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Woltjer Jr., J.

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## On the energy required for the ionisation of the interior of a star,

by *J. Woltjer Jr.*

The energy content of a star in the absence of rotation consists of contributions from various sources, that may be enumerated as follows:

*a*. potential gravitational energy;

*b*. heat energy;

*c*. radiation energy;

*d*. dissociation energy;

*e*. energy from subatomic origin and unknown sources.

The contributions from the first three sources may easily be calculated for a star in radiative equilibrium; the values are respectively: *)

$$a : -\tfrac{3}{2} f \frac{M^2}{r_0}; \quad b : \tfrac{3}{4} \beta f \frac{M^2}{r_0}; \quad c : \tfrac{3}{2} (1-\beta) f \frac{M^2}{r_0};$$

$M$ is the mass, $r_0$ the radius of the star, $f$ the constant of gravitation; $\beta$ is EDDINGTON's constant: the proportion of gas-pressure to total pressure. The computation of the energy from subatomic origin is impossible; the supply from unknown sources, though certainly to be kept in mind, is unaccessible even for a rough estimation.

The purpose of this note is to consider the dissociation energy more in detail. The dissociation that comes into play in the interior of a star is not the ordinary chemical dissociation, but the breaking up of the atoms: the ionisation. F. H. SEARES **) considered the energy required to reach a degree of ionisation that seemed probable for the atoms inside a star. However, I think the more important point is the computation of the energy required for the *change* in ionisation connected with the transition of a star from one state to another.

**1.** I start from the well known dissociation formula, also used by E. A. MILNE *), to determine the state of ionisation for the interior of a star. This formula connects $x_r$, the relative number of atoms, that have lost $r-1$ electrons, for two succeeding values of $r$, with the partial electron pressure $P_e$, the ionisation potential $\chi_r$, the absolute temperature $T$ and some constants by the equation:

$$(1) \qquad \frac{x_{r+1}}{x_r} P_e = T^{\frac{5}{2}} e^{-\frac{\chi_r}{kT}} \frac{(2\pi m_e)^{\frac{3}{2}} k^{\frac{5}{2}}}{q_r h^3} \sigma_r;$$

$m_e$ is the mass of an electron, $k$ and $h$ the usual constants relating respectively to the mean kinetic energy of a molecule and the energy-quanta; $\sigma_r$ is a number relating to the particular kind of atom in question, its symmetry number; $q_r$ is the "statistical weight" of the condition of this atom. The formula supposes each neutral or ionised atom to have only one stationary state, neglecting the variety of states possible for the bound electrons.

I consider a star in two stages of its evolution, distinguished by the indices 1 and 2. Neglecting the difference between electron-pressure and gas-pressure (that certainly will be small) $P_e$ varies for stars of the same mass as the fourth power of $T$. So we get the formula:

$$(2) \left(\frac{x_{r+1}}{x_r}\right)_1 : \left(\frac{x_{r+1}}{x_r}\right)_2 = \left(\frac{T_2}{T_1}\right)^{\frac{3}{2}} : e^{\chi_r \left(\frac{1}{kT_1} - \frac{1}{kT_2}\right)}.$$

Suppose the stages of evolution 1 and 2 to be so chosen that the corresponding effective temperatures have the ratio 1 : 2; then the corresponding radii will have the ratio 4 : 1 and the mean temperatures the ratio 1 : 4. So we may take in (2):

$$(3) \qquad \frac{T_2}{T_1} = 4;$$

*) Compare e. g. EDDINGTON, *Zeitschrift für Physik*, 7.

**) F. H. SEARES, The masses and densities of the stars, *Astroph. J.* 55 (*Contr. from the Mt. Wilson Observatory*, nr. 226).

*) E. A. MILNE, Statistical equilibrium in relation to the photo-electric effect, and its application to the determination of absorption coefficients, *Phil. Mag.* 47.

thus reducing (2) to:

(4) $$\left(\frac{x_{r+1}}{x_r}\right)_1 : \left(\frac{x_{r+1}}{x_r}\right)_2 = 8 : e^{\frac{3}{4}\frac{\chi_r}{kT_1}}.$$

**2.** Before proceeding to a discussion of equation (4) we must specify the conditions in the interior of the star we consider. Suppose the total mass $M$ equal to 5 times the mass of the sun. The total luminosity $L$ may be calculated from EDDINGTON's equations*), that furnish the absolute magnitude — 0.33 equivalent to:

(5) $$L = 4.64 \times 10^{35} \frac{\text{erg}}{\text{sec}}.$$

The supposed value of the effective temperature $T_e = 3000°$ furnishes the value of the radius $r_0$ in state 1:

(6) $$r_0 = 2.82 \times 10^{12}\, cM.$$

According to EDDINGTON **) we have:

(7) $$1 - \beta = 0.32.$$

The mean temperature follows from the formula:

(8) $$T_m = \tfrac{1}{2} \frac{m\beta f}{R} \frac{M}{r_0};$$

$m$ is the molecular weight, $f$ the gravitation constant and $R$ the absolute gas-constant. Taking $m = 2.11$ we get:

(9) $$T_m = 2.02 \times 10^6 \text{ degrees.}$$

The density corresponding to $T_m$ follows from the formula:

(10) $$\frac{\rho}{T^3} = \frac{am}{3R} \frac{\beta}{1-\beta};$$

$a$ is the energy-density of black-body radiation for $T = 1°$. Substituting numerical values we get:

(11) $$\rho = 1.13 \times 10^{-3} \frac{\text{gram}}{\text{cm}^3}.$$

Hence the value of the gas-pressure:

(12) $$p = 0.90 \times 10^{11} \frac{\text{dynes}}{\text{cm}^2}.$$

Substitution in (1) shows the relative concentration of succeeding stages of ionisation:

(13) $$\frac{x_{r+1}}{x_r} = 2.2 \times 10^4 \frac{\sigma_r}{q_r} e^{-0.0057\,\chi_r};$$

here $\chi_r$ is measured by the ionisation-potential in volts. In this way MILNE (*l. c.*) estimates the relative abundancy of different ionisation-stages within the star. Take e.g. $Fe$; the potential required for the ionisation of the $L$-level is 720 volts, for the $K$-level 7100 volts. Neglecting the difference between the potential required to expel an electron of the $L$-or $K$-level from the neutral atom and from the atom already ionised down to the $L$-or $K$-level, we get from (13):

(14) fraction of the atoms that have lost one $L$-electron and all outer electrons $= 4.10^2 \times$ fraction of the atoms that have lost all electrons outwards from the $L$-level; corresponding factor for the $K$-level $= 10^{-13.3}$.

Here we have neglected the deviation from unity of the value of $\frac{\sigma}{q}$. So we see that the $K$-electrons are practically intact.

**3.** Returning to our purpose, the comparison of the states 1 and 2 with respect to ionisation, we again consider the element $Fe$ as a representative element. In state 1 we may suppose nearly all the iron atoms to consist of a nucleus and the two $K$-electrons. In state 2 the fraction $x$ of these atoms will have lost one $K$-electron. Then applying formula (4) we may take $x_r = 1 - x$, $x_{r+1} = x$, $\chi_r$ the ionisation potential for the $K$-level. Substituting $T_1$ from (9) and $\chi_r = 7100$ volt (see 2) we get:

(15) $$\left(\frac{x}{1-x}\right)_1 : \left(\frac{x}{1-x}\right)_2 = 10^{-12.3}.$$

Hence applying the value of $\left(\frac{x}{1-x}\right)_1$ from (14), viz:

(16) $$\left(\frac{x}{1-x}\right)_1 = 10^{-13.3}$$

we get:

(17) $$\left(\frac{x}{1-x}\right)_2 = 10^{-1.0} \qquad x = \tfrac{1}{11}.$$

**4.** It might seem that the consideration of iron as a representative element is far too special. To meet this objection we must remember that although for the lighter elements the foregoing considerations do not apply because most of them already in state 1 consist of bare nuclei of the heavier elements will behave analogous to iron, only with a difference in the level of the electrons that are being detached by the transition from state 1 to state 2.

**5.** Let us consider the energy required for the discussed change in ionisation supposing the star to consist of iron. The number of iron atoms is equal to the mass of the star divided by the mass of an iron atom; as the latter is 56 times the mass of an $H$-atom, and therefore equal to $0.93 \times 10^{-22}$ gram; we get:

*) *M. N.* 84, 5, p. 310.
**) *l. c.* p. 310.