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derived independently, it is clear that there must exist a strong correlation between the Harvard catalogue and SEARES' list. Between + 57°·5 and + 65° the Harvard catalogue does not contain own observations, but gives mean reduced magnitudes from the Yerkes II and the Pulkowo catalogues. The magnitudes in the zone between + 15° and + 25° have been determined at Harvard, although for some stars the reduced magnitudes of the *Göttingen Aktinometrie* have also been used. It seems therefore inevitable that the accuracy of the Harvard catalogue will strongly depend on the region of the sky. The stars used by the writer are, with one exception, all situated south of + 80°. Consequently the mean error derived in this paper is not comparable with the average deviation given by SEARES. The mean error of nearly two tenths of a magnitude is not necessarily an accidental error. It may also be caused by seasonal or other systematic errors. A more detailed investigation will only be possible, when more particulars about the Harvard catalogue will have been published.

The periods of the variables 8, 9, 11 and 12 in the globular cluster M 92,

by *P. Th. Oosterhoff.*

The variable stars in M 92 are investigated by means of the observations which have been used by HACHENBERG and which were put at the disposal of the writer. The periods derived by HACHENBERG for the variables 8, 9, 11 and 12 are not confirmed. New elements are determined. The frequency-distribution of the periods is similar to that of ω Cen, M 15 and M 53. A discussion of the *a*- and *b*-type variables in 13 globular clusters shows that the clusters can be divided in two groups for which the mean period of these variables is ·54 and ·64 days respectively.

HACHENBERG has made an extensive study of the globular cluster M 92 or N.G.C. 6341, the results of which have been published in *Zeitschrift für Astrophysik*, **18**, 49, 1938. His work includes an investigation of 14 variable stars. The photographic plates at his disposal number 125 and were all taken with the 122 cm reflector of the Observatory at Neubabelsberg. One variable proved to be of the W Ursae Majoris type and is not physically connected with the cluster. The remaining 13 stars are cluster type variables and HACHENBERG derived periods for 12 of them. The frequency-distribution of these periods, although few in number, is rather remarkable. Eight variables have been assigned to BAILEY's subclass *a* or *b* by HACHENBERG. Six of them have periods longer than ·6 days, while the remaining two have periods of ·51 and ·40 days. This last period seems to be exceptionally short, as no such variable has been found in any of the globular clusters investigated for variable stars. The mean period for these eight *a*- and *b*-type stars is ·59 days, which value is increased to ·62 days by the omission of variable 8 with its very short period. This mean period suggests that M 92 resembles in this respect ω Cen, M 15 and M 53. The mean period of the variables of BAILEY's subclass *c* in these clusters is about ·37 days. It is therefore an interesting fact, contrary to expectation, that for two of these variables HACHENBERG derives a period of nearly ·23 days. It are the shortest periods known in any globular cluster with the exception of variable 65 in ω Cen with the ultra-short period of ·06 days, which probably should be assigned to a special class. A similar situation has been met in M 53, where two *c*-type variables had very short periods as compared with the remaining stars of this class. It was shown in *B.A.N.* No. 325 that these periods were erroneous, whereas the correct periods are in better harmony with the other *c*-type variables. It seemed therefore worthwhile to re-investigate some of the variables in M 92, but as no detailed observations have been published, this became possible only by the courtesy of Dr HACHENBERG, who sent me a copy of the individual observations. I am much obliged to him for his willingness to co-operate in this way.

The greater part of the observations was taken in the years 1925, 1933 and 1934; the remaining 14 observations date from the years 1935 and 1936. The following discussion of some of the variables will show that the material is insufficient for a satisfactory determination of some of the periods. More observations at large hour angles would have facilitated the investigation considerably. Four of the periods derived by HACHENBERG have not been confirmed. A discussion of the new periods is given in the following lines.

Variable 8: According to HACHENBERG this star is a cluster type variable of BAILEY's subclass *b*, with the very short period of ·401895 days. He derives two sets of elements, as the observations of 1925 do not fit in with the later observations. They seem to be satisfactorily represented by the same period, but are shifted in phase by nearly half a period. His diagram on page 52, in which the magnitudes have been plotted against phase, is therefore rather deceiving, as this constant difference in phase has been taken into account. An independent determination of the period by the writer indicates that the reciprocal period given by HACHENBERG should be diminished by one unit. Provisional phases have been computed with the formula:

$$\text{phase} = 1^{d^{-1}}\cdot485255 \ (\text{J.D.} - 2420000)$$

The light curve thus obtained is not unsatisfactory, although there seem to exist small systematic shifts between the observations of different oppositions. The mean epochs for a point of magnitude 15·00 on the

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rising branch of the light curve are given in the following table for four different years. The number of periods elapsed since the first epoch according to the above formula is given in the second column under the heading n_1.

epoch	n_1	n_2	n_3
d	P	P	P
2424427·036	·00	·00	·00
7350·407	4341·95	4340·97	4339·98
7590·144	4698·02	4697·03	4696·02
8308·488	5764·95	5763·99	5762·99

A slightly better representation of these epochs and a more satisfactory light curve are obtained with the following quadratic elements:

$$2424427{\cdot}036 + {\cdot}6735605\ E - {\cdot}000000028\ E^2$$

Phases have been computed with the corresponding formula:

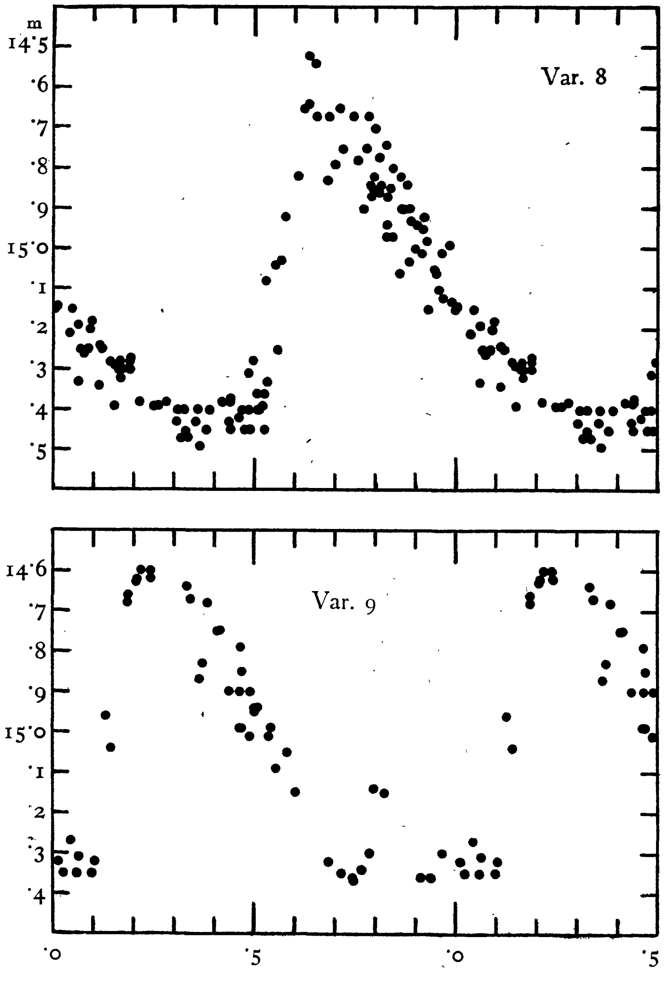
$$\text{phase} = 1^{\mathrm{d}-1}{\cdot}4846476\ (\text{J.D.} - 2420000) + {\cdot}0000000423\ E^2$$

The number of periods elapsed since the first epoch of the little table is given in the third column under the heading n_2. The light curve resulting from these elements is shown in the upper half of Figure 1. The mean error of a single observation, computed from the differences in magnitude between observations following each other in phase, is found to be $\pm^{\mathrm{m}}{\cdot}075$ for the light curve given here and $\pm^{\mathrm{m}}{\cdot}079$ for the light curve by HACHENBERG. The difference is hardly significant, but the longer period is to be preferred, as in this case all observations are represented by a single set of elements. But the exact value of this longer period cannot yet be determined with certainty. A nearly equally good light curve is obtained with the elements:

$$2424427{\cdot}036 + {\cdot}6738286\ E - {\cdot}000000055\ E^2$$

The representation of the four epochs for a point on the rising branch by these elements is shown in the last column under the heading n_3. It is impossible to decide from the present material which of the three solutions n_1, n_2, and n_3 is the correct one. The number of periods elapsed between the epochs of 1933 and 1936 is the same for these solutions, the only difference being the counting in the interval from 1925 to 1933.

FIGURE 1.



Variable 9: According to HACHENBERG this star is a cluster type variable of BAILEY's subclass *c* with a period of ·377949 days. His diagram however in which the magnitudes have been plotted against phase is very unsatisfactory and suggests that this period is erroneous. Although much time has been spent on this star, the writer has not been able to derive a period which gives a fair representation of all the observations and it even seems improbable that such a period can be found, as some of the observations are contradictory. From the observations during J.D. 2427602 and 2428308 it is learnt that the rising branch is steep and that the brightness of the variable at this phase increases with ·7 magnitudes in ·05 days. The observations of the years 1934, 1935 and 1936 are well represented by the period ·60863 days, the reciprocal of which is smaller than that of HACHENBERG by one unit. The shape of the light curve, which is shown in the lower half of Figure 1, is concordant with this longer period and the star should be classified as an *a*- or *b*-type cluster variable. The dispersion of the observations is not unduly large, although two observations near minimum show a considerable deviation. The phases used in the diagram have been computed according to the formula:

$$\text{phase} = 1^{\mathrm{d}-1}{\cdot}643034\ (\text{J.D.} - 2420000)$$

The remaining observations of the years 1925 and 1933 however do not comply with the above elements. During the night of J.D. 2427344 the variable remains of nearly constant brightness for more than ·05 days half way the rising branch. This is in clear contradiction with the observations of the following years and this difficulty exists not only for the period used

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here, but just as well for the period derived by HACHENBERG. We conclude that the present material is insufficient for this variable, that the star probably belongs to BAILEY's subclass *a* or *b* with a period of nearly ·61 days and that future observations must decide whether the star is strictly periodic or whether the light variation is more complicated.

Variable 11: According to HACHENBERG this variable belongs to BAILEY's subclass *c* and has a period of ·235734 days, which corresponds with the reciprocal period 4·242069. A redetermination of the period indicates that this value probably should be diminished by one unit. In the following table a number of epochs for a point on the rising branch of magnitude 14·9 and for a point of magnitude 15·0 on the descending branch have been tabulated together with their phases computed with the reciprocal periods 3·242117 and 4·242069.

rising branch	phase OOSTERHOFF	phase HACHENBERG
d	P	P
2424410·34	·00	·00
7343·31	9509·03	12441·86
59·36	9561·07	12509·95
7559·50	10209·95	13358·95
92·53	10317·03	13499·07
7601·45	10345·95	13536·91
8307·51	12635·08	16532·06
08·43	12638·06	16535·97
descending branch		
2424381·48	·00	·00
4426·25	145·15	189·92
33·31	168·04	219·87
7344·37	9606·04	12568·78
45·31	9609·08	12572·77
7574·48	10352·08	13544·93
90·50	10404·02	13612·88
7611·49	10472·07	13701·93
12·42	10475·09	13705·87

The difference between the two sets of phases is not decisive, but the smaller reciprocal period gives the better representation. In the left hand part of Figure 2 the light curves according to the writer and HACHENBERG respectively are shown. The phases have been computed by means of the formula:

$$\text{phase} = P^{-1}\ (\text{J.D.} - 2420000)$$

in which for P^{-1} the values given above were used. One observation, which has been marked as uncertain, of magnitude 14·88 and phase (HACHENBERG) ·787 is not given in the original diagram by HACHENBERG. It is nearly half a magnitude too bright for his light curve, but its position in the new light curve is entirely satisfactory. Figure 2 leaves little doubt that the new period is the correct one. The mean error of a single observation computed from the differences in magnitude between observations following each other in phase is found to be $\pm^{\rm m}{\cdot}075$ for OOSTERHOFF's and $\pm^{\rm m}{\cdot}100$ for HACHENBERG's period. The elements for a point on the rising branch of magnitude 14·9 are:

$$2427343^{\rm d}{\cdot}307 + {}^{\rm d}{\cdot}3084416\ E$$

Variable 12: According to HACHENBERG the period of this *c*-type cluster variable is ${}^{\rm d}{\cdot}225130$, which corresponds with the reciprocal period 4·441878. If the observations of the different oppositions are treated separately, it will be found that the best representation is obtained with the reciprocal period 2·43914. This value therefore differs by two units from that of HACHENBERG. For four oppositions a mean epoch could be derived for a point on the rising branch of magnitude 15·00. These epochs are given in the following table:

	E	O—C
d		d
2424427·152	0	·000
7360·131	7154	—·003
7595·068	7727	+·004
8308·062	9466	—·001

No satisfactory solution of these epochs could be found however when linear elements are used. The introduction of a positive quadratic term removes this difficulty completely. The resulting elements are:

$$\begin{array}{lllll} 2424427{\cdot}152 & + & {\cdot}4099586\ E & + & {\cdot}0000000027\ E^2 \\ & \pm & 35 & \pm & 4\ (\text{m.e.}) \end{array}$$

The residuals from this formula are given in the third column. Phases have been computed by means of the formula:

$$\text{phase} = 2{\cdot}439271\ (\text{J.D.} - 2420000) - {\cdot}0000000066\ E^2$$

The light curve thus obtained is compared with that by HACHENBERG in the right hand part of Figure 2. The mean error of a single observation, computed in the same manner as for the preceding star, is found to be $\pm^{\rm m}{\cdot}051$ for OOSTERHOFF's and $\pm^{\rm m}{\cdot}066$ for HACHENBERG's light curve.

In connection with the four variables discussed above it may not be superfluous to emphasize once more the importance of a well considered distribution of the observational material. If M 92 would have been observed during a number of nights for as many hours as possible in spite of large hour angles, it seems probable that in this case the correct periods would not have been missed.

An investigation about the frequency distribution of the periods of cluster type variables in five globular clusters has been published by the writer some years ago 1). The main result consisted in the fact that these five clusters could be divided into two groups, which differ in the length of the mean period for the

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1) *The Observatory*, **62**, 104, 1939.

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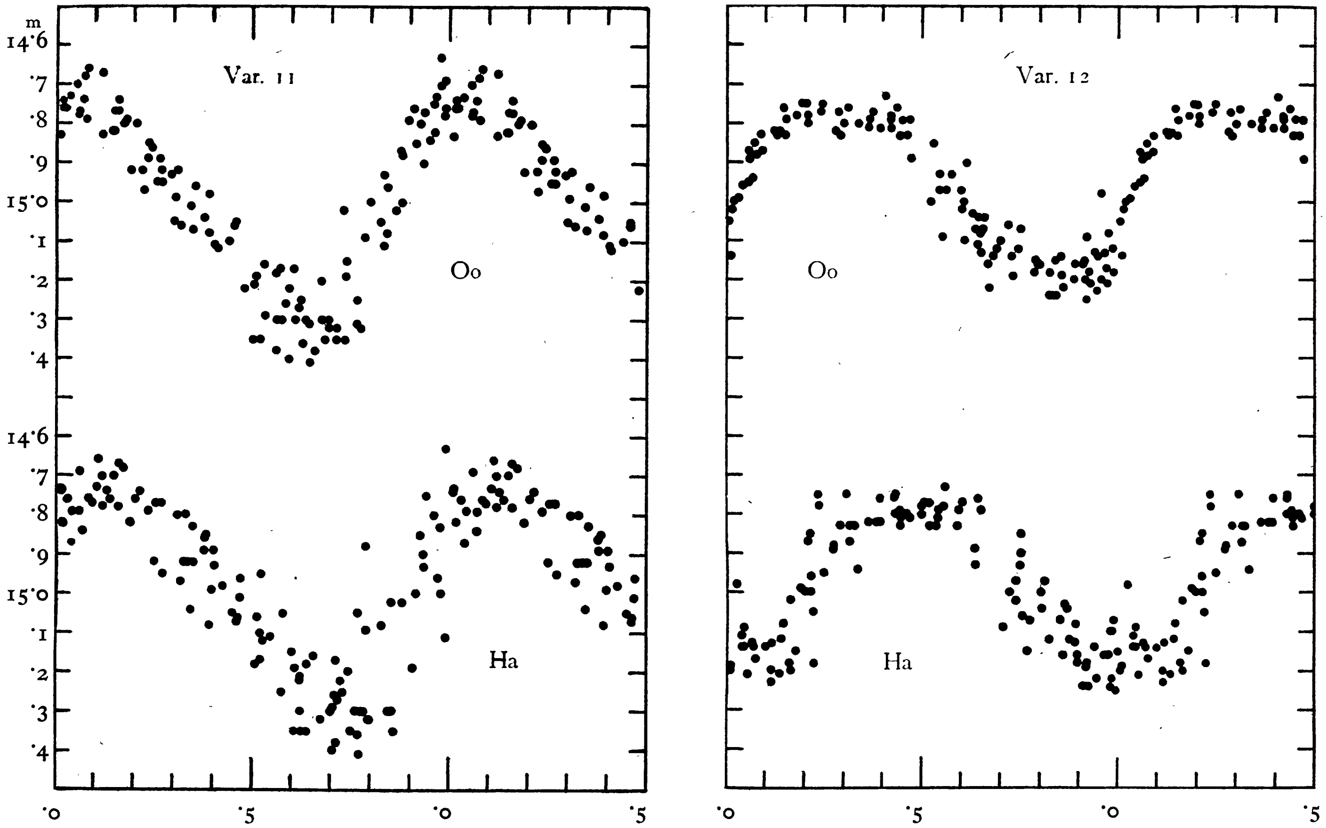
FIGURE 2.



a- and *b*-type as well as for the *c*-type variables. The periods in ω Cen, M 15, and M 53 are nearly 15 percent longer than those in M 3 and M 5. In the following table we repeat some of the data for these five clusters and for M 92 with a few slight changes, which are due to information only recently obtained.

	c-type		*a*- and *b*-type	
	mean period	number	mean period	number
	d		d	
ω Cen	·37	58	·65	77
M 15	·38	28	·65	31
M 53	·37	15	·62	17
M 5	·32	13	·54	63
M 3	·32	27	·55	124
M 92	·37	3	·63	9

Although the number of variables in M 92 is very small, the frequency distribution of the periods seems to be similar to that of the first three clusters.

Finally we may draw attention to the fact that this division of the globular clusters into two groups seems to be of a rather general validity. In order to examine more clusters in this connection, even if they have been less thoroughly investigated, only the *a*- and *b*-type variables have been taken into account in the following table. Because of their larger range, they will be more completely observed than the *c*-type variables. The table gives the mean periods of these variables for 13 clusters.

N.G.C.	M	mean period	*n*	N.G.C.	M	mean period	*n*
		d				d	
362		·54	7	5024	53	·62	17
3201		·56	55	5139	ωCen	·65	77
5272	3	·55	124	6341	92	·63	9
5904	5	·54	63	6656	22	·63	7
6121	4	·51	17	7078	15	·65	31
6723		·51	17	7089	2	·63	11
6981	72	·55	21				
		·545	304			·643	152

If the mean period is shorter than ·6 days the cluster has been entered in the left hand part of the table, the others are found in the right hand part. The division into two groups seems quite convincing. If the mean period would not have any preference for particular values between ·50 and ·65 days, the distribution of the observed values for 13 clusters would probably have a rather different aspect. Future observations should learn whether this result is corroborated by the *c*-type variables.

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