Proper motions of stars belonging to the Pleiades
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Proper motions of stars belonging to the Plejades *) by *Dr. E. A. Kreiken.*

1. *Measures and reduction.* This paper contains discussions of the results of my measurements of five pairs of plates in the region of the Plejades. The details regarding the plates are given in Table I. All plates were taken with *Carte du Ciel* instruments, so that 1 mm on the plate corresponds to 1′. The centre of all plates is Alcyone, with the exception of pair *D*, of which the centre is Maja.

TABLE 1. List of plates.

Pair	Early plates				Recent plates			
	Nr.	Date	Observatory	Exp.	Nr.	Date	Observatory	Exp.
				m				m
A	13	1891 Oct.	Potsdam	120	1106	1920 Oct. 18	Potsdam	30
B	738	1893 Jan. 24	Greenwich	39	833	1920 Dec. 28	Greenwich	40
C	3	1891 Oct. 2	Helsingfors	150	1	1919 Nov. 12	Helsingfors	120
D	7	1890 Sept. 14	Helsingfors	20	2	1919 Sept. 29	Helsingfors	12
E	755	1893 Feb. 7	Greenwich	180	1107	1920 Oct. 18	Potsdam	120

The recent plates were taken through the glass. The two plates of a pair were placed film to film in the measuring apparatus. The distances between the images of the same star on the two plates were then measured in both coordinates, in two positions of the plates differing by 180°. The means of the measured distances are denoted by Δ_x and Δ_y respectively. The equations of condition then are:

$$\Delta_x = a_x + b_x \cdot x + c_x \cdot y + \mu_x \qquad (1)$$
$$\Delta_y = a_y + b_y \cdot x + c_y \cdot y + \mu_y$$

where a_x, b_x, c_x, a_y, b_y, c_y are the plate constants, x and y the coordinates relatively to the centre of the plate expressed in millimeters, and μ_x, μ_y the effect of proper motion in the interval between the two plates, expressed in revolutions of the micrometer screw. The method of reduction is a slight modification of that explained in *Gron. Publ.* 26, p. 3. A first approximation to the plate constants was derived from 25 arbitrarily chosen stars of about the 13^{th} magnitude. Substitution of this first approximation in the equations (1) gives preliminary proper motions for all stars. The second approximation depends on a set *S* of 140 stars, which were so chosen that their magnitudes and preliminary proper motions were between the limits.

Mag:	12.05	and	13.55
μ_x:	$+''.034$	„	$-''.034$
μ_y:	$+''.034$	„	$-''.017$

The preliminary proper motion of the group being $\mu_x = +''.0150$, $\mu_y = -''.0365$, we can be sure that the set *S* contains no group stars.

This set *S* was used for the pairs *A*, *B*, *C*, *E*. For the pair *D* a few stars, which are outside the plate, are missed out.

From the 140 × 2 equations (1) given by the stars of the set *S* we derive the normal equations

*) As has been mentioned in the last two annual reports, the directors of the Observatories of Potsdam, Greenwich, Helsingfors, Bonn, Paris and Alger have had the great kindness on the request of Prof. HERTZSPRUNG to take several plates of regions in the Plejades, of which old plates taken at the same observatories exist, and to put the old and the new plates at our disposal. Some of these plates were measured by Dr. KREIKEN, and a discussion of his results is given in the present paper. Final results can only be given after the completion of the whole series (consisting of about 20 pairs), which are now being measured at the Observatory.

$$
(2)\quad \begin{aligned}
&[1]\,a_x+[x\,]\,b_x+[y\,]\,c_x=[\ \Delta_x]-[\ \mu_x]\\
&[x]\,a_x+[x^2]\,b_x+[xy]\,c_x=[x\,\Delta_x]-[x\,\mu_x]\\
&[y]\,a_x+[xy]\,b_x+[y^2\,]\,c_x=[y\,\Delta_x]-[y\,\mu_x]\ ,
\end{aligned}
$$

and similar equations replacing the suffixes x by y. As a consequence of the choice of the set S, the mean proper motion in x of this set will be very nearly zero, but the mean proper motion in y will have a certain positive value p. If now we introduce into (2) the conditions

$$
(3)\quad \begin{aligned}
&[\ \mu_x]=0 \qquad [\ (\mu_y-p)]=0\\
&[x\,\mu_x]=0 \qquad [x\,(\mu_y-p)]=0\\
&[y\,\mu_x]=0 \qquad [y\,(\mu_y-p)]=0\ ,
\end{aligned}
$$

and then solve for the plate constants a_x, b_x, ... c_y, then these plate constants introduced into (1) will give for all stars proper motions relatively to a system in which the mean proper motion of the set S is

$$(\mu_x)_S=0 \qquad (\mu_y)_S=p.$$

The meaning of the second and third line of (3) is that within the set S the values of μ_x and μ_y-p are assumed to be independent of x and y.

2. *Magnitude equation.* The reductions as explained above were carried out for each pair separately. The differences between the values of the proper motion as found from different pairs appeared to depend strongly on the magnitude. Table 2 contains the corrections to be applied to the proper motions derived from the other pairs to reduce them to the system of pair B. The correction for the argument 13.5 applies to magnitudes between 13.05 and 14.05, and similarly for the other magnitudes. For the interval 12.05 to 13.55, which is the interval within which lie the magnitudes of the set S from which the plate constants were determined, the corrections are small, but they increase rapidly for fainter as well as for brighter stars.

TABLE 2. First magnitude corrections.

m	Corr. to μ_x				Corr. to μ_y			
	A	C	D	E	A	C	D	E
14·5		− ″·0034		+ ″·0039		+ ″·0026		+ ″·0017
13·5	+ ″·0052	− 13	− ″·0013	+ 21	+ ″·0013	+ 13	+ ″·0009	+ 13
12·5	+ 30	0	− 9	− 9	+ 13	− 9	0	+ 9
11·5	+ 13	+ 21	+ 4	− 17	+ 13	− 17	0	+ 4
10·5	− 21	+ 13	− 13	− 52	+ 13	− 30	− 4	0
9·5	− 129	− 77	− 43	− 198	+ 13	− 56	− 43	− 13
< 9·05	− 258	− 120	− 129	− 275	+ 13	− 73	− 86	− 77

After the application of the corrections from Table 2, the results from the different pairs were combined with weights inversely proportional to the squares of the mean errors given below. These means are thus referred to the system of pair B. To find the magnitude equation of this system the condition was introduced, on the suggestion of Prof. HERTZSPRUNG, that the stars belonging to the group of the Plejades should for all magnitudes give the same proper motion for the group. In this way all magnitudes were reduced to $m=13.5$, for which the proper motion of the group is $\mu_x=+''.014$, $\mu_y=-''.036$. Table 3 contains the corrections so derived. Here again the corrections for the faintest and for the brightest stars are very large. I am therefore inclined to attach very little value to the proper motions of these stars, especially of those brighter than 9.5

3. *Mean errors.* The mean errors were derived from the final differences between the different plates, after the application of the first magnitude correction. The values found are given in Table 4. As has been already said, the final values of the proper motions

TABLE 3. Second magnitude corrections

m	correction to	
	μ_x	μ_y
15·5	− ″·0064	− ″·0043
14·5	− 21	− 13
13·5	0	0
12·5	0	0
11·5	0	0
10·5	+ 13	+ 13
9·5	+ 64	+ 47
8·5	+ 125	+ 95
7·5	+ 129	+ 86

were derived by combining the different pairs with weights inversely proportional to the squares of these mean errors. The last line of Table 4 gives the mean errors of these weighted means. For the brightest stars these mean errors probably are not a good measure of the real accuracy, on account of the large magnitude equations for these stars.

4. *Reduction to absolute proper motions.* For the system to which our proper motions should be reduced that of BOSS' Preliminary General Catalogue has been chosen. The Boss stars occuring on our plates

TABLE 4. Mean errors.

m	14·5	13·5	12·5	11·5	10·5	9·5	<9·05
A		± ″·012	± ″·012	± ″·009	± ″·013	± ″·012	± ″·015
B	± ″·013	10	11	9	11	13	11
C	12	10	9	9	9	7	12
D		13	12	10	11	10	11
E	12	11	13	14	10	13	16
Mean	± ·007	± ·005	± ·004⁵	± ·004	± ·005	± ·005	± ·006

TABLE 5. Stars of $m > 9.55$ belonging to the Plejades.

Nr Hertz-sprung	Phot. mag	$\mu_\alpha \cos\delta$	μ_δ	Nr of obs.	Nr Hertz-sprung	Phot. mag	$\mu_\alpha \cos\delta$	μ_δ	Nr of obs.	Nr Hertz-sprung	Phot. mag	$\mu_\alpha \cos\delta$	μ_δ	Nr of obs.
19	10.8	+ ″·017	— ″·045	5	298	10.2	+ ″·019	— ″·049	5	809	13.9	+ ″·028	— ″·045	5
22	12.0	+ 21	— 48	5	321	15.4	+ 34	— 49	3	844	11.8	+ 14	— 52	4
32	11.6	+ 23	— 51	4	322	11.0	+ 21	— 53	4	851	12.2	+ 23	— 45	4
47	10.9	+ 25	— 51	5	329	11.0	+ 20	— 55	5	862	11.4	+ 14	— 57	5
48	10.2	+ 24	— 56	5	332	13.6	+ 20	— 44	5	875	10.8	+ 19	— 59	5
49*	11.0	+ ·012	— ·046	5	343	11.8	+ ·026	— ·058	5	879	12.6	+ ·021	— ·050	5
55	12.2	+ 19	— 51	5	346	13.7	+ 25	— 49	5	890*	12.8	+ 29	— 48	1
68	10.6	+ 24	— 45	5	376	10.7	+ 28	— 51	5	899	13.3	+ 20	— 55	5
70	11.3	+ 22	— 53	5	378	13.3	+ 27	— 50	5	919	11.3	+ 26	— 52	5
71	12.7	+ 13	— 52	5	380	14.6	+ 21	— 51	3	922	11.8	+ 26	— 54	5
73	12.1	+ ·024	— ·052	4	385	10.6	+ ·023	— ·055	5	936	12.1	+ ·027	— ·052	5
74	11.0	+ 23	— 43	5	388	9.6	+ 27	— 58	5	946	11.4	+ 22	— 50	5
77	10.8	+ 31	— 55	5	390	13.1	+ 26	— 49	5	956	12.3	+ 27	— 49	5
80	11.5	+ 27	— 52	5	395	13.1	+ 23	— 51	5	971	11.1	+ 20	— 52	5
82	14.4	+ 15	— 47	3	420	9.9	+ 27	— 52	4	972	13.3	+ 25	— 56	5
93	12.5	+ ·021	— ·046	5	424	10.9	+ ·027	— ·048	5	986	12.3	+ ·021	— ·050	5
96	14.7	+ 14	— 45	3	428	10.9	+ 18	— 50	5	1001	10.9	+ 28	— 58	3
108	10.0	+ 25	— 52	5	452	12.1	+ 26	— 46	5	1002**	10.9	—	—	—
115	12.1	+ 25	— 52	5	466*	14.6	+ 35	— 46	3	1005	9.7	+ 33	— 52	4
129	11.3	+ 25	— 46	5	467	15.2	+ 25	— 53	3	1030	15.0	+ 22	— 59	3
133	10.9	+ ·019	— ·058	5	468	9.6	+ ·028	— ·054	5	1049	11.5	+ ·022	— ·052	4
141	12.6	+ 24	— 52	5	481	13.6	+ 18	— 53	5	1059	11.2	+ 22	— 60	4
154	15.2	+ 29	— 53	3	487	14.0	+ 26	— 54	5	1064	15.2	+ 21	— 44	3
176	9.8	+ 28	— 41	5	493	15.2	+ 20	— 50	3	1091	13.9	+ 18	— 51	4
181	13.3	+ 14	— 49	5	552	14.0	+ 25	— 58	5	1108	10.6	+ 23	— 55	4
197	14.9	+ ·027	— ·055	3	577	11.1	+ ·018	— ·052	5	1132	12.3	+ ·024	— ·049	4
202	14.1	+ 26	— 54	3	588	14.6	+ 18	— 46	3	1133	12.2	+ 30	— 45	4
213	10.5	+ 15	— 52	5	589	15.0	+ 21	— 52	3	1181	14.0	+ 23	— 46	3
219	10.1	+ 22	— 46	5	610	11.5	+ 27	— 50	4	1186	11.8	+ 19	— 49	4
223	13.3	+ 18	— 48	5	620	10.2	+ 20	— 49	5	1206	12.7	+ 23	— 57	4
225	9.9	+ ·031	— ·058	5	639	11.7	+ ·018	— ·051	5	1224	9.7	+ ·017	— ·050	4
227	9.9	+ 23	— 52	5	646	15.1	+ 24	— 57	3	1230	12.5	+ 26	— 50	3
228	11.9	+ 24	— 49	5	681	9.9	+ 18	— 57	5					
234	11.0	+ 20	— 49	5	698	11.6	+ 21	— 60	5					
268	13.7	+ 18	— 48	5	706	10.8	+ 20	— 53	5					
273	13.9	+ ·031	— ·054	4	708	10.3	+ ·022	— ·052	5					
277	13.6	+ 17	— 51	4	736	10.4	+ 23	— 51	5					
279	12.9	+ 29	— 56	5	744	13.5	+ 17	— 56	4					
294	15.1	+ 28	— 47	3	760	10.6	+ 24	— 48	5					
295	12.5	+ 29	— 58	5	807	11.6	+ 21	— 52	5					

*) 49, 466, 890: uncertain members. **) 1002: images overlap with 1001.

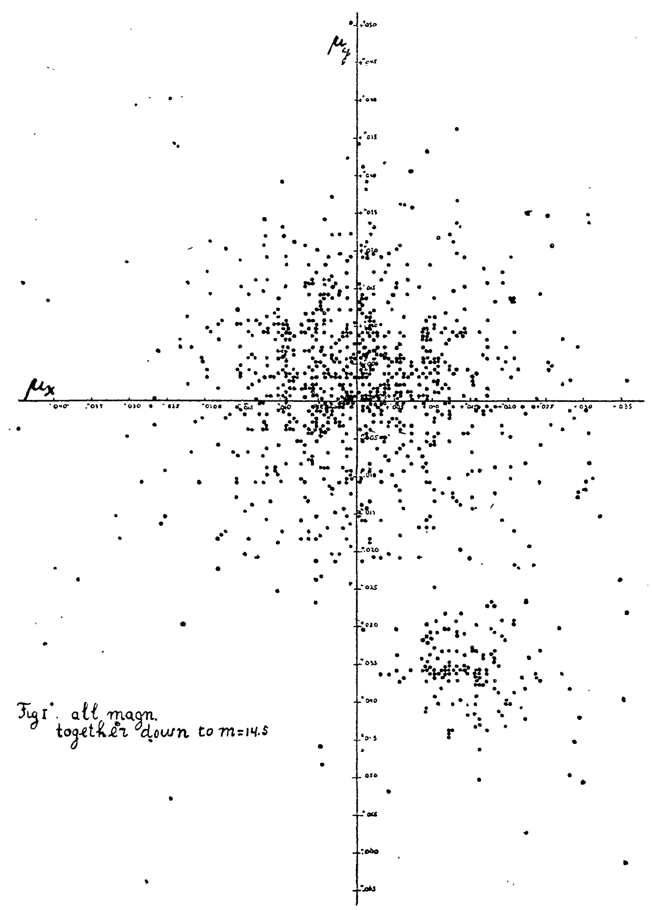
belong to the brightest stars, whose proper motions are very unreliable. It has therefore appeared preferable not to use individual stars, but the motion of the group. The twelve group stars occuring in Boss give for the group motion*)

$$\mu_\alpha \cos\delta = +\,''.023, \qquad \mu_\delta = -\,''.051.$$

Consequently our group motions, in order to be reduced to BOSS's system, require the corrections

$$\delta\mu_x = +\,''.009, \qquad \delta\mu_y = -\,''.015.$$

5. *The group stars.* To discriminate between the group stars and the other stars, diagrams were made of the components μ_x and μ_y for each magnitude separately **). In each of these diagrams a circle was drawn with a radius equal to twice the mean error given in the last line of Table 4, having as centre the point corresponding to the adopted motion of the group. The star whose proper motion points fall inside this circle were reckoned to belong to the group. The diagram for all magnitudes together from the brightest down to $m = 14.5$ is given in *Figure* 1. The diagram, which also contains the uncertain proper motions, gives relative proper motions. From the group stars so selected, those that are fainter than 9.55 are given in Table 5. The stars are referred to by their number in HERTZSPRUNG's Catalogue *), where also the right ascensions and declinations, the colour indices and the distances from Alcyone are given. The magnitudes given in Table 5 are taken from that catalogue.



HERTZSPRUNG's catalogue contains stars down to $m = 15.8$ photographically. The plates on which this catalogue is based have an exposure time of 30^m, while for the limit of completeness of plates taken at the same time and with the same instrument, but with an exposure time of 60^m, which were discussed in my dissertation **), I find a value between 14.9 and 15.5. In the discussion of my result I have therefore used only stars with $m < 14.5$.

6. *The apparent luminosity curve. Distribution of stars of different magnitudes.* Table 6 gives the numbers of group stars within a distante of 55′ from Alcyone between definite limits of magnitude, for all stars together, and for the A stars, which are those whose effective wavelength is between -0.04 and $+0.30$, separately. It will be seen that for the A stars the descending part of the curve ist not so smooth as the ascending part, but on accoun of the small number of stars it is doubtfull whether this has any real meaning. The most striking feature of the curve for all stars is

TABLE 6. Apparent Luminosity curve.

m	$A0$—$A9$	All spectra.
< 14·55		13?
13·55—14·55		12
12·55—13·55		12
11·55—12·55		17
11·05—11·55		21
10·55—11·05	3	
10·05—10·55	4	12
9·55—10·05	7	
9·05— 9·55	4	11
8·55— 9·05	7	
8·05— 8·55	10	16
7·55— 8·05	6	
7·05— 7·55	6	11
6·55— 7·05	3	
5·55— 6·55		4
4·55— 5·55		3
3·55— 4·55		4
2·55— 3·55		2

*) VAN MAANEN, *Mt. Wilson Contr.*, 167, p. 414.

**) These diagrams will be published in the March number of *Hemel en Dampkring*.

*) E. HERTZSPRUNG, Effective wavelengths of stars in the Pleiades, *Mém. Acad. Roy. de Danemark*, 8me série, t. IV, Nr. 4, pp. 349—406, 1923.

**) On the colour of the faint stars in the Milky Way and the distance of the Scutum-group, Groningen, 1922.

the drop*) in the observed numbers at the magnitude 9.5, which is the more surprising as the maximum at 8.0 cannot be ascribed to yellow giants, which do not exist in the Plejades.

Still this does not prove that KAPTEYN's luminosity curve does not apply to the Plejades. In judging the value of our results three points must be considered.

1. There is a certain arbitrariness in the separation of the stars in group and non-group stars, as some stars may be included which accidentally have the same proper motion as the group. Observations of radial velocity would be a great help in sorting out these pseudo group stars.

2. The observed nnmber of stars is small, so that there may be large accidental irregularities.

3. The group of the Plejades extends beyond the field covered by the plates. It seems to be a general characteristic of star clusters that the brightest members are more concentrated towards the centre, while the fainter stars are scattered over a larger field. If the distribution of the stars in the Plejades depends on the magnitude, so that the fraction of the group covered by our plates will be different for different magnitudes, then the observed irregularities in the numbers of stars may be explained, even if we assume KAPTEYN's luminosity curve to apply to the cluster as a whole. That such an unequal distribution of stars of different magnitudes does actually exist, is shown by Table 7, which

TABLE 7. Number of group stars in three zones.

m	I	II	III
$<4{\cdot}55$	7	0	0
$4{\cdot}55-6{\cdot}55$	3	$2\frac{1}{2}$	$\frac{1}{2}$
$6{\cdot}55-7{\cdot}55$	7	2	2
$7{\cdot}55-8{\cdot}55$	11	$2\frac{1}{2}$	$4\frac{1}{2}$
$8{\cdot}55-9{\cdot}55$	6	3	3
$9{\cdot}55-10{\cdot}55$	8	4	5
$10{\cdot}55-11{\cdot}55$	7	11	12
$11{\cdot}55-12{\cdot}55$	5	10	8
$12{\cdot}55-13{\cdot}55$	6	6	2
$13{\cdot}55-14{\cdot}55$	9	1	3
$<14{\cdot}55$	7	4	2

contains the number of stars in three zones: I within 35′ from the adopted centre $\alpha_{1900} = 3^h\,41^m\,11^s$, $\delta_{1900} = +23°\,50'$, II from 35′ to 50′, and III more than 50′ distant from this centre. It will be seen that all stars brighter than 4.5 fall in the first zone. The widest scattering takes place for the stars of mag. 11.5, which is also the maximum of the observed luminosity curve. The fainter stars are again mostly confined to the inner zones. This may however partly be due to incompleteness of our measures of these faint stars near the edges of the plate.

*) Cf. Also R. TRÜMPLER, *Lick Bulletin* 333.

To show that an unequal distribution of this kind can produce a drop in the luminosity curve of the kind as is shown by the observations, I have computed the number of stars of different magnitudes within a certain distance from the centre under the following assumptions:

a. The luminosity curve is KAPTEYN's:

$$\varphi(M) = \text{const.} \times e^{-h^2(M-M_0)^2},$$

with $h = 0.28$, $M_0 = 2.7$*). To reduce to apparent magnitudes I have used a parallax of 0″.013, derived by me from the brightness of the *B* and *A* stars.

b. The density at a distance r from the centre is given by

$$D(r) = \text{const.} \times e^{-k^2 r^2},$$

k being a constant depending on the magnitude. The values of k were so chosen that they form a smooth curve, and so as to reproduce the observed irregularities in the luminosity curve. The result of this computation is given in Table 8. The arguments

TABLE 8. Hypothetical numbers of stars.

m	M	$\varphi(M)$	k	Comp.[d] Nr.	Obs.[d] Nr.
15	5·6	40	0·23	12	13?
14	4·6	58	0·18	12	12
13	3·6	75	0·15	13	12
12	2.6	75	0·18	15	17
11	1·6	72	0·21	19	21
10	0.6	58	0·24	$15\frac{1}{2}$	12
9	−0·4	37	0·30	12	11
8	−1·4	22	0·60	14	16
7	−2·4	10	1·25	10	11
6	−3·4	5	1.25	5	4
5	−4·4	$2\frac{1}{2}$	1.25	$2\frac{1}{2}$	3
4	−5·4	0	1.25	0	4

15.0 etc. stand for 14.55 to 15.55 etc., and similarly 5.6 etc. for 5.15 to 6.15, etc. It will be seen that the computed numbers show the same drop near $m = 9.5$ as the observed ones. It is, of course, not intended to consider this as a proof that the actual distribution of the stars in the Plejades corresponds to the values of k of Table 8. I only wanted to show that, so long as *not all stars are included in the counts*, an irregularity in the apparent luminosity curve of the observed character is compatible with the assumption that KAPTEYN's luminosity curve holds for the cluster as a whole. It is easily seen that by chosing k still smaller for m about 11.5, we can effect a division of the luminosity curve into two parts, one containing the giants and the other the dwarfs.

Haarlem, January 1924.

*) J. C. KAPTEYN and P. J. VAN RHIJN *Ap. J.* 52, pages 23—38