A determination of the longitude of the vertex and the ratio of the axes of the velocity ellipsoid from the dispersions of the proper motions of faint stars measured at the Radcliffe Observatory

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BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS.

1939 January 14. Volume VIII. No. 315.

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

A determination of the longitude of the vertex and the ratio of the axes of the velocity-ellipsoid from the dispersions of the proper motions of faint stars measured at the Radcliffe Observatory, by *A. Blaauw*.

The validity of the ellipsoidal hypothesis for the description of the distribution of the proper motions of faint stars has been shown mainly by VAN DE KAMP and VYSSOTSKY [1]) from the McCORMICK material, and by KNOX-SHAW and SCOTT BARRETT [2]) from the Radcliffe proper motions. However, the data for the elements of the velocity-ellipsoid are still rather uncertain. In both investigations SCHWARZSCHILD's method has been applied, in the first case the "first" method, and in the second case the "second" one, both depending on the distribution of the directions of the absolute proper motions. The results for the ratio of the axes obtained in this way are systematically influenced by the relatively large accidental errors of the proper motions of the faint stars. Rough estimates of this effect were made by VAN DE KAMP and VYSSOTSKY [3]) and by OORT [4]), from which it is clear that considerable deviations are due to this cause. As to the values for the longitudes of the vertex two results deserve attention, viz. the systematic deviation of the longitude found, from the direction to the galactic centre, and the increase of this deviation for stars at low latitudes as apparent from VAN DE KAMP and VYSSOTSKY's work.

In view of these points it seemed justifiable to attack the problem once more along a quite different line, using the dispersions in the proper motion components. If we assume that one of the axes of the velocity-ellipsoid is directed to the galactic pole, we can determine the longitude of the vertex and the ratio of the two axes in the galactic plane in the following way: In each area the mean without regard to sign of the residual proper motion components in the direction parallel to the galactic plane, $|\mu_l|''$, is computed for a certain range in brightness, and corrected for accidental error as well as for the influence of the sun's motion. (The term residual component refers to the difference between the observed and the average component.) Suppose we are able to reduce the results from different areas to the same mean parallax; we can then consider the dependence of these reduced values, u_l, on the galactic longitude towards which they point, the longitude of the vertex being indicated by the largest values of u_l, and the variation with the longitude yielding the ratio of the axes. Various zones of latitude may be treated separately. In addition values $|\mu_b|''$, computed in the same way from the proper motion components in the direction of increasing latitude, may give some information about the size of the third axis in the zones of low latitude, while in the areas near the galactic pole, where they are directed nearly parallel to the galactic plane, they can be used for the same purpose as the μ_l-components.

In this way results are obtained, independent from the reduction to absolute motion, and the influence of the accidental errors can be taken into account in a convenient way. On the other hand, some trouble arises from the relatively large weight given to the large proper motions, and, in low latitude zones, from the reduction to the same mean parallax. The first difficulty has been met by distributing the effect of the large motions within each zone over the components pointing between certain limits of galactic longitude, and by rejecting a few very large motions. For the reduction to the same mean parallax I have made use of the relation between the star counts and average values of the mean parallaxes, to which OORT has drawn attention in his article on mean parallaxes of faint stars (l.c.).

If, in a certain area, $\bar{p}$ is the mean parallax of the stars concerned and $|\mu_l|$ the corrected mean residual proper motion component in the chosen direction, we

1) *Publications of the Leander McCormick Observatory*, Vol. VII, Section V. 2) *Radcliffe Catalogue of Proper Motions* (Introduction). 3) L. c. page 22. 4) *B.A.N.* No. 290, page 81.

can compute for each area $\overline{|\mu_i|}/\overline{p} = u_i$, and then make a comparison between the values of u_i obtained from different areas.

Strictly we should apply corrections for the dependence of the average velocity on the distance from the sun and take into account the probable variation of the velocity with the distance from the galactic plane. These corrections correspond to the factors f and γ in OORT's paper (l.c. pages 84 to 86). However, they are small, and, varying little within each zone of latitude, they were not applied.

It is pointed out by OORT that the linear relation between the logarithm of the number of stars per square degree $A(m)$, and the logarithm of the mean parallax must be considered as still rather uncertain. However, it is the only way in which we can obtain individual mean parallaxes for areas at the same galactic latitude, and being in a high degree independent of the magnitude, the relation is a valuable help in our investigation. For the sake of comparison two plots have been made for the zone 0° to ± 20° latitude. The second (Figure 1*b*) shows the mean residual proper motions reduced to the same mean parallax with the aid of star counts, while in the first (Figure 1*a*) no individual values of the star counts have been used. The proper motions have been plotted against the galactic longitude (for details see below).

The numbers of stars per square degree were derived from *Groningen Publication* No. 43. In the low latitude zones the relation between star counts and mean parallaxes seems to deviate somewhat from the linear formula. Therefore, I drew a slightly deflecting curve through the corresponding dots in the figure in OORT's paper and read the values of log p for the areas in the zone between 0° and ± 20° latitude from this curve.

It is clear that the relation only holds for mean values of log $A(m)$, and real fluctuations in the numbers $A(m)$ will occur where the mean parallaxes are the same. Particularly in the areas at high latitude almost the whole variation in the star counts must be accounted for in this way. In the zone 70°—90° latitude I thus used a fixed mean parallax, corresponding with the mean value of log $A(m)$ for all areas.

One might object to the application of these mean parallaxes, derived on the assumption of certain values for the directions and ratios of the ellipsoidal axes, to our investigation, where it is aimed to determine these very constants. Evidence for the assumption is advanced in OORT's paper. Moreover, only a pronounced relation of the star counts with galactic longitude could make our results dependent on the assumed ratios. These ratios were ·63 for the galactic axes, and ·50 for the z-and the principal axes. The values found in the present paper agree with the ratios mentioned. Somewhat doubtful are those obtained for the third axis, where most uncertainty arises from the increase of the velocities of the stars at greater distances from the galactic plane.

As to the possibility of a variation of the mean velocity in the direction perpendicular to the galactic plane with galactic longitude, direct information might be derived from the b-components in the low latitude zone. However, these mean residual proper motion components being very small for these regions, the results are very uncertain; the spread of the values of u_b for the various intervals of longitude is very large, and a considerable percentage has even been found to be imaginary.

If l_1 and m_1 denote the cosines of the angles between u_l (the average velocity in the direction of increasing galactic longitude) and the two galactic axes of the ellipsoid, and if a and b are the values of u_l in the directions of these two axes, we have the equation

$$u_l^2 = a^2\, l_1^2 + b^2\, m_1^2.$$

Relative to an arbitrary system of coordinates in the same plane we have

$$u_l^2 = \alpha\, l'^2 + \beta\, m'^2 + 2\, \gamma\, l'\, m'.$$

EDDINGTON and HARTLEY [1]) have shown that it is preferable to replace, in such a case as this, these equations by, respectively:

$$u_l = a\, l_1^2 + b\, m_1^2 \qquad (1)$$

and

$$u_l = A\, l'^2 + B\, m'^2 + 2\, C\, l'\, m' \qquad (2)$$

which for the actual ratio of the axes represent the observed values sufficiently accurately, while this procedure prevents exaggerating the influence of the large values of u_l by squaring. For three zones of latitude a least squares solution has been made and from the values A, B and C the unknowns a, b and the directions of the galactic axes were solved.

In the determination of the ratio of the third to the principal axis two ways may be followed. In both we adopt the longitude of the vertex as derived from the l-components. In the first method values u_b are computed in the same way as u_l. Denoting by c the value of u_b in the direction of the galactic pole, we have:

$$u_b = a\, l^2 + b\, m^2 + c\, n^2 \qquad (3)$$

where l, m, n are now the known direction cosines of the b-components in a given area, and a and b the values obtained in the solution of the l-com-

1) *Monthly Notices R.A.S.* **75**, 521 (1915).

ponents. Thus c can be solved immediately. In the second method the ratio μ_b/μ_l is computed for each area separately. Denoting this by ζ, we can write:

$$\zeta = \frac{a\,l^2 + b\,m^2 + c\,n^2}{a\,l_1^2 + b\,m_1^2}$$

and, as the longitudes of the two components differ 90°:

$$\zeta = \frac{k^2\,(m_1^2 + \beta l_1^2) + \gamma\,n^2}{l_1^2 + \beta\,m_1^2}\,, \qquad (4)$$

where:

$$\beta = b/a,\ \gamma = c/a, \text{ and } k^2 = 1-n^2.$$

The value β may be derived again from the l-components. This method is to be preferred, because the ratios ζ are practically independent of the somewhat problematic distance reduction, which only enters into the correction for the sun's motion.

The areas in the *Radcliffe Catalogue* were divided into three zones of latitude in each hemisphere: 0° to ± 20°, ± 20° to ± 50°, and ± 50° to ± 90°, the corresponding zones in the two hemispheres being combined. In each area a range in brightness of either $13^m \cdot 1 - 14^m \cdot 5$ or $12^m \cdot 6 - 14^m \cdot 0$ was chosen, according to the limiting magnitude m_l being fainter than $14^m \cdot 5$, or between 14·0 and 14·6. Areas for which m_l is brighter than 14·1 were rejected.

The proper motions between these limits were plotted on squared paper. From Ohlsson's tables (galactic pole $12^h 40^m$, + 28° (1900)) position angles of the components were derived. The distributions of the components were then read off the graphs. Only proper motions of full weight were used. The algebraic average and the mean residual were then computed and the latter multiplied with a factor $\sqrt{n/n-1}$ in order to take into account the deviation of the average motion from the true mean. Let us call this mean residual $\overline{|\mu|}''$, the average error of the proper motions in the area η; the true mean residual proper motion component $\overline{|\mu|}'$ may then be found by the equation:

$$\overline{|\mu|}'^2 = \overline{|\mu|}''^2 - \eta^2,$$

which is inferred from the similar equation for the mean square motion and the mean error, on the assumption that the ratio between mean square motion and mean residual motion is the same as that between mean error and average error. The first ratio was determined for a number of areas in each zone, the averages being 1·38, 1·40 and 1·40 for the three zones respectively. In this computation the large proper motions were already smoothed in the way described below. As the mean error is 1·48 times the probable error r, the values for η used in the equation were obtained by multiplying the probable errors in the *Radcliffe Catalogue*, Table I, by 1·07 and 1·06 for the zones 0°— 20° and 20°– 90° respectively.

The values of $\overline{|\mu|}'$ thus obtained are to be corrected for the increase of the dispersion, caused by the motion of the sun. A similar line of reasoning as followed in Oort's paper has been applied here. If $\overline{|\mu|}$ is the mean residual proper motion component after this correction, and σ the reflected motion of the sun at the distance of a given star and in the direction of the observed component, we can write with a sufficient degree of approximation:

$$\overline{|\mu|}'^2 = \overline{|\mu|}^2 + \overline{|\sigma-\bar{\sigma}|}^2$$

(bars denoting average values). If, further, we denote by $V_\circ$ the velocity of the sun (in astronomical units per year),
by c_A the fraction of this velocity in the direction of the component,
and by p the parallax of a given star,
we have:

$$\overline{|\mu|}'^2 = \overline{|\mu|}^2 + (c_A\,V_\circ)^2\,\overline{|p-\bar{p}|}^2.$$

It has been noticed already by Oort that the ratio $\overline{|p-\bar{p}|}/\bar{p}$ is nearly the same for faint stars of different magnitudes and at different galactic latitudes, viz. ·55. As in our case the range in brightness is somewhat greater, I have computed it once more for the stars of magnitude 12·5—14·5 from the distance distributions shown in Table 24 of Oort's article for ε = ·0011, assuming the number of stars between $13^m \cdot 5$ and $14^m \cdot 5$ to be 1·5 times the number between $12^m \cdot 5$ and $13^m \cdot 5$. The resulting value is ·57 for both $b = 7°$ and $b = 25°$. Thus it seemed quite safe to use the value ·56. The formula is then:

$$\overline{|\mu|}^2 = \overline{|\mu|}'^2 - (\cdot 56\,c_A\,V_\circ)^2\,\bar{p}^2.$$

In the computation of c_A for each component I have adopted $18^h 0^m$, + 30° for the coordinates of the apex. The velocity $V_\circ$ originally used was 20 km/sec. Later on it seemed preferable to choose a somewhat higher value for the areas at high latitudes (see below).

In the course of the investigation it appeared that it would be impossible to obtain reliable results from the b-components in the zone 0°— 20°. Thus, information about the third axis was only obtained from the b-components in the zone 20° — 50°. In the zone 70° — 90°, where the observed components are almost parallel to the galactic plane, it seemed worth while to use more than two components in each area. Of course, the observed distributions for the various components in one area are not entirely independent, but allowance was made for this in the weights given to these results. In each of the areas between 70° and 90° latitude four components were chosen: the *b*- and *l*-components and the two making angles of 45° with them, denoted by *A* and *B*. The lowest latitude for any area in this zone is 72°. Assuming the ratio of the third to the principal axis to be as 1 to 2, we find that the maximum difference between the observed value u_i and the value of u_l in the direction with the same longitude in any area amounts to less than 4 percent. Thus, it appears quite admissible to treat these components as though they were parallel to the galactic plane.

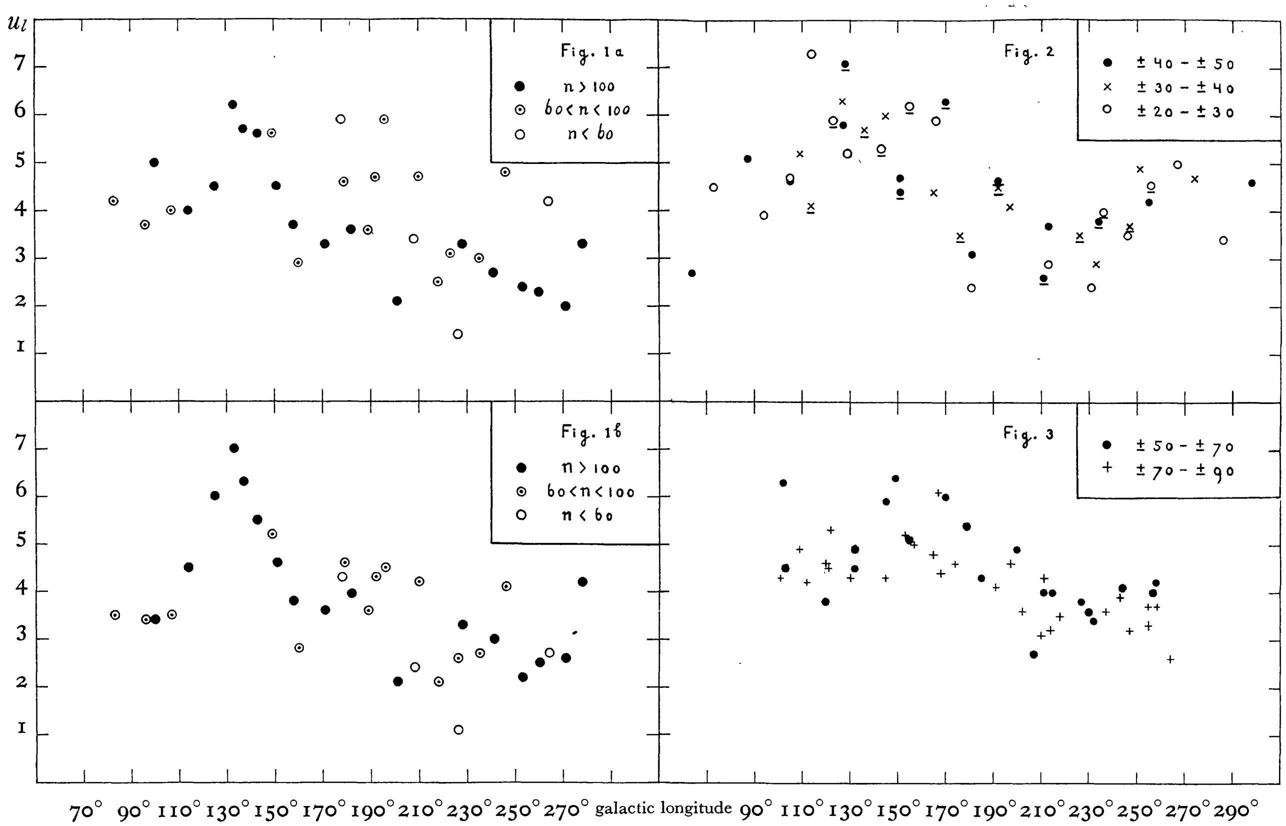
The large motions were treated in the following way. The longitudes were grouped in intervals of 30° and the areas for which the components μ_l pointed to the same interval grouped together. Within each interval the components surpassing a certain limit were added and the sum distributed over the areas in the group, the share for each area being propor-

TABLE 1*a*.

area	l	b	m	c_A	r¹)	n	$\overline{\lvert\mu_l\rvert}''$¹)	$\overline{\lvert\mu_l\rvert}$¹)	u_l
				0° to ± 20°					
24	128°	0°	f	·90	3·8	68	5·62	3·04	2·1
40	53	0	b	·46	4·6	114	8·93	7·33	5·5
87	17	0	f	·11	3·4	90	6·08	4·86	3·5
19	81	− 1	f	·76	3·7	151	5·95	3·95	3·6
64	35	− 1	b	·19	4·0	226	7·34	5·94	6·0
74	163	+ 1	f	·60	4·6	134	5·95	2·94	2·2
98	181	+ 1	f	·36	3·7	279	4·71	2·36	2·6
8	92	− 2	f	·86	4·4	128	6·81	4·35	3·9
49	145	− 2	f	·79	3·6	94	5·70	3·63	2·7
9	106	+ 3	f	·92	3·4	68	8·30	7·04	4·5
18	68	+ 6	f	·65	4·1	107	6·56	4·54	3·8
23	120	− 7	f	·92	3·1	80	7·16	5·82	4·2
41	61	− 8	b	·57	4·5	140	7·99	6·15	4·6
25	133	+ 9	f	·88	3·2	73	5·68	3·84	2·6
37	47	+ 9	f	·37	4·1	130	8·29	6·96	6·3
50	151	+ 10	b	·74	4·0	115	6·22	3·85	3·0
73	156	− 11	f	·68	3·8	81	7·63	6·16	4·1
97	174	− 11	f	·46	4·3	36	7·19	5·37	2·7
111	6	− 11	f	·28	4·2	82	6·64	4·83	3·4
48	136	− 12	f	·86	4·0	51	5·28	1·87	1·1
65	43	− 12	f	·31	4·0	145	9·09	7·97	7·0
88	24	− 12	b	·01	5·2	143	8·01	5·77	4·5
2	88	+ 13	b	·84	4·0	49	9·80	8·40	4·3
10	118	+ 13	f	·93	4·6	63	6·95	4·13	2·4
22	111	− 13	f	·93	4·0	118	5·73	2·73	2·2
42	70	− 13	f	·67	4·0	90	6·00	3·75	2·8
86	10	+ 13	f	·22	3·8	126	7·69	6·49	3·4
75	170	+ 14	f	·51	3·3	146	4·86	3·00	2·5
109	353	+ 14	b	·47	3·3	63	7·13	6·01	3·5
99	188	+ 15	f	·25	3·0	152	5·64	4·59	4·2
20	89	− 17	f	·85	3·2	93	7·85	6·49	4·6
21	99	− 17	b	·89	3·7	78	7·71	5·74	3·6
3	102	+ 18	f	·91	4·0	77	8·45	6·54	4·3
26	138	+ 18	f	·85	4·2	101	7·06	4·66	3·3
17	59	+ 19	b	·54	4·5	65	10·40	8·96	5·2
				± 20° to ± 50°					
7	76	+ 20	f	·74	4·6	115	12·05	10·57	5·9
66	53	− 21	b	·46	5·1	131	10·20	8·48	5·3
51	156	+ 22	f	·69	4·2	99	7·37	5·32	3·5
62	24	+ 23	f	·01	4·2	116	10·91	9·94	7·3
89	33	− 23	f	·16	3·4	101	9·74	9·03	5·9
72	146	− 24	f	·78	3·6	39	12·36	10·72	4·0
96	166	− 25	f	·57	3·3	78	8·94	7·90	4·6
112	15	− 25	b	·14	3·7	115	8·67	7·72	4·7
11	123	+ 27	f	·92	4·6	90	8·19	5·25	2·9
76	177	+ 27	f	·42	4·1	105	9·20	7·98	5·0
85	4	+ 27	b	·31	2·8	71	8·17	7·46	3·9
1	91	+ 28	f	·86	4·2	30	9·17	6·09	2·4
37	39	+ 28	b	·25	3·2	103	9·74	9·07	5·1
67	65	− 28	f	·61	4·5	128	10·76	9·39	6·2
100	196	+ 28	f	·12	3·1	122	6·30	5·34	3·4
108	343	+ 28	f	·60	2·6	60	9·07	8·24	4·5
27	141	+ 29	f	·83	3·5	89	7·26	4·20	2·4
				± 20° to ± 50° (continued)					
4	107°	+ 32°	f	·92	4·7	84	9·98	7·66	4·1
44	86	− 32	f	·83	3·0	92	7·51	6·04	3·5
45	102	− 32	f	·91	3·8	76	10·27	8·57	4·5
16	55	+ 33	f	·48	4·0	70	11·87	10·91	6·0
61	19	+ 34	f	·07	4·5	83	10·63	9·48	5·2
71	136	− 34	f	·86	3·2	47	9·79	8·00	3·5
90	46	− 34	f	·35	3·9	73	11·75	10·90	5·7
52	161	+ 35	f	·63	3·8	80	10·33	9·13	4·9
6	75	+ 36	f	·72	3·4	54	10·70	9·42	4·4
95	157	− 37	f	·68	3·1	34	11·00	9·66	3·7
113	24	− 38	b	·01	3·5	51	9·65	8·91	4·1
28	143	+ 39	f	·81	3·5	51	7·78	5·73	2·9
36	37	+ 39	f	·21	4·4	62	13·13	12·22	6·3
77	184	+ 39	b	·31	3·3	58	10·78	10·08	4·7
84	357	+ 40	b	·42	4·4	91	11·97	10·81	5·1
107	334	+ 40	b	·71	3·5	36	8·63	6·62	2·7
12	123	+ 41	f	·92	5·3	90	10·76	7·98	3·7
101	208	+ 41	b	·07	4·8	69	12·96	11·91	4·6
5	91	+ 42	f	·86	4·3	45	10·36	7·83	3·1
70	121	− 42	f	·92	3·6	34	10·65	7·72	2·6
91	61	− 43	b	·57	4·8	49	13·55	12·01	4·4
53	165	+ 47	f	·58	4·5	70	11·23	9·66	4·2
60	15	+ 47	f	·14	4·5	64	11·73	10·68	4·7
68	80	− 47	f	·77	3·7	94	14·84	13·77	6·3
69	102	− 47	b	·91	4·9	50	14·73	12·50	4·5
15	61	+ 48	b	·57	4·8	59	14·15	12·70	4·7
94	144	− 48	f	·80	2·7	52	10·83	9·37	3·8
35	37	+ 49	f	·21	4·1	89	13·79	13·06	5·8
114	38	− 49	f	·23	3·2	87	15·82	15·41	7·1
				± 50° to ± 70°					
29	142	+ 50	f	·82	3·0	47	10·17	8·42	3·4
106	320	+ 51	f	·84	2·8	57	10·65	9·00	3·6
102	222	+ 52	b	·29	4·0	50	13·26	12·46	4·9
13	110	+ 53	f	·93	3·2	37	13·65	12·08	4·9
83	347	+ 53	f	·55	3·4	60	10·51	9·87	4·0
78	193	+ 54	f	·17	3·8	53	11·85	11·11	4·5
14	80	+ 57	f	·77	5·0	62	16·37	14·84	6·0
93	124	− 58	f	·91	2·9	48	11·57	9·83	4·0
115	59	− 58	f	·54	3·3	70	16·43	15·75	6·4
34	42	+ 59	b	·29	5·1	37	13·95	12·72	4·5
59	12	+ 59	f	·19	4·8	54	16·50	15·65	6·3
30	137	+ 60	f	·85	5·1	50	12·78	10·30	3·8
54	168	+ 60	b	·54	3·9	31	13·39	12·21	4·2
103	245	+ 60	f	·61	2·6	48	14·47	13·59	5·0
105	297	+ 60	f	·93	5·0	57	10·64	7·18	2·7
92	95	− 62	b	·88	4·9	42	14·79	12·49	4·3
104	269	+ 63	b	·84	5·5	57	17·77	15·76	5·4
82	334	+ 65	b	·71	3·9	31	13·63	12·07	4·1
33	55	+ 67	f	·48	4·0	24	16·58	15·76	5·9
79	210	+ 67	f	·11	2·9	52	10·65	10·17	3·8
31	122	+ 68	f	·92	3·9	44	12·93	10·84	4·0

¹) unit ″·001 per annum.

FIGURES 1*a*, 1*b*, 2, 3.



Relation between the values of u_l (ordinates) and the galactic longitude towards which they are directed (abscissae) for different zones of galactic latitude. u_l is the quotient of the mean residual proper motion component parallel to the galactic plane, and the mean parallax of the stars in the area, $\bar{p}$. Values of $\bar{p}$ based on star counts have been used in the computation of the points in Figures 1*b*, 2, and 3; for those in Figure 1*a* $\bar{p}$ depends only on the brightness and the galactic latitude of the area concerned. In Figure 2 points from areas at negative latitudes have been underlined. In Figures 1*a* and 1*b* the points have been distinguished roughly according to the numbers of stars *n* on which the values of u_l are based.

tional to the number of stars in it. In this way the large motions are prevented from influencing too greatly the results for the single areas, whereas only a few very great ones need to be excluded and the preference of the large motions for certain directions is taken into account. No distinction between the two ranges of brightness was made, and areas at negative and positive latitudes in the corresponding zones were combined.

In the zone − 20° to + 20° all components exceeding ″·040 were treated in the way described; the largest component occurring was ″·078.

In the zone ± 20° to ± 50° the areas between ± 20° and ± 30°, ± 30° and ± 40°, and ± 40° and ± 50° were treated separately. The limit was chosen at ″·050 in each of them for both the *l*- and *b*-components. As far as the *l*-components are concerned, components larger than this value were smoothed except 7, exceeding ″·140. These are ″·198 (7), ″·149 (71), ″·181 (28), ″·189 (70), ″·374 (70), ″·223 (70), and ″·193 (69); the numbers between parentheses indicate the areas. Of the *b*-components the following 7 were excluded: ″·120 (62), ″·203 (96), ″·138 (28), ″·208 (52), ″·147 (53), ″·143 (91) and ″·240 (107).

The zones ± 50° to ± 70° and ± 70° to ± 90° latitude were also treated separately; the limit adopted was ″·060. No components were excluded in the first zone, the largest being ″·150. In the zone 70° to 90°, where each area contains four components, one large motion, with components ″·096, ″·305, ″·149 and ″·205 was excluded; no other components larger than ″·104 occurred.

Table 1*a* shows the results for the *l*-components in the areas between 0° and ± 70° latitude. The columns contain: the number of the area, its galactic coordinates, the range in brightness, *m*, where b and f mean respectively $12^{\mathrm{m}}{\cdot}6-14^{\mathrm{m}}{\cdot}0$ and $13^{\mathrm{m}}{\cdot}1-14^{\mathrm{m}}{\cdot}5$, the cosine of the angle between this component and the

TABLE 1*b*.

area	l	b	c	l_c	c_A	r[1])	n	$\overline{\|\mu''\|}$[1])	$\overline{\|\mu\|}$[1])	u
32	84°	+ 72°	*l*	174°	·81	3·3	37	13·97	12·51	4·6
			A	218	·79			11·41	9·63	3·5
			b	264	·34			8·32	7·23	2·6
			B	310	·34			12·51	11·81	4·3
58	11	+ 72	*l*	101	·20	3·7	42	12·60	11·91	4·4
			A	145	·38			12·67	11·81	4·3
			b	191	·74			12·89	11·35	4·1
			B	237	·67			11·50	9·95	3·6
55	168	+ 74	*l*	258	·54	3·5	44	11·36	10·18	3·7
			A	302	·21			15·09	14·57	5·3
			b	348	·82			13·74	12·17	4·4
			B	34	·96			11·26	8·71	3·2
81	301	+ 75	*l*	31	·92	2·6	55	13·51	11·86	4·3
			A	75	·64			10·41	9·16	3·3
			b	121	·02			12·72	12·41	4·5
			B	167	·65			17·33	16·60	6·1
80	243	+ 76	*l*	333	·59	2·5	43	15·00	14·27	5·2
			A	17	·97			14·31	12·63	4·6
			b	63	·79			12·12	10·70	3·9
			B	109	·13			13·86	13·57	5·0
56	165	+ 80	*l*	255	·58	4·2	46	11·81	10·28	3·7
			A	300	·15			13·40	12·59	4·6
			b	345	·77			14·71	13·12	4·8
			B	30	·96			11·45	8·64	3·1
57	22	+ 84	*l*	112	·03	3·7	50	12·18	11·54	4·2
			A	157	·60			14·63	13·58	5·0
			b	202	·82			11·88	9·94	3·6
			B	247	·65			10·49	8·82	3·2

direction of the sun's motion, c_A, the probable error of the proper motions, r, the number of stars between the chosen limits of magnitude, n, the mean residual proper motion component, $\overline{|\mu_l|}''$, the same corrected for accidental error and solar motion, $\overline{|\mu_l|}$, and u_l, obtained by dividing the latter by the mean parallax, computed in the way described above. Table 1*b* shows the data for the areas between 70° and 90° latitude, where four components were chosen in each area, denoted by *l*, *A*, *b*, *B* (see the 4th column). The first three and the last six columns are analogous to those in Table 1*a*. Under l_c the longitude of the direction is given for each component. The longitude towards which the *l*-component points is obtained by adding 90° to the longitude of the area. For the *A*-and *B*-components the values l_c correspond to the position angles obtained by adding 45° and 135° to the position angle of the *l*-component. The values u_l are represented in Figures 1*b*, 2 and 3. Ordinates are u_l, abscissae the galactic longitudes towards which the components are directed; in the zone 50°— 90°, 180° has been added to values of l_c lying outside the interval 90° to 270° longitude. Figure 1*a* shows values of u_l computed with the mean parallaxes corresponding to the latitudes and brightness concerned, but not corrected with the aid of star counts. The spread of the dots seems larger in the upper diagram, and the use of the star counts in the computation of the mean parallaxes appears

[1]) unit ″·001 per annum.

TABLE 2.

area	c_A	$\overline{\|\mu_b\|''}$[1])	$\overline{\|\mu_b\|}$[1])	u_b	ζ
		± 20° to ± 30°			
7	·15	8·45	6·85	3·8	·65
66	·63	7·05	3·74	2·4	·46
51	·57	5·80	2·92	1·9	·58
62	·02	6·70	4·96	3·6	·50
89	·70	5·78	3·61	2·4	·42
72	·14	8·07	7·05	2·6	·66
96	·03	5·48	4·22	2·4	·53
112	·73	6·85	4·71	2·9	·63
11	·40	5·82	2·52	1·4	·50
76	·70	7·09	4·86	3·0	·63
85	·06	9·00	8·48	4·4	1·14
1	·16	8·60	7·25	2·8	1·19
37	·09	7·24	6·38	3·6	·70
67	·66	8·15	6·75	4·5	·73
100	·77	7·06	5·40	3·4	1·04
108	·00	9·00	8·55	4·7	1·04
27	·52	6·87	5·30	3·0	1·28
		± 30° to ± 40°			
4	·25	8·17	6·34	3·4	·83
44	·53	6·62	5·26	3·1	·89
45	·41	7·18	5·59	2·9	·66
16	·12	9·61	8·62	4·7	·79
61	·21	8·52	6·96	3·8	·74
71	·11	7·32	6·45	2·9	·81
90	·78	7·81	5·33	2·8	·52
52	·69	9·26	7·62	4·1	·85
6	·04	8·20	7·36	3·4	·78
95	·08	10·50	9·95	3·8	1·03
113	·87	8·46	5·65	2·6	·69
28	·58	8·43	6·92	3·5	1·24
36	·28	10·92	9·75	5·0	·80
77	·84	8·52	6·11	2·9	·65
		± 40° to ± 50°			
84	·25	11·04	9·88	4·7	·92
107	·11	12·94	12·38	4·9	1·87
12	·38	13·20	11·74	5·5	1·48
101	·89	9·54	4·97	1·9	·51
5	·04	9·02	7·75	3·1	·99
70	·19	12·35	11·65	4·0	1·51
91	·77	9·81	5·96	2·2	·57
53	·78	12·47	10·36	4·5	1·11
60	·42	10·34	8·76	3·8	·83
68	·62	9·72	8·06	3·7	·60
69	·39	12·12	10·53	3·8	·85
15	·29	11·18	9·71	3·6	·77
94	·10	10·89	10·47	4·2	1·16
35	·44	10·66	9·35	4·1	·73
114	·92	10·52	8·28	3·8	·57

to be justified. The only disadvantage is the possibility that, for certain longitudes, systematic deviations of the star numbers from the numbers fitting the relation with $\bar{p}$ might occur; such may be the cause of the large values of the u_l's pointing to about 130° longitude in the zone 0°—20°, the mean parallaxes in the areas near 40° longitude and at low latitudes presumably being larger than should be expected from the great surface density in these regions. In general, the mutual agreement of the points in this zone is far from satisfactory, though it could hardly be expected to be better, in view of the small proper motions at low latitudes, and the consequently large influence of the corrections applied.

In the zone 20°—50° the spread of the numbers n is less than in the former zone, and, consequently, there is less difference between the values $\bar{p}$ computed in the two ways. The data from the three zones 20°—30°, 30°—40°, and 40°—50° are in good agreement, as may be seen from the diagram, and the variation with galactic longitude comes out satisfactorily. The same holds for the zone 50°—90°. In the zones 0°— 50° the abscissae reflect immediately the distribution of the longitudes of the areas. In the zone 50°—90°, where the directions of the areas do not diverge so much, this discrimination is of no importance, so the points from areas at widely different longitudes are mixed in the diagram.

Up to the present the areas at positive and negative latitudes have been treated together. In the investigation by VAN DE KAMP and VYSSOTSKY the mean parallaxes for the southern latitudes come out considerably smaller than for the northern ones. Consequently one should expect the values u_l obtained from southern areas to be smaller than those from the northern areas, provided the real mean linear velocity is the same for stars of the same brightness. No trace of this effect appears in Figure 2, where dots from southern areas have been underlined.

For each of the three zones a least squares solution has been made in the way described above. Within each zone relative weights were assigned, according to the numbers n, but not proportional to them, the spread in the values u_l being in large part due to erroneous corrections and reduction for mean parallax. Therefore, relative weights were chosen roughly proportional to the square root of the numbers n. It has already been mentioned that the four components in an area in the zone 70°—90° cannot be considered as independent. As in the areas between 50° and 70° only one component was used per area, the weight per area in the first mentioned zone will lie between two and four times the weight in the latter. The numbers of stars being almost the same per area in the two zones, I adopted the ratio 1 : 3 and so attached to each component in the zone 70°—90° a weight 3/4.

Table 3 shows the results. l_V is the longitude of the vertex, a and b are the same as in formula (1).

Considering the size of their mean errors the values of l_V agree satisfactorily. The weighted mean:

$$322°.7 \pm 3°.4 \text{ (m.e.)}$$

can be considered as representative for the three zones. This value deviates considerably from those found by KNOX-SHAW and SCOTT BARRETT from the

TABLE 3.

	0° to ± 20° $V_o = 20$	± 20° to ± 50°		± 50° to ± 90°	
		$V_o = 20$	$V_o = 23$	$V_o = 20$	$V_o = 25$
a	5.10 ± .40 (m.e.)	5.55 ± .41 (m.e.)	5.51 ± .41 (m.e.)	5.24 ± .26 (m.e.)	5.17 ± .26 (m.e.)
b	2.58 ± .43 „	3.19 ± .38 „	3.03 ± .36 „	3.51 ± .27 „	3.25 ± .25 „
b/a	.51 ± .06 „	.58 ± .08 „	.55 ± .08 „	.67 ± .06 „	.63 ± .06 „
l_V	320°.2 ± 6°.0 (m.e.)	318°.8 ± 6°.2 (m.e.)		327°.7 ± 5°.5 (m.e.)	

same material. As the mean errors of both results are small compared to the difference, it is very probable that it is not accidental. In connection with this it may be stated that the result obtained in the present work satisfies somewhat better the identity of the directions of the vertex and the galactic centre as predicted by simple dynamical considerations.

The values b/a in the table show an increase for the higher latitudes. But the mean errors are rather large, so that one might doubt if this increase can be considered as real. Moreover, part of it may be due to the fact that in the higher latitudes the solar motion is probably larger than 20 km/sec, so that the corrections for solar motion should be increased.

An estimate of the sun's velocity at different latitudes for faint stars has been made by OORT in *B.A.N.* No. 298. Referring to this I have estimated sufficiently accurate corrections to the values of a and b, assuming V_o to be 23 km/sec and 25 km/sec, respectively, for the zones 20°—50° and 50°—90°. The results are in Table 3, in the fourth and sixth columns. It will be observed that the values b/a have been somewhat reduced.

An increase of the ratios for high latitudes would also occur if too large values had been used for the probable errors of the proper motions; but OORT and VAN HOOF [1]) have accurately confirmed the published probable errors by a study of B- and A- stars.

The weighted mean value of b/a in the first computation is:

[1]) *B.A.N.* Nos 290 and 289.

$$\cdot 61 \pm \cdot 04 \text{ (m.e.)},$$

and in the case of increasing V_0:

$$\cdot 58 \pm \cdot 04 \text{ (m.e.)}.$$

The data used in the computation of the third axis are shown in Table 2. The first three columns indicate: the number of the area, the fraction c_A for the component in the direction of ascending latitude, and the uncorrected mean residual proper motion components. The latter were corrected for accidental error and the sun's motion, which, in the three zones 20°—30°, 30°—40° and 40°—50°, was assumed to be 22, 23 and 24 km/sec respectively. These data and the resulting values u_b are in the next columns. The last columns show the ratio ζ of the b- and l-components for the same value V_0. A least squares solution for the third axis was made for each of the three zones of 10° width, using formula (3). The same weights were used as in the case of the l-components. The values of a and b given in the fourth column of Table 3 were substituted; the longitude of the vertex was adopted at 323°. The results are in the first line of Table 4. The second line shows the results obtained in the solution from the ratios ζ. As in the determination of ζ the accuracy depends mainly upon the numbers n, weights were assigned equal to n.

For each of the zones the results in the two lines agree satisfactorily. The smaller mean errors in the second solution confirm the suspected better re-

TABLE 4, c/a.

	±20° to ±30°	±30° to ±40°	±40° to ±50°
First solution (from u_b)	·51±·07 (m.e.)	·57±·07 (m.e.)	·63±·08 (m.e.)
Second solution (from ζ)	·47±·06 ,,	·49±·05 ,,	·62±·07 ,,

liability of the second method, which is independent of the mean parallaxes. However, the increase of the values c/a towards higher latitudes, shown in both, makes it doubtful if the b-components in a high latitude zone, which point in directions between 40° and 50° from the galactic pole, can yield reliable information about the third axis. The rapidly increasing number of large motions may be the origin of the observed effect.

I am much indebted to Professor OORT for his kind guidance and advice, given during this work.