TABLE 4.
Mean distances and mean parallaxes
(computed with $M_0 = -2{\cdot}1$, dispersion $\pm 1^{M}{\cdot}0$)

m \ b	$\bar{r}$ (parsecs) 10°	30°	55°	$\bar{\pi}$ (unit $''{\cdot}001$) 10°	30°	55°
7·0	900	1010	940	1·32	1·17	1·24
8·0	1400	1440	1300	·86	·81	·88
9·0	2060	1980	1770	·57	·58	·65
10·0	3070	2670	2360	·39	·43	·48

It seems to us that the above results for M_0 — and especially for the mean distances and parallaxes, which are practically independent of the absorption assumed — deserve considerable confidence. They are probably better determined than the values which can be derived from the present material of proper motions.

The various results in Table 3 for the absolute magnitudes are quite accordant. It should be noted that the quantity M_0 given here is the mean absolute magnitude of stars in an element of volume. The mean absolute magnitude M_m for stars of a certain apparent magnitude will be brighter. Using MALMQUIST's relation $M_m = M_0 - \frac{\sigma^2}{\text{Mod}} \frac{d \log A(m)}{dm}$, in which σ is the dispersion, and $A(m)$ the number of stars of magnitude m, the difference between M_m and M_0 may be estimated at — ·9, so that M_m would become — $3^{M}{\cdot}0$. In order to compare with the value derived from proper motions the quantity $m + 5 + 5 \log \bar{\pi} - a$ found on page 332 should be reduced to M_m by subtracting about $^{m}{\cdot}2$; we thus obtain roughly — $2^{M}{\cdot}0$ from proper motions. We do not know the cause of the considerable difference between the two determinations. Possibly, the dispersion of the absolute magnitudes at maximum light is less than the value of $\pm 1^{M}{\cdot}0$ which has been quite arbitrarily assumed; with a dispersion of $\pm 0^{M}{\cdot}5$ the values of M_m would become: from the galactic concentration — $2^{M}{\cdot}3$, from proper motions — $1^{M}{\cdot}8$.

The computations of the mean distances used in the preceding article for the derivation of the density gradient of the long-period variables were made with the compromise values $M_m = -2^{M}{\cdot}0$, dispersion $\pm 0^{M}{\cdot}5$, $M_0 = -1^{M}{\cdot}8$. This is for the average of all variables; we have tentatively assumed that for the periods less than 300 days M_0 would be $0^{M}{\cdot}4$ brighter, and for those greater than 300 days $0^{M}{\cdot}4$ fainter than this average. The good agreement found with the gradient determined dynamically lends some additional support to these absolute magnitudes and distances.

The constants of differential rotation and the ratio of the two galactic axes of the velocity ellipsoid in the case when peculiar motions are not negligible, by *J. H. Oort*.

Summary.

It is shown that for stars with considerable dispersion in the velocities, like long-period variables, the ratio of the galactic axes of the velocity ellipsoid, as well as the coefficients of differential galactic rotation, may become very different from the values applying to low-velocity objects. The table on p. 335 shows the possible limits for these quantities, corresponding to reasonable variations in the run of the density within 1 kps from the sun; limits more appropriate for ordinary-type stars are shown at the end of the article.

For stars with an average velocity along the major axis of the ellipsoid of over 25 km/sec, rotation effects can no longer be reliably used as a criterion for the average distance, unless the ratio of the axes of the velocity ellipsoid is also known with some accuracy. The small rotation effects found for the long-period variables do not conflict with the large distances derived for these stars; they indicate that the density gradient grows less steep outwards. The amount of the decrease in the absolute value of the density gradient is still very uncertain; it is estimated as ·22 ± ·06 (m.e.) per kps.

The lack of asymmetry in the motions of ordinary later-type stars proves that, within a region of about 1 kps radius around the sun, the systematic variation of the density with distance from the centre must be negligible; the variation is certainly much smaller for these stars than for the long-period variables.

When we have to do with stars which do not strictly follow circular orbits, the differential rotation as well as the elongation of the velocity ellipsoid may become very different from the values usually quoted.

If, for simplicity, we limit ourselves to the case that the distribution of velocities is of the form $c e^{-h^2 \Pi^2 - k^2(\Theta - \Theta_0)^2 - l^2 Z^2}$, and if we assume that the system is in dynamical equilibrium, we have $h^2 = c_1$, $k^2 = c_2 \varpi^2 + c_1$, $\Theta_0 = c_3 \varpi / (c_2 \varpi^2 + c_1)$ [1], in which ϖ is the distance to the axis of rotation of the galactic system, and c_1, c_2, c_3 are constants.

Theoretically these constants might have any positive value, but in practice, for any one value of the

[1] *B.A.N.* **4**, 276, 1928.

mean peculiar velocity along the axis of Π, the possible range in the constants c_2 and c_3 is limited, because considerable deviations from the normal values would generally be accompanied by large variations in star density, which we know not to exist. In the case of the long-period variables, for instance, we know that the density variation in the direction of ϖ cannot be much greater than a factor of 2 within a distance of 1 kps in either direction. For the common types of stars the maximum systematic variation of the density within this range is probably considerably smaller. In the following I want to investigate the limits imposed upon the constants c_2 and c_3, or upon h/k and Θ_o, by the restrictions which we know to exist for the density variation.

If ν represents the density, and $\Delta\varpi$ the difference in ϖ between the stars considered and the sun, we may develop $\partial \log \nu/\partial\varpi$ according to powers of $\Delta\varpi$. Let us put

$$\partial \log \nu/\partial\varpi = \alpha + \beta\,\Delta\varpi;$$

for the present purpose no further terms are needed. If all distances are expressed in kps the above condition implies that $|\alpha| < \cdot 30$. It seems likely that the absolute value of β cannot much surpass this same limit, for otherwise we would be led to excessive negative and even positive gradients at distances of which we have still sufficient knowledge to discard such extremes.

We have the following relation between $\partial \log \nu/\partial\varpi$ and the constants of the velocity distribution [1]),

$$\frac{1}{\text{Mod}}\frac{\partial \log \nu}{\partial\varpi} = \frac{2h^2\Theta_o^2}{\varpi} + 2h^2 K_\varpi - \frac{k^2-h^2}{k^2\varpi}.$$

With the aid of the above expressions for h^2, k^2 and Θ_o this may be developed up to the first power of $\Delta\varpi$. In this development the constants c_1, c_2, c_3 may be replaced by the values of h^2, k^2 and Θ_o pertaining to the neighbourhood of the sun, which in the following will simply be written as h^2, k^2, Θ_o without further index; similarly, in the remainder of this note the distance of the sun to the centre will be denoted by ϖ. Θ_c denotes the circular velocity in the neighbourhood of the sun. After further substituting $\overline{\Pi^2}$ for $1/2h^2$ we obtain the following formula

$$\frac{\overline{\Pi^2}}{\text{Mod}}\frac{\partial \log \nu}{\partial\varpi} = \frac{\Theta_o^2}{\varpi} - \frac{\Theta_c^2}{\varpi} - \frac{\overline{\Pi^2}}{\varpi}\left(1-\frac{h^2}{k^2}\right) + \left\{\left(-3+4\frac{h^2}{k^2}\right)\frac{\Theta_o^2}{\varpi^2} + \frac{\Theta_c^2}{\varpi^2} - 2\frac{\Theta_c}{\varpi}\frac{\partial\Theta_c}{\partial\varpi} + \frac{\overline{\Pi^2}}{\varpi^2}\left(1-\frac{h^2}{k^2}\right)\left(1-2\frac{h^2}{k^2}\right)\right\}\Delta\varpi.$$

For ordinary stars the terms with $\overline{\Pi^2}$ in the right-hand member will be negligible compared to those with Θ. Assuming $\Theta_c = 250$ km/sec, $\varpi = 8$ kps, $\partial\Theta_c/\partial\varpi = -5$ km/sec. kps[1]) we may first compute Θ_o for various values of the average velocity in the direction of the major axis of the velocity ellipsoid (a). The following table shows the results for $\partial \log \nu/\partial\varpi = \cdot 00$ and $-\cdot 30$ respectively.

a km/sec	$\overline{\Pi^2}$	Θ_o in km/sec for		Possible limits for					
		$\frac{\partial \log \nu}{\partial\varpi} = \cdot 00$	$\frac{\partial \log \nu}{\partial\varpi} = -\cdot 30$	h/k		A' km/sec.kps		B' km/sec.kps	
10	157	250·2	248·4	·67	·62	+ 17	+ 19	− 14	− 12
20	628	250·8	243·7	·72	·54	+ 15	+ 22	− 16	− 9
30	1414	252	236	·81	·31	+ 10	+ 27	− 20	− 3
40	2513	253	224	·94	·00	+ 3	+ 34	− 25	+ 6
50	3927	255	208	1·13	·00	− 7	+ 45	− 33	+ 19
60	5655	257	186	1·40	·00	− 22	+ 61	− 46	+ 38

For ordinary G- and K-type giants and dwarfs a is about 25 km/sec. If the density gradient were of the order of $-\cdot 30$, we see that Θ_o would be 240 km/sec, corresponding to an increase of the ordinary solar motion by about 10 km/sec. Actually, the solar motion found for these stars deviates but little from the standard velocity of about 20 km/sec derived from small-velocity objects [2]). We may conclude that for these ordinary stars the systematic decrease of density in the direction of the anti-centre is considerably less than a factor of 2 per kps as found from the long-period variables. It is unlikely to amount to more than a factor of 1·25 per kps.

Up to $a = 40$ km/sec the percentage difference between Θ_o and Θ_c is small, so that for all practical purposes we can replace Θ_o by Θ_c in the coefficient of $\Delta\varpi$. Neglecting the last term in the brackets, which is of a smaller order, we can then write this coefficient as

1) *M.N.* **99**, 374, 1939.

2) Cf. for instance the results found by VAN HOOF, who has computed the solar motion without excluding large velocities (*B.A.N.* **7**, 272, 1935). From 1061 G5—K2 stars he derived a solar velocity of 19·5 km/sec ± ·9 m.e., and from 727 K3—Mc stars 23·1 km/sec ± 1·2 m.e.

1) *B.A.N.* **4**, 276, 1928.

$$- 2\frac{\Theta_c}{\varpi}\left\{\left(1 - 2\frac{h^2}{k^2}\right)\frac{\Theta_c}{\varpi} + \frac{\partial\Theta_c}{\partial\varpi}\right\},$$

$$\text{or } -1900\left(1 - 2\frac{h^2}{k^2}\right) + 310.$$

Now the absolute value of this expression should be smaller than ·30 $\overline{\Pi^2}$/Mod = ·69 $\overline{\Pi^2}$. If either $\overline{\Pi^2}$ or the density gradient becomes very small the form between brackets must approach to zero, so that

$$\frac{h^2}{k^2} = \frac{1}{2}\left(\frac{\Theta_c}{\varpi} + \frac{\partial\Theta_c}{\partial\varpi}\right) : \frac{\Theta_c}{\varpi} = \frac{-B}{A-B},$$

which is a well-known relation [1]) (A and B are the constants of differential rotation applying to circular motions). The ratio h/k derived from this relation is ·65.

For a = 20 km/sec the formula indicates that the value of $-1900(1-2h^2/k^2) + 310$ should lie between + 433 and − 433, or h/k between ·73 and ·55. The rigorous formula, with the value of Θ_0 for $\partial \log \nu / \partial \varpi$ = − ·30, gives ·72 and ·54. These latter limits have been calculated for various values of a, and have been inserted in the preceding table; the first limit quoted in the table corresponds to the case that the density gradient becomes less steep for larger values of ϖ, while in the case of the second limit the steepness of the gradient increases outwards. It will be seen that already for a = 40 km/sec the value of h/k is wholly indeterminate if we allow for the possibility of density variations of the proposed size. There thus appears to be not much reason to expect a priori that the long-period variables, especially those with periods smaller than 300 days, give the same ratio between the axes of the velocity ellipsoid as the more common slowly moving stars. The same indeterminacy holds for the constants of differential rotation as derived from the systematic motions of these objects. Let us denote these by A' and B', in order to distinguish from the ordinary quantities A and B relating to circular motions. A' and B' are connected with h/k by the relation [1])

$$\frac{h^2}{k^2} = \frac{-B'}{A'-B'},$$

while we have the further relation

$$A' - B' = \Theta_0/\varpi.$$

The limits of A' and B' corresponding to the two limits of h/k in the 5th and 6th columns of the table are given in the last columns. In each line they have been computed with the smaller of the two values of Θ_0 given in that line. The first number in both columns corresponds to the first value in the column h/k, that is, to a density gradient which flattens out for greater values of ϖ.

For the long-period variables with periods which are smaller than 150 days or between 200 and 299 days a = 44·0 km/sec; for those with periods in excess of 300 days it is 29 km/sec (cf. p. 328, Table 1). From the values of $\bar{r}A$ in Table 2, p. 328, A' is found to be + 6·5 ± 2·9 m.e. on the average, indicating a considerable deviation from the normal value of + 18·0. An inspection of the table on p. 335 shows that with the large velocity dispersion of these stars such a deviation is easily within the bounds of possibility. In order to estimate the most probable value of A' we should combine the above value with that derived from the ratio of the two galactic axes of the velocity ellipsoid. The average ratio for the last three groups of Table 1, p. 328, weighted proportionally with the numbers in the various subgroups is b/a = ·720 ± ·074 (m.e.). From this value I derive B' = − 15·0 ± 3·1 (m.e.) and A' = + 13·9 ± 3·1 (m.e.). The weighted average of this and the above value is A' = + 10·0 ± 2·1 (m.e.). The corresponding value of β is + ·22 ± ·06 (m.e.), indicating that the density gradient of the long-period variables grows less steep outwards, its absolute value decreasing by ·22 per kps; the actual amount remains uncertain, however. The small value of A' should be accompanied by a larger absolute value of B'; we may estimate that B' might become about − 19 km/sec.kps or − ″/a·0040.

For objects with average peculiar velocities exceeding about 25 km/sec the rotation effects can no longer be used for determining average distances, unless the value of h/k is accurately known for these same objects; if reliable distances are available from other sources the rotation terms may, however, be used to estimate the second derivative of log ν with respect to ϖ.

It should be noted that for the common-type stars the limits shown in the table are likely to be too extreme; as remarked above, the density gradient is probably small for these stars, and it would thus seem unlikely that the coefficient β could become as large as ·30 in this case. If we compute the limits for a range of β from + ·15 to − ·15 we find the following ranges:

a	h/k	A'	B'
20	·69 to ·60	+ 16 to + 19	− 14 to − 11
30	·74 to ·54	+ 13 to + 21	− 16 to − 9

It appears that for these stars the values of h/k, A' and B' should be rather close to the normal values of ·65, + 18 and − 13 holding for slowly moving objects.

1) Compare, for instance, *B.A.N.* **4**, 277, 1928.

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