Astron. Astrophys. 124, L1–L3 (1983)

ASTRONOMY AND ASTROPHYSICS

Letter to the Editor

The final state of a solar flare

C. A. Norman[1] and J. Heyvaerts[2]

[1] Sterrewacht, Leiden, Wassenaarseweg 78, Postbus 9504, 2300 RA Leiden, The Netherlands and Institute of Astronomy, Cambridge, England
[2] Institut d'Astrophysique, Paris and University of Paris VII, France

Received March 31, accepted May 20, 1983

Abstract

It is shown that the final state of a solar flare is a force free field with constant α where α is determined by the boundary conditions. This result is independent of the details of magnetic energy build up and release. The crucial assumptions are that during the period of rapid magnetic energy release the effects of slow photospheric foot point motions can be neglected and that magnetic field reconnection occurs in regions whose total volume is small compared to that of the overall configuration.

1. INTRODUCTION

The solar flare phenomenon is, in general associated with the release of magnetic energy in a complex, low β, coronal magnetic structure. Many of the details of the build up and release of magnetic energy are as yet unclear. The work of Heyvaerts et al. (1982) and Birn et al. (1978) proposed the idea that photospheric foot point motions shear a force free field configuration thereby building up energy, and eventually cause some catastrophic saturation to be reached, where energy release occurs. Alternatively, the energy may already be stored in magnetic flux loops emerging from the photosphere. The release of magnetic energy is probably achieved via the tearing mode (Spicer 1977) or in neutral or current sheet systems (Syrovatskii 1969). It has not been possible up to now to follow the flare evolution through its non-linear stage to the final state since this involves treatment of the kinetic instabilities that affect transport coefficients such as anomalous electrical and thermal conductivity (Norman and Smith, 1978). It is shown here that, with certain plausible assumptions, the immediate post flare magnetic structure is a force free magnetic field, $\underline{B}$, given by

$$\text{curl } \underline{B} = \alpha \underline{B} \qquad (1)$$

where α is a constant determined by the boundary conditions - linear force free field! This result is essentially independent of the details of the energy build up and release. There are two basic assumptions made here concerning the process of magnetic field reconnection in solar flares and the behaviour of the boundary conditions during the flare.

This result and its generalisations follow directly from results obtained by Woltjer (1958) in an astrophysical context and from studies of plasma laboratory devices where variational principles have been used to estimate the final state of the Z-pinch (Taylor, 1974) and tokamaks (Kadomtsev, 1977, Bhattacharjee and Dewar, 1982). In both these plasma devices, magnetic field reconnection and tearing modes play a significant role. The principal differences with flares is that laboratory devices are enclosed by conducting walls. The above work can be applied to flares because of the observational evidence that:

1. The time scale for the rapid magnetic energy release in a solar flare is typically ~ 10^2s (the Alfven crossing time) and the time scale for any significant shearing due to photospheric eddy turn over time scale. Thus, during rapid energy release the feet points located in the photosphere can be regarded as fixed and thus there is no transfer of energy into the flaring volume on this time scale through the photospheric boundary.
2. For standard flares where there is no evidence for significant hydrodynamic surging motions (Pallavicini et al. 1977, Spicer, 1977, Heyvaerts et al. 1982) the basic magnetic configuration is a closed one with very little, if any, evidence for open structures. Thus, above the photosphere we can assume for the standard Class I flares (Pallavacini et al. 18977) the configuration is bounded by a closed magnetic flux surface and its photospheric foot points before, during and after the flare.

With these two remarks we prove the basic result and some generalisations in II, give some specific calculations to justify our main approximations in III and discuss implications in IV. We emphasize that the results that follow only apply to flares where the basic configuration remains bounded and the bulk kinetic energy is negligible compared to the magnetic energy.

II. THE MINIMUM ENERGY STATE

Variational principles are only useful if one can write down a physically meanongful quantity to be varied, a well-defined configuration over which one can vary, and can write down the physically relevant constraints using Lagrange multipliers. We choose here to minimise the magnetic energy of this low-β plasma over a fixed volume V_0 bounded by a surface S_0 comprising the photospheric surface and a bounding surface at infinity. It is the Lagrange multiplier contrainsts that are most interesting. In principle there are, for an ideal MHD fluid, an infinite number of conserved quantities (Woltjer 1958, Taylor 1974, Moffatt 1969) since the helicity $\int_V \underline{A}.\underline{B}\, dV$ is conserved for any volume bounded by flux surfaces.

Send offprint requests to: C. A. Norman

Clearly we cannot choose any volume if topology changes are allowed since magnetic surfaces can be drastically changed. Taylor (1974) proposed that changes of topology due to magnetic field reconnection can be incorporated if we use only averages over the total volume of the reconnecting region. His success in explaining the Zeta pinch with these arguments gives very considerable plausibility to this basic assumption that we adopt here. We assume that the flare is encapsulated by a flux surface with photospheric feet points for which $\delta \underline{A} = 0$ during the flare and, consequently, there is no flow of helicity across the flare boundary during the flare. Field line reconnection and snapping can, and do, occur inside the flare loop but, for the system loop as a whole, the total helicity is conserved (see III).

We then proceed to minimise the free energy

$$F = \int_{V_o} \frac{B^2}{8\pi}\, dV - \lambda_o \int_{V_o} \underline{A} \cdot \underline{B}\, dV \qquad (2)$$

with λ_o a Lagrange multiplier, and find that varying with respect to $\underline{A}$ we have

$$\delta F = 2 \int_{V_o} (\mathrm{curl}\, \underline{B} - \lambda_o \underline{B}) \cdot \delta\underline{A}\, dV + 2 \int_{S_o} (\delta\underline{A} \times \underline{B}) \cdot \underline{dS} + \lambda_o \int_{S_o} \underline{A} \times \delta\underline{A} \cdot \underline{dS} \qquad (3)$$

and since $\delta\underline{A}$ vanishes on the surface S_o during the time scale over which the free energy is changing we find that a force free field "with constant α" (1) extremises the free energy functional.

Consideration of the second variation of F shows that this is indeed a minimum if the final state contains no mode rational surface (Spicer, 1981).

Of course the assumption of conservation of only the total helicity out of all the infinite number of conserved partial helicities is an extreme one. Recent work on tearing modes in tokamaks (Kadomtsev 1977, Bhattacharjee and Dewar, 1982) indicates that a second quantity may be treated as conserved. In particular, these authors hypothesise that for a disruption dominated by a particular tearing mode with mode number ratio $q_s = m/n$, the quantity $\int_{V_o} \underline{A}.\underline{B}\, \chi\, dV$ is conserved where χ is the helical flux function. Here $\chi = q_s \Psi - \phi$, and Ψ and ϕ are poloidal and toroidal flux functions. Flare loops are not of course tokamaks but applying their principle with two invariants, which can also be shown to give a true minimum, to the free energy functional one finds that for a local and suitable normalised helical flux function

$$\mathrm{curl}\, \underline{B} = \lambda_o\, \underline{B}\, [1 + \chi] \qquad (4)$$

thus the final current profile is significantly modified by the dominant energy releasing tearing mode.

In a flare the tearing modes are likely to interact and therefore, unlike the tokamak situations, the reconnection will not in general be dominated by a single mode. Therefore we henceforth adopt the variational principle where only total helicity is conserved.

It is appropriate to remark here that the magnetic relaxation proces will occur only in a finite region of the corona which may otherwise be dominated by, for example, a low current or potential field.

III. DISSIPATIVE EFFECTS ON HELICITY CONSERVATION

The invariants discussed in the previous section hold strictly for the pure MHD case. We shall now give some estimates of the effect of dissipation on the conservation of total helicity $H_o = \int_{V_o} \underline{A}.\underline{B}\, dV$. It is easy to show that, when boundaries are fixed

$$\frac{dH_o}{dt} = \int \frac{\underline{A} \times J \cdot \underline{dS}}{\sigma} - 2 \int \frac{\underline{B} \cdot J\, dV}{\sigma} \qquad (5)$$

where $\underline{J}$ is the curent density and σ is the conductivity Considering the surface term first we assume that there are no current concentrations on this flux surface. Then, if B_θ is typical of the azimuthal field in the loop, the normal value of J is $\sim B_\theta/\mu_o d$ where d is the loop diameter. Assuming the Kruskal-Shafranov kink instability threshold is not exceeded, we have $B_\theta = \alpha_{KS} dB_o/L$ where B_o is the value of the field strength along the loop of length L. We estimate the typical value of $\underline{A},\underline{J}$ and H_o to scale as $A \sim B_o d$, $K \sim B_o^2 d^3 L$, $J \sim \alpha_{KS} B_o/\mu_o L$ and using these typical values the surface term in eq. (5) has a time-scale for helicity change $\sim LS/v_A$ S where $S = \mu_o \sigma d v_A$ is the magnetic Reynold's number that has a numerical value $\sim 10^{12}$–10^{13} for the flare region. Helicity changes due to the surface term are therefore negligible.

The volume term in (5) is more difficult to estimate because of the current concentrations that may occur in dissipative regions such as X-points, shock fronts, or tearing modes. At any given time during the flare there may be a large number of these structures. Let us consider each separately.

Tearing modes develop sheets of thickness $\ell \sim d/S^{\frac{1}{4}}$ carrying a current $\sim \delta B/\mu_o \ell$ where δB is the amplitude of the perturbation. If $\mathcal{N}$ is the number of resonant surfaces we find $B \cdot J/\sigma \sim H_o(\delta B/B_o)(\mathcal{N}/\mu_o \sigma d^2)$ giving a helicity change timescale of $\simeq (B_o/\delta B)(d/\ell)(L/v_A)(S/\mathcal{N})$

For well developed reconnection into long, thin separatrices of widths of order the Petschek dissipation region, we find using the usual relations:

$$v_o L = v_A \ell, \qquad \frac{v_o}{v_A} \sim \frac{\pi}{8} [\ell n(\frac{S}{10})]^{-1},$$

$$\ell \sim (\mu_o \sigma v_o)^{-1} \quad \text{and} \quad J \sim B_\theta/\mu_o \ell$$

where v_o is the plasma inflow velocity in the long dimension, that the helicity change timescale

$$\tau_H \sim \frac{1}{\alpha_{KS}} \frac{L}{v_A} \left(\frac{\pi}{8 \ell n\, (\frac{S}{10})}\right)^{-2} S^2 . \qquad (6)$$

For reconnection that is a completely developed Petcheck flow helicity will be destroyed in shocks with the same characteristic dimensions as the reconnection region with a time scale $\sim \frac{1}{\alpha_{KS}} \frac{L}{v_A} S/\mathcal{N}$.

These estimates indicate that for the hypothesis to hold $\mathcal{N}$ should be less than $< S \sim 10^{13}$ and although it seems quite reasonable to suppose this we cannot prove it rigorously. We can however strengthen our argument by two remarks. Firstly in coronal loops the backgrounds potential field may be significant and owing to the Kruskal Shafronov limit may even dominate. If so, the time scale for helicity change is $\sim \frac{1}{\alpha_{KS}} (L/d) S \frac{L}{v_A} >> \frac{L}{v_A}$. Secondly, if the internally generated fields dominate the background we need to discuss turbulence theory.

The generation of helicity by turbulence is given by

$$\frac{dH_{o,t}}{dt} = \int_{V_o} \frac{B_t \cdot J_t}{\sigma}\, dV \simeq S^{-1} H_o \left(\frac{B_t}{B}\right)^2 \frac{A}{d}$$

where B_t and J_t are the turbulent part of B and J and Λ is the length of the complicated line where current concentrations occur on the cross section of the loop. These current concentrations have width ℓ. If the curve becomes a fractal in the limit of large S then its length $\Lambda \sim d^{1+f} \sim S^f d$ and the helicity change timescale $\sim (\frac{B_o}{B_t})^2 \frac{d}{V_A} S^{1-f}$ which is very long if $f<1$.

Finally, assume a magnetic turbulence spectrum $k^2 B_k^2 \sim k^{-\alpha}$ This spectrum extends between k_o and k_D, where $k_o \simeq 1/d$ and k_D is a dissipation wave number. Computing the characteristic timescale for energy dissipation τ_E and helicity generation, τ_H, we find

α	τ_E/τ_H
$\alpha > 3$	1
$\alpha = 3$	$[\ell n k_D/k_o]^{-1}$
$2<\alpha<3$	$(k_o/k_D)^{3-\alpha}$
$1<\alpha<2$	(k_o/k_D)
$0<\alpha<1$	$(k_o/k_D)^{\alpha}$

we conclude that here $\tau_H \gtrsim \tau_E$ in all cases and often $\tau_H \gg \tau_E$ so that our hypothesis would be well satisfied.

IV. IMPLICATIONS

It should be possible to test this result by inspecting immediate post flare structure for constant α behaviour. Techniques for doing this have been discussed by Nakagawa and Raadu (1972) and Tanaka and Nakagawa (1973) who show that post flare magnetic configurations can be represented by force free fields with constant α. The actual change in field strengths should be small $\lesssim 1 \sim 10\%$. Topology changes should occur. On post flare time scales over which foot point motions may occur it is not expected that the field will keep a constant α configuration (see Heyvaerts et al. 1983 for a discussion of the effect of slow motion).

Our discussion allows a realistic estimate of how much energy can be released from a given magnetic configuration. This is the difference between the magnetic energy in a sheared configuration at the point of catastrophe (Heyvaerts et al. 1982) and the linear force free field configuration with the same photospheric boundary conditions. Such estimates can also be very useful in coronal heating models (Heyvaerts et al. 1983).

Obvious further developments will occur if a well defined second invariant emerges from tokamak studies that is important in determining current profiles in post-flare loops. The entire subject has wider ramifications for astrophysics. The physical understanding of why the Crab filaments are force free (Woltjer 1958) and of how force free field analysis should be applied to, say, pulsar magnetospheres or black hole electrodynamics is becoming a little clearer by these considerations of laboratory experiments and our own local solar MHD laboratory.

It is a pleasure to acknowledge many stimulating discussions with J. Birn, D. Callebaut, C. Chiuderi, M. Kuperus, D. Spicer and E.R. Priest. We are extremely grateful to Bram Achterberg, Joachim Birn and Dan Spicer for their comments on the manuscript which added significant improvement and clarification. This work was begun at the Paris-Meudon CECAM workshop in August 1977

REFERENCES

Bhattacharjee, A. and Dewar, R.L. 1982, Phys. Fluids 25, 887.
Birn, J., Goldstein, H. and Schindler, K. 1978, Solar Physics, 57, 81.
Heyvaerts, J., Lasry, M., Schatzman, M. and Witomsky, G. 1982, Astron. Astr. 111, 104.
Heyvaerts, J., Priest, E.R. and Chiuderi, C. 1983, submitted to Astron. Astrophys.
Kadomtsev, B. 1977, Proceedings 6th Controlled Nuclear Fusion Conf., Brechtesgaden (IAEA, Vienna), I, 555.
Moffatt, H.K. 1969, J. Fluid Mech. 35, 117.
Nakagawa, Y. and Raadu, M.A. 1972, Solar Physics, 25, 127.
Norman, C.A. and Smith, R.A. 1978, Astron. Astrophys. 68, 145.
Pallavicini, R., Serio, S. and Vaiana, G.S. 1977, Ap. J. 216, 108.
Spicer, D.S. 1977, Solar Physics 53, 305.
Spicer, D.S. 1981, NRL Report, 4550.
Syrovatskii, S.I. 1969 in C. de Jager and Z. Svetska, XI COSPAR Symposium "Solar Flares and Space Research", p. 346, (North Holland).
Tanaka, K. and Nakagawa, Y. 1973, Solar Physics, 33, 187.
Taylor, J.B. 1974, Phys. Rev. Letters 33, 1139.
Woltjer, L. 1958, Proc. Nat. Acad. Sci. 44, 489.