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COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN.

Additional notes concerning the rotation of the galactic system, by *J. H. Oort.*

Velocities of faint δ Cephei variables. As a supplement to the investigations of *B. A. N.* **120** and **132** the radial velocities of the δ Cephei variables observed by ADAMS, JOY and SANFORD *) have been examined for effects of the galactic rotation. These stars had previously been left out of consideration on account of their small number and because the data did not allow to make more than rough estimates of the true radial velocities. However, upon closer inspection these drawbacks appeared to be outweighed by the very large distances of these stars and I add therefore the following results.

In table 2 of the paper quoted there are eleven Cepheids whose parallaxes according to SHAPLEY **) are smaller than 0″.0010. Six of these have parallaxes between 0″.0005 and 0″.0008, the rest have parallaxes smaller than ″.0003. To the first group we add the velocity of T Monocerotis which has recently been determined by SANFORD ***). Adopting as the true radial velocities the averages between the two extreme velocities published by ADAMS, JOY and SANFORD, and correcting for a solar motion of 20 *km/sec* toward the apex 18^h0^m; + 34°, I derive for the stars with parallaxes between ″.0005 and ″.0008 $\overline{r}A = +29$ *km/sec* ± 8 (m. e.), thus $\overline{r} = 1530$ parsecs and $\overline{\pi} =$ ″.00065. From the five stars with smallest parallaxes I find $\overline{r}A = +28$ *km/sec* + 10 (m. e.), $\overline{r} = 1470$ parsecs, $\overline{\pi} =$ ″.00068. Excluding the doubtful star W Virginis which has a galactic latitude of + 58° the last value of $\overline{r}A$ is decreased to + 22 *km/sec* ± 10 (m. e.).

Thus the faint Cepheids also seem to show the effects of the galactic rotation ****). From these few stars the longitude of the centre can be determined with a mean error of only 7°. We find 320° ± 7° (m. e.) (W Vir excluded), agreeing within its mean error with the average longitude 324° derived from eight different groups of stars in *B. A. N.* **132** (p. 80).

For the first group the average parallax derived on the assumption that $A = +0.019$ agrees well with the average of SHAPLEY's parallaxes. In this connection I should mention that the value of A derived from the radial velocities of 13 bright Cepheids in *B. A. N.* **132**, p. 81, has erroneously been given as + 30 ± 17. In the computation of this result I have used the reciprocal of the average parallax instead of the average of the reciprocals. Using the latter expression for $\overline{r}$ we find $A = +0.021 \pm 0.012$ (m. e.). From the radial velocities it would thus appear that SHAPLEY's parallaxes for long-period Cepheids are very nearly correct: the correcting factor derived from the above results for the bright stars and for those with parallaxes between ″.0005 and ″.0008 is 1.07 ± 0.26 (m. e.). At first sight this conclusion does not seem to remain valid for the four Cepheids with parallaxes smaller than ″.0003. These should be some three times more distant than the former and yet the value of $\overline{r}A$ is only + 22 *km/sec.* However it is probable that in this case the approximation used is no longer valid. If SHAPLEY's parallaxes are right, these stars are situated at distances comparable to the distance of the centre as derived in *B. A. N.* **132**, p. 88, so that the rotation effect cannot any longer be computed on the assumption that r/R is small.

With the data given in *B. A. N.* **132** it is possible to compute the expected radial velocities in a more or less rigorous way. The results are shown as unbracketed numbers in the last column but one of the following table (headed "computed I"). In the computation it was assumed that the four stars were at a distance of 4000 parsecs and that $\partial V/\partial R$ was a constant. (With the notation adopted in *B. A. N.* **120** and **132** $\partial V/\partial R = -A - B = +0.005$ *km/sec. parsec* ± 0.006 (m. e.)). In practice $\partial V/\partial R$ will increase towards the centre and decrease in a direction away from it. The bracketed velocities were therefore computed with a larger value of $\partial V/\partial R$ (viz: + 0.017) and these are intended to represent the lower limit of the rotational effect. The last column, headed "computed II", shows

*) *Publ. Astr. Soc. Pacific*, 36, 139, 1924.

**) *Astrophysical Journal*, 48, 282, 1918; *Mt Wilson Contr.* No. 153.

***) *Publ. Astr. Soc. Pacific*, 39, 236, 1927.

****) For a comparison of the individual velocities of the 7 variables with parallaxes between ″.0005 and ″.0008 with those computed by the formula + 29 sin $(l - l_0)$ see the 4th large column of Table 2 of the next paper.

the velocities computed in the ordinary way by the formula 4000 A sin 2 $(l-l_0)$. A comparison of the observed radial velocities, which are given in the sixth column and which have been corrected for solar motion, with the computed values, shows that for the last two stars, where the difference between the two computed values is considerable, the velocities computed in the first way agree rather better with the observations than those in the last column. The velocity of SZ Aquilae deviates strongly from either computation, but the velocity range for this star is very uncertain as it rests on only three plates.

For these very distant stars the zero velocity is reached at only 60° distance from the direction to the centre, whereas for the nearer stars it occurs at a distance of 90°. Thus, if the assumed average parallax is correct, these four stars would seem to indicate that the centre is really in the direction of 324° longitude and not in the opposite direction, an inference which so far could not be made from the velocities alone. They would then further seem to confirm, though extremely weakly, that the centre is really as near as was found in *B. A. N.* **132**.

These deductions are evidently so uncertain that it would not have been useful to communicate them, if it were not for the fact that they can serve as an illustration of how much importance may be attached to the determination o fonly a few more radial velocities of very distant δ Cephei variables.

Star	Gal. long.	π (Shapley)	Velocity Range (*km/sec*)	Number of plates	Corrected radial velocity	Comp. I (*km/sec*)	Comp. II (*km/sec*)
WZ Sgr	339°	".00028	− 4 to + 80	3	+ 50	+ 40 (+ 27)	+ 38
SZ Aql	4	29	− 20 to + 2	3	+ 7	+ 64 (+ 44)	+ 74
R Sge	25	13	− 28 to + 32	8	+ 19	+ 23 (+ 27)	+ 65
V Vul	35	20	− 29 to + 4	20	+ 5	+ 2 (− 2)	+ 47

The result of the determination of the velocity of the sun with respect to Cepheids as given in *B. A. N.* **120**, p. 278, is not correct. If, instead of introducing a general rotation term into the equations, we apply to each star a correction for rotation proportional with the distance of that star, the solution for the velocity of the sun is changed from 13 *km/sec* to 19.3 *km/sec* ± 5 (m. e.).

Concentration of mass in the great system. From the example used in *B. A. N.* **132** to illustrate under what sort of mass distribution the first derivative of the force could take the form found in that article, it might be erroneously inferred that a very strong *central* condensation of mass is a necessary result from the observations. The addition of another numerical example representing the observed quantities just as well as the one given in *B. A. N.* **132**, may help to remove any misunderstanding of this point.

Let us suppose that the matter of the galactic system is arranged in a series of concentric and rather flattened spheroids, the axial ratios being as 10 to 1 for instance. For the sake of simplicity we shall further suppose that the density is uniform up to the spheroid whose semi-major axis is equal to nine tenths of the distance of the sun to the centre and that the density has again a constant, but smaller, value outside this ellipsoid up to some distance beyond the sun. In order to get such results for the gravitational force and its derivative as found in the paper quoted, the density in the inner ellipsoid must then be equivalent to 0.66 times the sun's mass per cubic parsec, whereas the density in the outer shell would have to be 0.30 *), in the same units. This corresponds to a decrease of density with a factor of 2 over a distance of roughly 500 parsecs.

Had we assumed, for instance, that the inner ellipsoid extended to only three quarters of the distance from the centre to the sun, the density in the inner ellipsoid would have had to be increased to 1.19 and that of the outer shell would have come out 0.25. The latter density does not depend very materially upon the distribution of the mass in inner ellipsoids; it varies mainly with the axial ratio assumed for the outer shell itself, being, for large values of this ratio, roughly proportional to it.

Since the publication of *B. A. N.* **132** my attention has been drawn to the fact that in my references to authors who had previously suspected a rotational term in the proper motions I have left unmentioned SCHILT's paper on "Statistical properties of Cepheids" in which the author derives an annual galactic rotation of −".0075 ± ".0032 (m. e.) **) for these stars.

*) The total mass per cubic parsec of the stars which are luminous enough to be observed, is about 0.04 times that of the sun, so that the density estimated above leaves enough room for fainter stars and dark matter.

**) *Astrophysical Journal*, 64, 161, 1926; *Mt Wilson Contr.* No. 315.