged ones. Then the transition is monotonic, and we define the structure function ϕ of the mixed shock such that the velocity u is the solution of a differential equation of the form
$$\phi(u) = \kappa_r\frac{\partial u}{\partial x} \ . \quad (2)$$

If the turbulence level is not uniform in the precursor, κ_r depends on x. However, this would introduce a smooth dependence of κ_r in term of u in Eq. (2), which we will disregard.

The function ϕ is derived from the hydrodynamical equations and is always negative. It describes the structure of the mixed shock and is an essential ingredient in order to calculate the spectral index of the associated solution. Note that if the subshock oscillates, in the limit $\delta \to 0$ the detailed structure of the subshock is unimportant. The stationary equation, using the variable u instead of the space variable x, is then
$$\frac{E^3}{3}\frac{\partial}{\partial E}(\frac{\rho_r}{E^2}) - u\frac{\partial \rho_r}{\partial u} + \frac{\partial}{\partial u}(\phi\frac{\partial \rho_r}{\partial u}) = 0 \ . \quad (3)$$

In this article, we will suppose that the Alfvenic Mach number is infinite, so that the magnetic field does not contribute to the hydrodynamical transport across the shock. Thus the velocity u of the scattering centers is identical to the velocity v of the flow. We will investigate the influence of the Alfvenic Mach number in a future article, especially through the change on the structure function leading to significant corrections to the solutions $\theta_2(\theta_1)$ and to the spectral index (Lehoucq & Roland 1992).

2.2. Basic equations

2.2.1. The Rankine-Hugoniot relations and the compression ratio

When the Mach number is great enough, say $M \geq 2$, the shock is called a turbulent shock, and even if $T_{e2} < T_{i2}$, the downstream distributions are isotropized and quasi-Maxwellian (T_{e2} and T_{i2} are respectively the temperature of the thermal electrons and of the thermal protons in the downstream flow). Thus we can write the Rankine-Hugoniot relations, i.e. the conservations of the mass flux, of the momentum flux and of the energy flux (Tidman & Krall 1971). We will suppose that the shock is turbulent and denoting by ρ the mass density, we have
$$J = \rho_1 v_1 = \rho_2 v_2 \ , \quad (4)$$
$$F = \rho_1 v_1^2 + p_{c1} + p_{r1} = \rho_2 v_2^2 + p_{c2} + p_{r2} \ , \quad (5)$$
$$\frac{v_1^2}{2} + \frac{1}{\rho_1}(\frac{\gamma_r p_{r1}}{\gamma_r - 1} + \frac{\gamma_c p_{c1}}{\gamma_c - 1}) = \frac{v_2^2}{2} + \frac{1}{\rho_2}(\frac{\gamma_r p_{r2}}{\gamma_r - 1} + \frac{\gamma_c p_{c2}}{\gamma_c - 1}) \ , \quad (6)$$
where $\gamma_r = 4/3$, $\gamma_c = 5/3$.

2.2.2. The compression ratio

Let r be the compression ratio of the shock, defined by $r = \rho_2/\rho_1 = v_1/v_2$. Equations (5), (6) and the definition of the pressure jump Δ lead to the compression ratio

$$r = \frac{\frac{7+4\theta_2}{1+\theta_2} + \frac{1}{\Delta}}{1 + \frac{1}{\Delta}\frac{7+4\theta_1}{1+\theta_1}} . \tag{7}$$

Defining the square of the Mach number by

$$M^2 \equiv Jv_1/(p_{c1} + p_{r1}) , \tag{8}$$

leads to a relation between Δ and M:

$$M^2 = (\Delta - 1)\frac{r}{r-1} . \tag{9}$$

Equations (7) and (9) give the compression ratio of a mixed shock for arbitrary values of θ_1, θ_2, Δ or M.

2.2.3. The structure equations

From the definition of the relativistic pressure, namely $p_r = (1/3)\int \rho_r(E)E\mathrm{d}E$ and from Eq. (3), it is possible to derive the equation governing the pressure of the relativistic particles

$$\gamma_r p_r + v\frac{\mathrm{d}p_r}{\mathrm{d}v} = \frac{\mathrm{d}}{\mathrm{d}v}(\phi\frac{\mathrm{d}p_r}{\mathrm{d}v}) . \tag{10}$$

We will describe the evolution of the pressure of the thermal component by the same kind of equation, i.e.

$$\gamma_c p_c + v\frac{\mathrm{d}p_c}{\mathrm{d}v} = \frac{\mathrm{d}}{\mathrm{d}v}(\delta\phi\frac{\mathrm{d}p_c}{\mathrm{d}v}) , \tag{11}$$

where $\delta \ll 1$. This equation describes a monotonic transition of the averaged velocity field in the subshock that satisfies the jump conditions ; it properly accounts for the heating of the thermal component on a prescribed width δ_c very short compared to δ_r (Roland et al. 1988).

The description of mixed shocks, i.e. the determination of ϕ, p_c and p_r can be obtained by adding the equation

$$F = Jv + p_c + p_r . \tag{12}$$

Hereafter, Eqs. (10), (11) and (12) will be named "the structure equations".

3. The influence of the Mach number

The system depends on four input parameters, namely θ_1, θ_2, δ and Δ or M. The parameter δ, which is much less than unity, has been fixed to 10^{-2}. This value is reasonable for mixed shocks responsible for the formation of hot spots in extragalactic radio sources (see Roland et al. 1988 in the case of Cygnus A).

We solved the system using the pressure jump Δ instead of the Mach number. We introduced the dimensionless variable $X \equiv v/v_2$ with $X \in [1, r]$, and defined the dimensionless functions $Y \equiv p_c/Jv_2$ and $Z \equiv p_r/Jv_2$. We used a 2–stage implicit Runge–Kutta method (Butcher 1964). For given values of θ_2 and Δ, we have to determine the parameter θ_1, such that $\phi(r) = 0$ and $Y(r) + Z(r) = r/M^2$. Then, we have checked that $Y(r)/Z(r) = \theta_1$.

We started the calculation at $X = v/v_2 = 1$, with the initial conditions $\phi(1) = 0$, $\theta_2 = Y(1)/Z(1)$, $Y(1) + Z(1) = r(1 + 1/M^2) - 1$ and used the definition of the Mach number given by Eq. (9). The derivative $\phi'(1)$ is obtained from Eqs. (10) and (11). We obtained the solutions corresponding to the structure equations for various values of Δ. For a given value of Δ we obtained a curve $\theta_2(\theta_1)$. The solutions are plotted in Fig. 1.

We drew the line $\theta_2 = \theta_1$ because the solutions with $\theta_1 \geq \theta_2$ correspond to mixed shocks which accelerate efficiently the relativistic particles, whereas those with $\theta_1 \leq \theta_2$ are not able to do so. In the following, we will consider and discuss only the solutions with $\theta_1 \geq \theta_2$.

In order to estimate the acceleration efficiency of the different solutions, we will define the efficiency ϵ of the solutions by

$$\epsilon = \frac{p_{r2}}{p_{c2} + p_{r2}} = \frac{1}{1+\theta_2} . \tag{13}$$

3.1. The stationary solutions

3.1.1. The pressure jump is greater than the critical value ($\Delta > \Delta_c$)

From Fig. 1, one finds that there is a critical value for the pressure jump, namely $\Delta_c \simeq 50$. The corresponding critical Mach number is $M_c \simeq 7.7$ for $\theta_2 \simeq 1$.

If the pressure jump is greater than the critical value, i.e. if the Mach number is greater than about 7.7, there exist generally several stationary solutions if $\theta_1 \leq 10^5$, i.e. even if the pressure of the relativistic particles is very small compared to the thermal pressure in the upstream flow.

For a given value of θ_1 greater than about 300, there are three solutions for each value of θ_2. Two solutions correspond to $\theta_2 > 1$ and the other to $\theta_2 < 1$. Let us note the behavior of these three solutions θ_2, in the strong shock limit (i.e. $M \to \infty$):

1. solution (1), with $\theta_2 > 1$, $\theta_2(\theta_1 = 10^5) \to 961$.
2. solution (2), with $\theta_2 > 1$, $\theta_2(\theta_1 = 10^5) \to 48.4$.
3. solution (3), with $\theta_2 < 1$, $\theta_2(\theta_1 = 10^5) \propto 1/M^2 \to 0$.

When $10 \leq \theta_1 \leq 300$, there are only the solutions (2) and (3), and when $\theta_1 \leq 10$, the solution (3) remains alone.

3.1.2. The pressure jump is smaller than the critical value ($\Delta < \Delta_c$)

When the pressure jump becomes smaller than the critical value or when $M < M_c \simeq 7.7$ for $\theta_2 \simeq 1$, the situation becomes more complicated, and one has to distinguish several intervals of θ_1 and Δ to find solutions to the structure equations.

1. The solution (1) exists if $\theta_1 \geq 300$ for all the values of Δ.

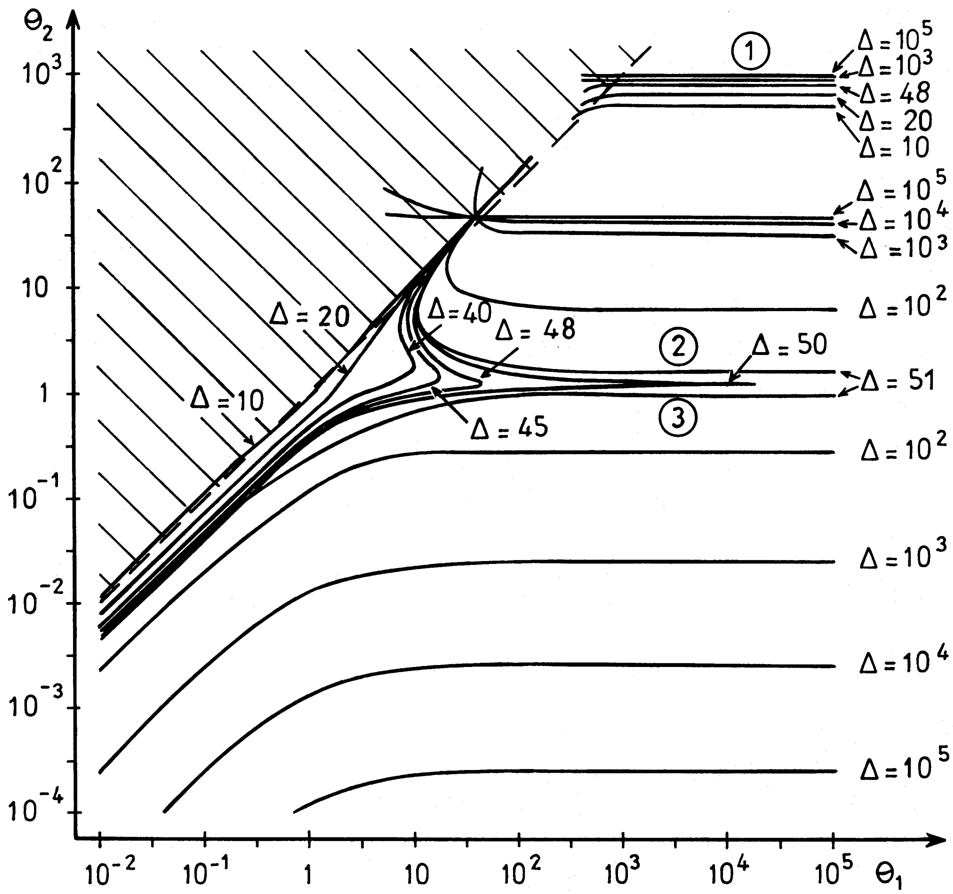

Fig. 1. The solutions $\theta_2(\theta_1)$ corresponding to stationary adiabatic mixed shocks for various values of the pressure jump Δ. We drew the line $\theta_2 = \theta_1$, the hatched area, where $\theta_1 \leq \theta_2$, corresponds to mixed shocks which does not accelerate efficiently relativistic particles.

2. If $30 \leq \theta_1 \leq 10^4$ and $45 \leq \Delta \leq 50$ there are two solutions which are different from solution (1).
3. If $\theta_1 \simeq 20$ and $35 \leq \Delta \leq 45$, there are three solutions different from solution (1).
4. If $10 \leq \Delta \leq 30$ and $\theta_1 \leq 100$ there is only one solution different from solution (1).

It was already found by Drury & Völk (1981) that there exist several solutions to the structure equations when the Mach number is greater than a critical value.

3.2. Influence of the parameter δ

We found the solutions $\theta_2(\theta_1)$ for various values of the pressure jump Δ assuming that $\delta = 10^{-2}$. However, when this parameter varies, there are several interesting remarks concerning the solutions $\theta_2 > 1$.

3.2.1. Behavior of the solution (1)

The solution (1) moves upward slowly when δ decreases. For instance, when $\Delta = 10^5$, $\theta_2(\theta_1 = 10^5) \simeq 687$ if $\delta = 3 \times 10^{-2}$, $\theta_2(\theta_1 = 10^5) \simeq 961$ if $\delta = 10^{-2}$ and $\theta_2(\theta_1 = 10^5) \simeq 1100$ if $\delta = 3 \times 10^{-3}$. The variations of the solution (1) for different δ are not the same for different Δ. Indeed, when $\Delta = 10$, $\theta_2(\theta_1 = 10^5) \simeq 390.4$ if $\delta = 3 \times 10^{-2}$, $\theta_2(\theta_1 = 10^5) \simeq 518.3$ if $\delta = 10^{-2}$ and $\theta_2(\theta_1 = 10^5) \simeq 586.86$ if $\delta = 3 \times 10^{-3}$.

3.2.2. Behavior of the solution (2) in the strong shock limit

When $M \to \infty$, $\theta_2(\theta_1 = 10^5) \propto 1/\delta$. For instance, when $\Delta = 10^5$, $\theta_2(\theta_1 = 10^5) \simeq 15.25$ if $\delta = 3 \times 10^{-2}$, $\theta_2(\theta_1 = 10^5) \simeq 48.4$ if $\delta = 10^{-2}$, and $\theta_2(\theta_1 = 10^5) \simeq 162.6$ if $\delta = 3 \times 10^{-3}$. Thus in the strong shock limit, this numerical computation confirms the law obtained analytically by Pelletier & Roland (1986), namely $\theta_2 = 1/(2\delta) - 16/15$. In the strong shock limit, the different values $\theta_2 > 1$ can be obtained with different values of δ.

3.2.3. The critical value Δ_c

If $\delta \leq 0.01$, the critical value Δ_c does not depend very much on δ. Indeed, if $\delta = 3 \times 10^{-3}$ we have $\Delta_c \simeq 48$, but if $\delta = 3 \times 10^{-2}$ we find $\Delta_c \simeq 58$.

3.3. The structure functions of the different solutions

Pelletier & Roland (1986) found an approximate analytical solution of the structure function ϕ of mixed shocks. Their result can be summarized as follow:

The structure function of adiabatic mixed shocks can be approximated by two pieces of parabola, i.e.

1. when $u_2 \leq u \leq u_0$, $\phi \simeq \phi_c \simeq a_c(u - u_1^*)(u - u_2)$ where $a_c = (\gamma_c + 1)/(2\delta)$ and $1 \leq u_1^* \leq 4$,
2. when $u_0 \leq u \leq u_1$, $\phi \simeq \phi_r \simeq a_r(u - u_1)(u - u_2^*)$ where $a_r = (\gamma_r + 1)/2$ and $u_2^* \simeq u_1/7$.

The deepest parabola ϕ_c corresponds to the subshock while the other ϕ_r corresponds to the precursor.

As $\phi(u)$ vanishes at u_2 and u_1, it is convenient to define a function $g(u)$ by

$$\phi(u) = g(u)(u - u_1)(u - u_2) . \tag{14}$$

Let us note that in the two extreme cases $\theta_2 = 0$ and $\theta_2 \to \infty$, the structure function is given by a single parabola. Indeed,

1. when $\theta = 0$, we have $p_c \equiv 0$ and the structure equations reduce to Eq. (10) and $F = Jv + p_r$. Consequently, the structure function is given by
$$\phi \equiv \phi_r = a_r(u - u_1)(u - u_2) , \tag{15}$$
where $a_r = (\gamma_r + 1)/2$.
2. when $\theta \to \infty$, we have $p_r \equiv 0$ and the structure equations reduce to Eq. (11) and $F = Jv + p_c$. Then, one can verify that the structure function is again a parabola given by
$$\phi \equiv \phi_c = a_c(u - u_1)(u - u_2) , \tag{16}$$
where $a_c = (\gamma_c + 1)/(2\delta)$.

3.3.1. The structure functions of the three solutions in the strong shock limit

We plotted in Figs. 2 to 7, the functions $\phi(u)$ and $g(u)$ corresponding to the solutions (3), (2) and (1) respectively computed for $\Delta = 10^5$ and $\theta_1 = 10^4$.

i) The solution (3) with $\theta_2 \simeq 0.00261$ can be called a smooth transition without subshock. The structure function $\phi(u)$ corresponds mainly to ϕ_r given by Eq. (15) and the function $g(u)$ is mainly constant and equal to $(\gamma_r + 1)/2 \simeq 1.167$. It is when $u \simeq u_2$, that one observes a small variation of $g(u)$.

ii) The solution (2) with $\theta_2 \simeq 48.4$, corresponds to a mixed shock and is well represented by two pieces of parabola. Note that $g(u_1) \simeq (\gamma_r + 1)/2 \simeq 1.167$ as predicted by Pelletier & Roland (1986), this corresponds to $u_2^* \simeq u_1/7$, and $g(u_2) \simeq 131 \leq (\gamma_c + 1)/(2\delta) \simeq 133.3$.

iii) The solution (3) with $\theta_2 \simeq 961$ corresponds mainly to a pure subshock described by Eq. (16). Indeed, $g(u_2) \simeq 133.3$. However, it is not represented by a single parabola, especially when $u \simeq u_1$, there is still a small piece of parabola describing the precursor and one has $g(u_1) \simeq 1.333$.

As previously indicated, it is only when $p_r \equiv 0$ that $\phi \equiv \phi_c$ and $g(u) \equiv a_c$.

3.3.2. The structure function of the three solutions for $\Delta = 51$

When Δ becomes close to the critical value Δ_c, it is interesting to compare the structure functions with those found in the strong shock limit.

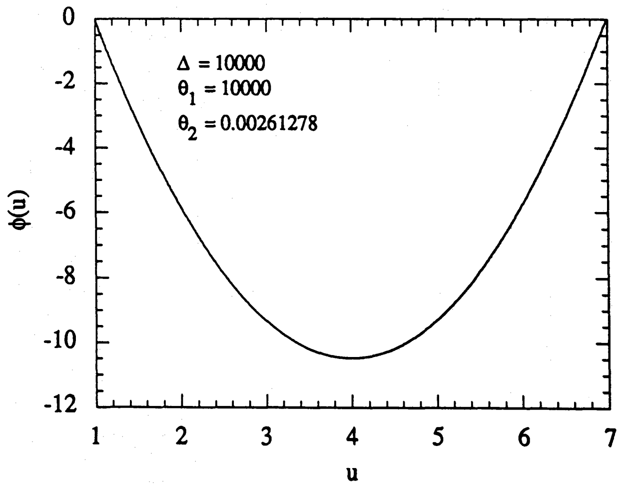

Fig. 2. The structure function $\phi(u)$ of the solution (3)

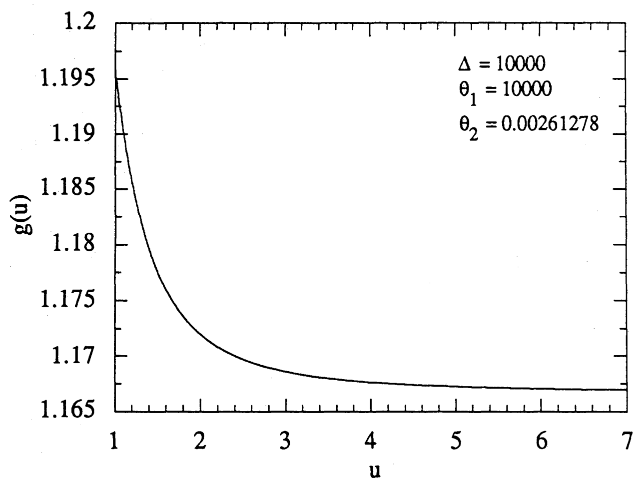

Fig. 3. The function $g(u)$ of the solution (3)

To do so, we plotted in Figs. 8 to 13, the functions $\phi(u)$ and $g(u)$ of the three solutions, assuming $\theta_1 = 10^4$ and $\Delta = 51$.

If there are important changes with the strong shock limit case, the two parabola approximation remains valid. For the three solutions, when $u \simeq u_1$, $\phi \simeq \phi_r$, i.e. $g(u_1) = 1.167$. This indicates that we still have $u_2^* \simeq u_1/7$.

However, when $u \simeq u_2$, only the solution (1) has $g(u_2) \simeq (\gamma_c+1)/(2\delta)$, because it corresponds to $\theta_2 \gg 1$. For the solutions (2) and (3), one has $\theta_2 \simeq 1$ and then $g(u_2) \leq 80$. Moreover, contrarily to the strong shock limit case, $g(u)$ has important variations when $u_2 \leq u \leq u_0$.

Finally, the solution (3) is well represented by two pieces of parabola instead of one in the strong shock limit.

4. Spectral theory of mixed shocks

On the one hand, mixed shocks in collisionless plasma are able to accelerate relativistic particles and on the other hand, they provide power law distributions of the accelerated particles in

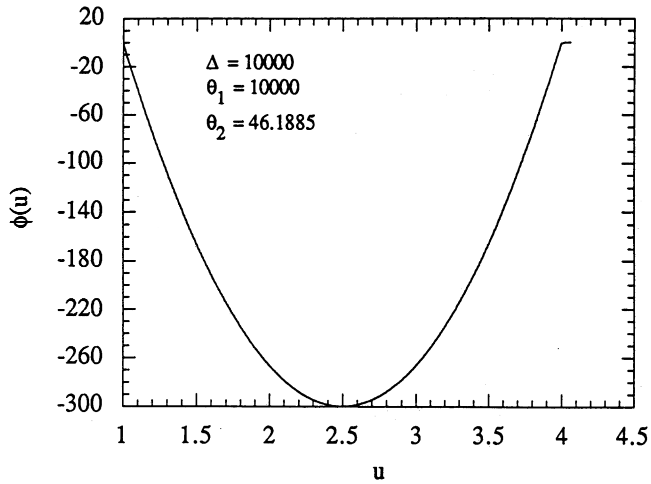

Fig. 4. The structure function $\phi(u)$ of the solution (2)

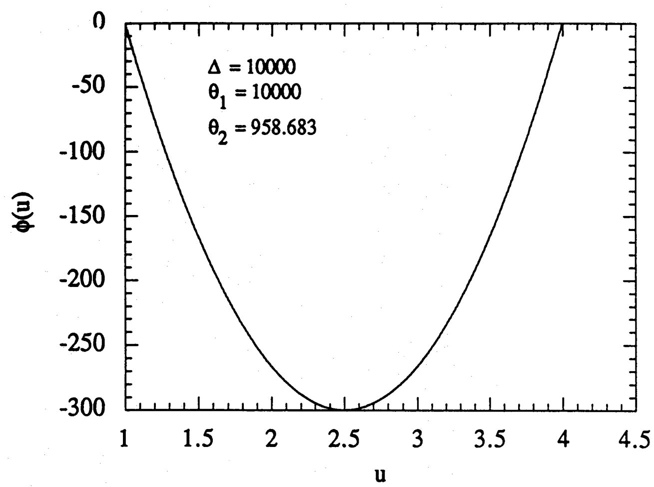

Fig. 6. The structure function $\phi(u)$ of the solution (1)

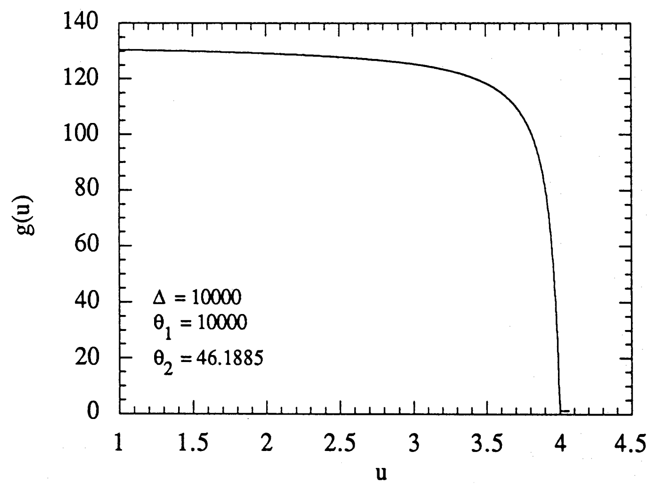

Fig. 5. The function $g(u)$ of the solution (2)

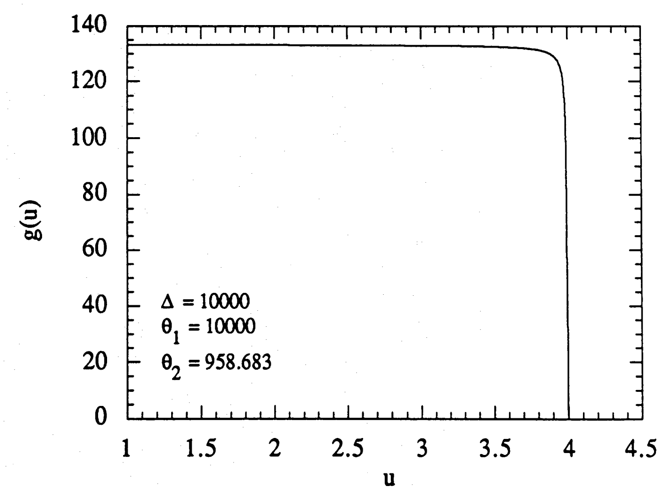

Fig. 7. The function $g(u)$ of the solution (1)

the downstream flow, i.e. at high energy the distribution $\rho_{\rm r}$ of the relativistic particle can be written

$$\rho_{\rm r} \propto E^{-\eta}\,, \tag{17}$$

where E is the energy of the relativistic particles.

It has been shown by Pelletier & Roland (1986), how it is possible to calculate the spectral index η when the structure function ϕ is known. Thus, we will use the numerical determination of the structure function that we obtained in the previous section to determine the spectral index of the different solutions.

4.1. Determination of the spectral index

As shown in Pelletier & Roland (1986), the spectral index is given by

$$\eta = \frac{3r}{r-1}\left(1+\frac{1}{a(r-1)}\right) - 2\,, \tag{18}$$

with

$$a = \frac{\displaystyle\int g(u)(u-u_2)^{\alpha}(u_1-u)^{\beta}\mathrm{d}u}{\displaystyle\int (u-u_2)^{\alpha}(u_1-u)^{\beta}\mathrm{d}u}\,, \tag{19}$$

where $g(u)$ is related to the structure function by Eq. (14), $\alpha = 1/[a(r-1)]$, $\beta = r/[a(r-1)]$ and r is the compression ratio given by Eq. (7).

4.2. Results

In order to compare with radio observations, we calculate the radio spectral index $\alpha_{\rm r}$ related to the particle distribution spectral index η by

$$\alpha_{\rm r} = \frac{\eta - 1}{2}\,. \tag{20}$$

4.2.1. $\Delta > \Delta_{\rm c}$

a) Solution (1)

The radio spectral index is $\alpha_{\rm r} > 0.5$. The parameter a from Eq. (18) is $a \simeq 132$, independently of Δ and θ_1. Then, the radio spectral index is

$$\alpha_{\rm r}(1) \simeq \frac{3}{2(r-1)}\,, \tag{21}$$

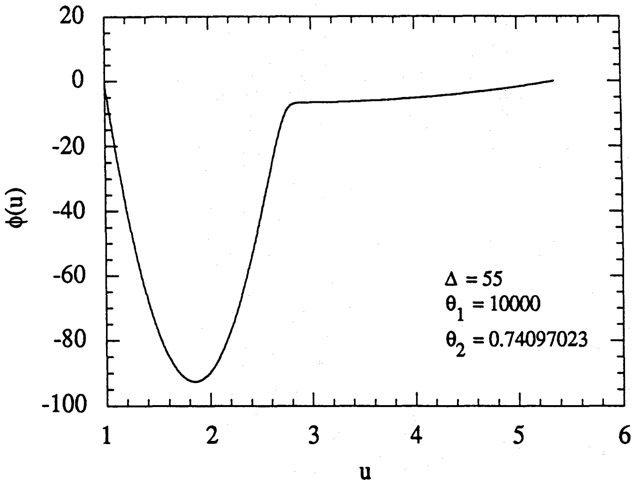

Fig. 8. The structure function $\phi(u)$ of the solution (3)

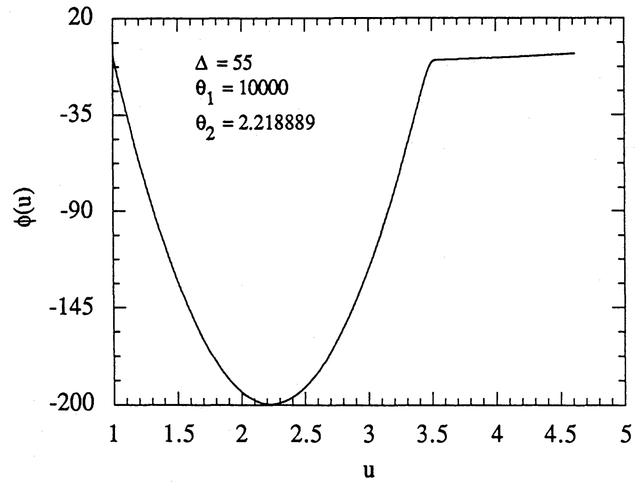

Fig. 10. The structure function $\phi(u)$ of the solution (2)

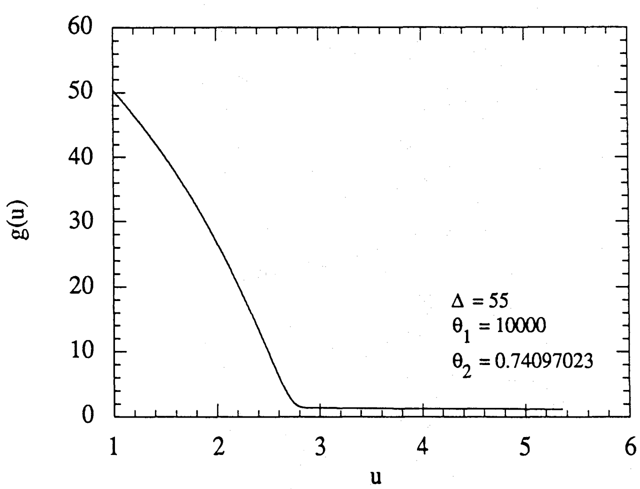

Fig. 9. The function $g(u)$ of the solution (3)

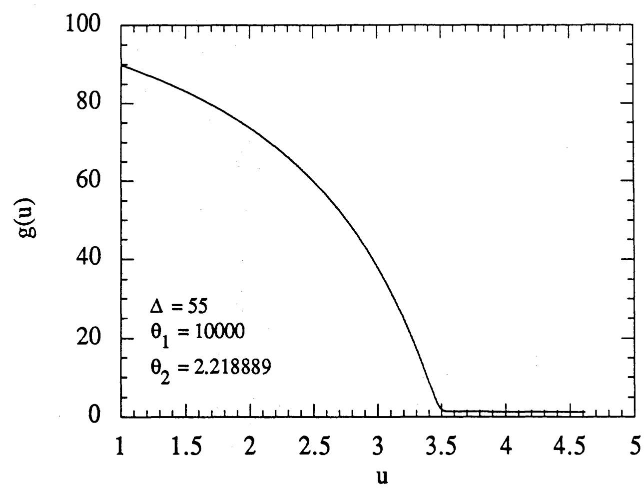

Fig. 11. The function $g(u)$ of the solution (2)

where r is the compression ratio given by Eq. (7). The results corresponding to solution (1) are presented in Table 1. In the strong shock limit, i.e. when $\Delta \to \infty$, $\alpha_r \to 0.50^+$ and $a \to 132.35 \simeq (\gamma_c + 1)/(2\delta)$ as expected in the case of the purely thermal shock.

Table 1. $\alpha_r(1)$ for $\theta_1 \geq 500$.

Δ	10^5	10^3	10^2	55	40	20	10
$\alpha_r(1)$	0.505	0.51	0.53	0.55	0.57	0.64	0.79

b) Solution (2)

The radio spectral index is $0.40 \leq \alpha_r(2) < 0.5$. As long as $\theta_1 \geq 100$, $\alpha_r(2)$ does not depend on θ_1 and when $\theta_2 \simeq 10$, $\alpha_r(2) \simeq 0.50$.

The parameter a from Eq. (18) is $30 \leq a \leq 120$, indicating that the non-linear effects in the structure equations are not negligible for solution (2). In the strong shock limit, i.e. when $\Delta \to \infty$, $\alpha_r \to 0.50^-$.

The results corresponding to solution (2) are presented in Table 2.

Table 2. $\alpha_r(2)$ for $\theta_1 \geq 100$.

θ_1 \ Δ	10^5	10^4	10^3	10^2	55	51
10^4	0.495	0.495	0.493	0.467	0.42	0.41
10^2	0.495	0.495	0.495	0.471	0.43	0.42

c) Solution (3)

When $\theta_1 > 1$, the radio spectral index is $0.37 \leq \alpha_r(3) \leq 0.5$. As long as $\theta_1 \geq 100$, $\alpha_r(3)$ does not depend on θ_1 and when $\theta_1 \leq 1$, $\alpha_r(3) \geq 0.50$.

The parameter a from Eq. (18) is $1.167 \leq a \leq 19$, indicating that non-linear corrections are essential for solution (3).

In the strong shock limit, i.e. when $\Delta \to \infty$, $\alpha_r \to 0.50$ and $a \to 1.167 \simeq (\gamma_r + 1)/2$ as expected in the case of the purely radiative shock (Drury et al. 1982).

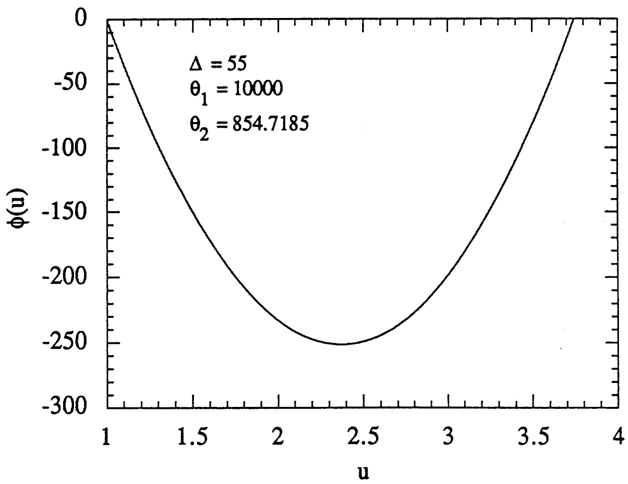

Fig. 12. The structure function $\phi(u)$ of the solution (1)

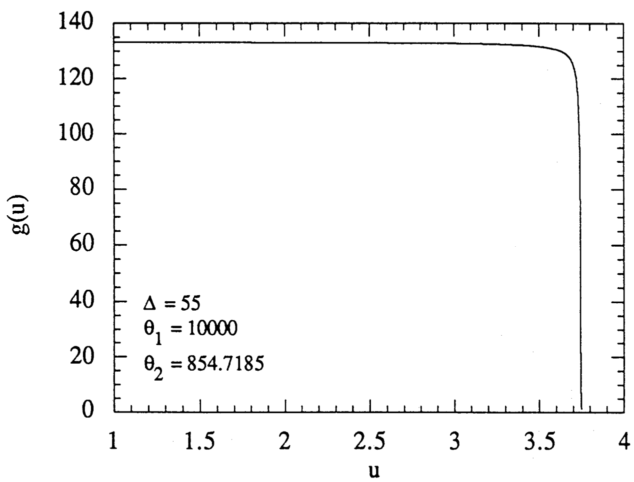

Fig. 13. The function $g(u)$ of the solution (1)

The results corresponding to solution (3) are presented in Table 3.

Table 3. $\alpha_r(3)$ for $\theta_1 \geq 1$.

θ_1 \ Δ	10^5	10^4	10^3	10^2	55	51
10^4	0.500	0.500	0.500	0.405	0.377	0.388
10^2	0.500	0.500	0.500	0.407	0.377	0.385
1	0.500	0.500	0.502	0.504	0.473	0.467

4.2.2. $\Delta < \Delta_c$

When $\theta_2 \simeq 1$, $\alpha_r < 0.50$ if $20 \leq \Delta < \Delta_c$. However, when $\Delta = 10$ and $\theta_2 \simeq 1$, one has $\alpha_r \simeq 0.62$. More generally, $\alpha_r > 0.50$ in the area where $\theta_2 \geq \theta_1$. When $\theta_2 \geq 5$ and $\theta_2 \leq 0.2$, $\alpha_r > 0.50$.

The results corresponding to $45 \leq \Delta \leq 50$ are presented in Table 4.

Table 4. α_r for Δ = 50, 45 and $\theta_1 \geq 10$.

θ_1	θ_1	θ_2	α_r
Δ = 50	1000	1.3721	0.403
	1000	1.1906	0.396
	10.219	5.623	0.489
	10	0.6682	0.382
Δ = 45	10	7.8983	0.511
	10	3.1903	0.461
	10	0.8789	0.391

When there are several solutions θ_2 for a given value of θ_1, the solutions with the smaller θ_2 have the flatter spectral indices.

4.3. Selection of the different solutions

The different solutions obtained describe adiabatic mixed shocks. However, all these solutions do not occur in the nature and it is possible to compare their efficiencies and their spectral indices with those of the observed collisionless adiabatic shocks.

Assuming $\theta_1 = 10^5$, in the strong shock limit, i.e. for $\Delta = 10^5$, the efficiencies, given by Eq. (13) and the radio spectral indices of the three solutions are:

- Solution (1): $\epsilon(1) \simeq 1/\theta_2 \simeq 10^{-3}$ and $\alpha_r \geq 0.50$,
- Solution (2): $\epsilon(2) \simeq 2 \times 10^{-2}$ and $\alpha_r \leq 0.50$,
- Solution (3): $\epsilon(3) \simeq 1 - \theta_2 \simeq 0.999$ and $\alpha_r \leq 0.50$ when $\theta_1 \geq 1$.

Observations of extragalactic radio sources indicate that the efficiency of the acceleration is low when the Mach number is very large. This effect can be observed in the case of the acceleration due to the bow shock in front of the hot spots associated with the extended lobes of asymmetrical powerful radio sources similar to 3C 263.

The radio source 3C 263 is characterized by asymmetrical hot spots and extended lobes (Leahy et al. 1989), i.e.

1. one lobe contains a strong hot spot and the other lobe without hot spot is weaker,
2. the weaker lobe is more distant from the galaxy than the strong lobe.

The advance speed, v_{ad}, of the hot spots in the intergalactic medium is

$$v_{ad} \propto 1/n_{ex}^{1/2}, \tag{22}$$

where n_{ex} is the density of the intergalactic medium.

In the case of Cygnus A, the external density is $n_{ex} \simeq 5 \times 10^{-3}$ e$^-$ cm^{-3} and the advance speed is $v_{ad} \simeq 0.05$ c. Thus, for a radio source with the same characteristics as those of Cygnus A, when the external density is $n_{ex} \leq 10^{-4}$ e$^-$ cm^{-3}, the advance speed becomes $v_{ad} \simeq 0.35$ c $\simeq v_j$, and the strong shock at the end of the jet disappears. Only the bow shock accelerates the relativistic electrons. The sound speed in the intergalactic medium is

$$c_{s,ex} \simeq 1.7 \times 10^4 \, T_{ex}^{1/2} , \quad (23)$$

If $T_{ex} \simeq 5 \times 10^7 \, K$, $c_{s,ex} \simeq 1.2 \times 10^8 \, \mathrm{cm\,s^{-1}}$. Then, the Mach number of the bow shock is $M_{bs} \simeq 88$ and the pressure jump through it, is $\Delta_{bs} \simeq 7700$.

Then, if the two jets have the same characteristics, i.e. the same speeds, mass ejected and Mach numbers, an asymmetry of the density of the intergalactic medium produces an asymmetrical radio source similar to 3C 263 and one observes that the weak lobe is more distant from the nucleus than the strong lobe. The asymmetry of the intergalactic medium has been recently found by Liu & Pooley (1991) in the case of 3C 299 which has a morphology similar to that of 3C 263.

Radio observations show that the extended lobe is weak when the hot spot disappears. This indicates that, in the strong shock limit, the efficiency of the acceleration due to the bow shock corresponds to the solutions $\theta_2 > 1$ and not to the solution $\theta_2 < 1$. Indeed, if it was the case, the solution $\theta_2 < 1$ will correspond to $\theta_2 \simeq 1/M_{bs}^2$ and the resulting extended lobe will be bright, which is not the case.

A similar result is obtained from the observation of the efficiency of the acceleration of the shocks due to the interaction of supernova remnants with the interstellar medium (Blandford & Eichler 1987). More generally, observations of strong shocks in collisionless plasmas indicate that ϵ is always much less than unity.

This result is not surprising, because in the strong shock limit, the solution $\theta_2 < 1$ assumes that the kinetic energy of the thermal protons in the jet is converted into the energy of the relativistic particles mostly without heating the thermal component. In the strong shock limit, the solution (3) corresponds to a shock with the smallest entropy increase and has never been observed.

The first consequence, when we describe the formation of the hot spots in extragalactic radio sources, is the importance of the thermal pressure in the downstream flow inside the hot spots. Indeed, it has been shown by Pelletier & Roland (1986) and Muxlow et al. (1988) that the formation of Cygnus A hot spots can be explained by the interaction of a non-relativistic supersonic jet with the intergalactic medium, if the speed of the jet powering the hot spots has a velocity $v_j \leq 0.45$ c. A similar result has been found for the jet powering hot spots of seven radio sources by Meisenheimer et al. (1989). In that case, one has $\theta_2 > 1$ inside the hot spots, i.e. the downstream pressure inside the hot spots is dominated by the pressure of the non-relativistic component.

As it is observed in collisionless shock waves, the temperature of the thermal protons is greater than the temperature of the thermal electrons. When the speed of the jet is non-relativistic, i.e. $v_j \simeq 0.35$ c, it has been found for Cygnus A hot spots that the dominant pressure is the pressure of the non-relativistic protons, whose mean energy per particle is $E_{th,p} \simeq 8.5$ MeV. The mean energy per thermal electrons is $E_{th,e} \simeq 1.2$ MeV, so they are mildly relativistic (Roland et al. 1988; Lesch et al. 1989).

Assuming that the magnetic field is quasi-perpendicular to the jet, we have $\beta_2 = p_{par}/p_m \simeq 2.5$, where p_{par} and p_m are respectively the pressure of the particles and the magnetic pressure. The corresponding Alfvenic Mach number is $M_A \simeq 6$, showing that it is necessary to study magnetized mixed shocks to describe the formation of hot spots (Pelletier & Roland 1988).

Moreover, when the speed of the jet is non-relativistic, inside the hot spots the pressure p_{rp} of the relativistic protons is smaller than the pressure p_{re} of the relativistic electrons, namely $\chi = p_{rp}/p_{re} < 1$. When the speed of the jet becomes relativistic, i.e. $v_1 \simeq c/\sqrt{3}$, the dominant pressure inside the hot spots is the pressure of the relativistic gas, i.e. $\theta_2 < 1$, but in that case the relativistic pressure is dominated by the pressure of the relativistic protons, namely $\chi \gg 1$ (Muxlow et al. 1988, see also Fedorenko 1988 for $\chi \leq 1$ when the jet is non-relativistic).

Finally, let us compare the spectral indices α_r of the solutions (1) and (2) with the observed spectral index of the radio supernovae which do not contain a pulsar.

The synchrotron emission of supernovae which do not contain a pulsar is due to the relativistic electrons accelerated during the interaction of the ejected envelope with the interstellar medium or with the material previously ejected during the supergiant phase (see Weiler & Sramek 1988; Chevalier 1992, and references therein).

Two well observed examples of radio supernovae which do not contain a pulsar are SN 1979c and SN 1980k (see Weiler & Sramek 1988). The interaction of the ejected envelope with the external medium produces a strong shock which heats the plasma. A weak reflected shock is generated and it accelerates the particles whose energy is greater than the threshold energy given by $E_{th} = m_p V_A c$ (Pelletier & Roland 1986) and which have been heated by the strong shock. Their radio spectral indices are $\alpha_r \geq 0.50$ and correspond to the radio spectral index of the solution (1). The solutions (2) and (3) have spectral indices which do not agree with observations. It is also the case for SN 1987a (Staveley-Smith et al. 1992; Chevalier 1992). The observed radio spectral index $\alpha_r \simeq 0.8$ corresponds to $\Delta \simeq 10$ (see Table 1), i.e. to a Mach number $M \simeq 3$. Note that for radio supernova remnants that are a few years old, synchroton losses do not have the time to affect the radio spectral index.

The solution (1) is the only solution where pressure of the relativistic particles do not diverge at high energy. In practice, the distribution of the relativistic particles is characterized by a high energy cut–off due to losses (see Pelletier & Roland 1986; Roland et al. 1988, for the application to Cygnus A hot spots).

5. Conclusion

We investigated the influence of the Mach number on the acceleration of relativistic particles by stationary adiabatic mixed shocks assuming non-relativistic hydrodynamics. More particularly, we studied the number of solutions, their efficiencies and their power law spectral indices.

We solved the structure equations corresponding to mixed shocks supposing that there exists a small pressure of relativistic particles, i.e. $\theta_1 = p_{c1}/p_{r1} \leq 10^5$ in the upstream flow. We found that there exists a critical value for the pressure jump $\Delta_c \simeq 50$ to which corresponds a critical Mach number $M_c \simeq 7.7$ for $\theta_2 \simeq 1$. Moreover,

1. when $\Delta > \Delta_c \simeq 50$: if $300 \leq \theta_1 \leq 10^5$, there exist three solutions to the structure equations. Two solutions correspond to $\theta_2 > 1$, i.e. the pressure in the downstream flow is dominated by the non-relativistic component and has an efficiency of about 10^{-3} and 10^{-2}, respectively. The radio spectral indices of these solutions are $\alpha_r(1) \geq 0.50$ and $\alpha_r(2) \leq 0.50$, respectively. The third solution corresponds to $\theta_2 < 1$, i.e. the pressure in the downstream flow is dominated by the relativistic component and has an efficiency greater than 0.5. The radio spectral index of the third solution is $\alpha_r(3) \leq 0.50$ when $\theta_1 > 1$. In the strong shock limit, observations of bow shocks in strong extragalactic radio sources and of supernova remnant shocks indicate that we observe the solutions with $\theta_2 > 1$, whose efficiencies are smaller than few percents, instead of the solution $\theta_2 < 1$ whose efficiency is greater than 0.99. Moreover, the spectral indices of radio supernovae which do not contain a pulsar correspond to the radio spectral index of the solution (1).
2. when $\Delta \leq \Delta_c$: in addition to solution (1) which exists for all Δ when $\theta_1 > 300$, there exists from one to three solutions depending on both the fraction of the relativistic gas in the upstream flow and the pressure jump.

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