Note on the distribution of luminosities of K and M giants

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BULLETIN OF THE ASTRONOMICAL INSTITUTES
OF THE NETHERLANDS.

1932 August 17 Volume VI. No. 239.

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Note on the distribution of luminosities of K and M giants, by *J. H. Oort.*

Recently STRÖMBERG has made a valuable contribution to our knowledge concerning the absolute magnitudes of K and M giants among stars brighter than the 6th apparent magnitude.[1])

Briefly the method followed by STRÖMBERG consists in a comparison of the distribution of proper motions with the distribution of radial velocities. Very similar procedures have previously been followed by other investigators of the problem [2]), but as STRÖMBERG's computations are more direct and somewhat more refined than former investigations and as they rest on a rather larger material of radial velocities, it seems of interest to use his figures for a re-determination of the distribution of luminosities in a unit of volume near the sun.

It is somewhat to be regretted that STRÖMBERG uses the υ components of the proper motions without regard to sign, thereby losing much of the characteristic feature of their frequency curve. I believe that the weight of his solution might have been appreciably increased if positive and negative υ values had been discriminated.

VAN RHIJN has drawn my attention to the fact that the ellipsoidal character of the velocity distribution causes a rather noticeable difference between the distribution of linear velocities in the τ-direction and that of the radial velocities. As STRÖMBERG starts from the assumption that these distributions are equivalent we might fear an error on this account. The matter is discussed in some detail in a note appended to the present article. It appears probable that the error is not very important.

The reduction of STRÖMBERG's frequencies of stars brighter than the 6th magnitude to total numbers per unit of volume would, of course, be very simple if the space density were the same at all distances: we should only have to divide the number of stars of a certain absolute magnitude, M, as found by STRÖMBERG, by the total volume of the sphere in which these stars would appear brighter than $6^m{\cdot}0$. For the brighter absolute magnitudes, however, this sphere extends to distances where the density is considerably less than that in the neighbourhood of the sun, so that if we wish to know the number per unit of volume near the sun our division factor must be less than the total volume of the sphere. Let us call this correct division factor the effective volume, V_e. If $\Delta(\rho)$ is the star density at a distance ρ we have evidently

$$V_e = \frac{4\pi}{Mod} \int_{\log \rho = -\infty}^{\log \rho = +2{\cdot}20 - 0{\cdot}20\,M} \rho^3 \, \Delta(\rho) \, d(\log \rho) \quad .$$

I have inferred the density $\Delta(\rho)$ partly from the data given by KAPTEYN and VAN RHIJN, partly from my own data for K and M stars. Table 1 shows the figures underlying the computation of the density. The numbers for the zones 0° to 20° and 20° to 40° galactic latitude shown in the 2nd and 3rd columns have been interpolated from Table VI of *Mt. Wilson Contributions* No. 188 [1]). The 4th column shows the density in the zone 40° to 90° latitude as computed with the data given in the preceding article:

$$\Delta = \theta \, 10^{2{\cdot}67.10^8 l_1^2 \int_0^z K(z)\,dz} + (1-\theta)\, 10^{2{\cdot}67.10^8 l_2^2 \int_0^z K(z)\,dz},$$

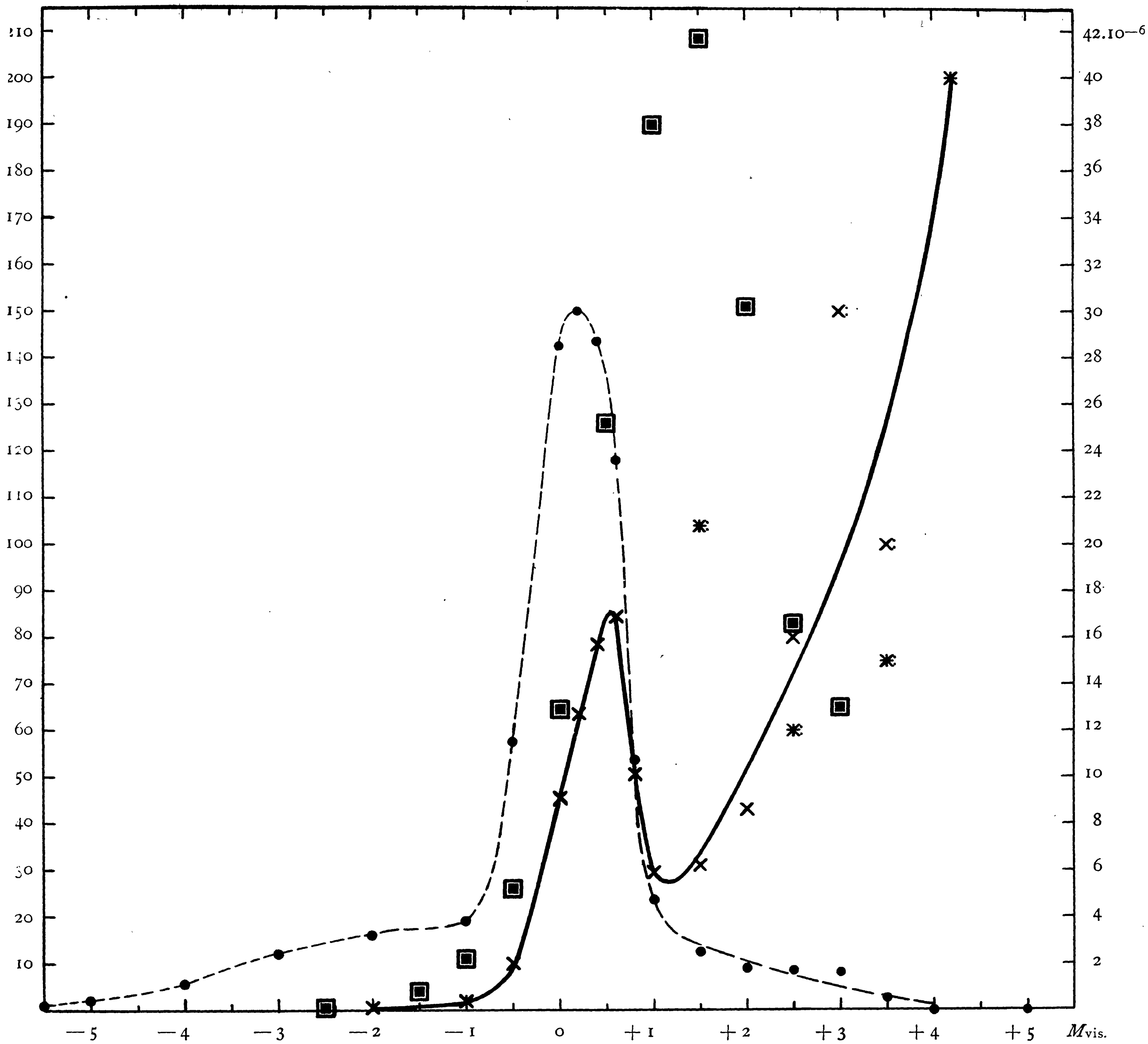
$l_1 = 0{\cdot}039$, $l_2 = 0{\cdot}020$ and $\theta = 0{\cdot}915$; $z = \rho \, \overline{\sin b} = 0{\cdot}825\,\rho$.

[1]) *Mt. Wilson Contributions*, Nos. 395, 410, 411 and 418; *Astrophysical Journal*, **71**, 163; **72**, 111 and 117 and **73**, 40 (1930-31).

[2]) On pp. 18 and 19 of *Groningen Publications*, No. 34 VAN RHIJN derives the distribution of parallaxes (in this case equivalent with that of absolute magnitudes) of stars of the 6th apparent magnitude in a numerical way entirely similar to the method employed by STRÖMBERG. Another interesting effort to determine the distribution of the luminosities of K giants was made by HERTZSPRUNG (*A.N.* Bd. **208**, p. 271, 1919; compare also *A.N.* Bd. **185**, p. 92).

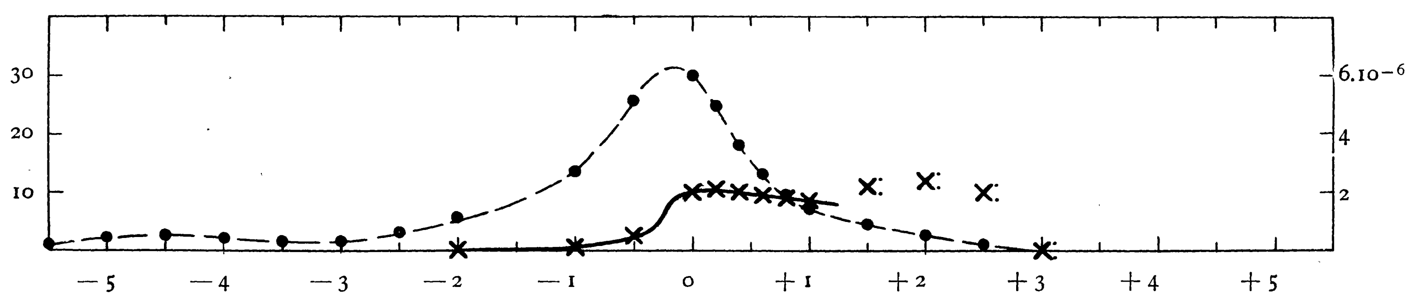
[1]) *Astrophysical Journal*, **52**, 36, 1920.

FIGURE 1.



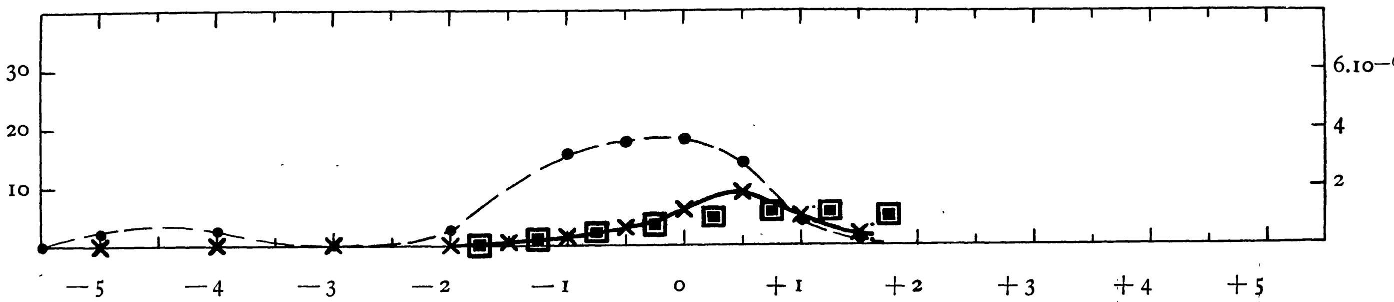
Numbers of K0—K9 stars between $M - 0{\cdot}1$ and $M + 0{\cdot}1$.
Crosses, full-drawn curve and numbers on the right indicate numbers per cubic parsec near the sun.
Dots, dotted curve, and numbers on the left indicate numbers of stars brighter than 6·0 apparent magnitude.
Black squares show the luminosity curve derived in *Groningen Publications*, No. 38.

FIGURE 2.



K3—K9 stars (for explanation see Figure 1).

FIGURE 3.



M stars (for explanation see Figure 1).

For $K(z)$ the average of K_a and K_b was taken (see the preceding article, *B. A. N.* No. 238, Figure 7). For the zone 20°—40° KAPTEYN and VAN RHIJN's densities come out smaller than those derived from $K(z)$ for the zone 40°—90°. As this is rather improbable I have used slightly modified values for the 20°—40° zone, as shown in the 5th column. If these densities had been computed with the aid of $K(z)$, assuming plane parallel equidensity layers, they would have come out still considerably larger. As the densities have only little influence on the following computations I have not thought it worth while to repeat them with these new densities though they are certainly to be preferred to those used below. The last column of Table 1 was obtained by combining the densities of the three zones with weights proportional to the number of K stars brighter than $6^{m}\cdot0$ in each of these zones.

TABLE 1.

Space densities.

log ρ	0°—20°	20°—40°	40°—90°	20°—40° corrected	0°—90°
2·0	0·98	0·78	0·89	0·92	0·94
2·2	·96	·65	·84	·88	·90
2·4	·89	·48	·60	·70	·75
2·6	·71	·32	·37	·48	·54
2·8	·51	·19	·18	·27	·34
3·0	·309	·098	·071	·098	·177
3·2	·166	·046	·029	·046	·090
3·4	·083	·019	·011	·019	·043

KAPTEYN and VAN RHIJN's data are provisional and it is not certain whether the relative density for K and M giants in the galactic zone will change in exactly the same manner as that of the stars in general. Nevertheless I feel confident that the uncertainty in these densities cannot have had an appreciable effect on the luminosity distribution for stars fainter than $-1\cdot0$ absolute magnitude [1]). Even at this magnitude the „effective volume" is only 16 °/₀ smaller than the total volume covered, so that the uncertainty is not likely to exceed 10 °/₀. It may be remarked that the correction applied above to the densities in the 20°—40° zone makes a difference of only about 3 °/₀ in the main parts of the final luminosity curves.

With the densities in the last column of Table 1 we can now compute V_e by numerical integration of the function $\rho^3 \Delta(\rho)$ with respect to $\log \rho$. The values of V_e obtained (in cubic parsecs) are shown in the second column of Table 2. For comparison the total volume of the sphere in which the corresponding stars would appear brighter than $6^{m}\cdot0$ is given under V in the third column. The next three columns show the number of stars per interval of $0^{M}\cdot2$ found from STRÖMBERG's tables (Table IV of *Mt. Wilson Contribution* 411 and Table V of *Contribution* 418). These tables refer to a total of 1000 stars in each subdivision. In the present table the numbers have been reduced to the total number brighter than $6^{m}\cdot0$ existing in the whole sky, viz. 1240 K stars and 225 M stars (using Draper spectral types) [1]). Dividing the numbers in these three columns by the effective volumes V_e we

[1]) Absolute magnitudes throughout this note are magnitudes at a distance of 10 parsecs: $M = m + 5 - 5 \log \rho$.

[1]) These numbers have been interpolated from Table 11, *Groningen Publications*, No. 30. The two subdivisions of the K stars used by STRÖMBERG contain respectively 1058 and 375 stars. Assuming the same relative proportion of the two sub-types among the 1240 K stars mentioned we obtain 915 K0 — K2 stars and 325 K3 — K9 stars. The numbers in Table 2 have been reduced to these totals.

Extrapolating the numbers of stars to different limits of visual magnitude in the Draper Catalogue as given by SHAPLEY and miss CANNON (*Harvard Circulars*, No. 226, 1921) I arrive at somewhat different total numbers, viz. 1316 K0 — K5 stars and 204 Ma — Mc stars brighter than $6^{m}\cdot0$.

It should be noted that the subdivision into the K0 — K2 and K3 — K9 groups adopted is neither representative of the Draper Catalogue, where the number of K0 — K2 stars is more than five times higher than the number of K5 stars, nor of the Mt. Wilson spectral classification, in which the stars are almost equally distributed over the two intervals. The present peculiar subdivision is due to the way in which STRÖMBERG has selected his groups.

now find the true numbers per cubic parsec and per interval of $0^{M}\cdot2$ as shown in the last four columns of Table 2 and by the crosses and full-drawn curves in the accompanying figures (ordinates to be read on the right). The dotted curves and black circles in these figures indicate STRÖMBERG's results for the numbers of stars brighter than $6^{m}\cdot0$ (ordinates to be read on the left side).

TABLE 2.

M	V_e	*V*	Numbers brighter than $6^{m}\cdot0$			Numbers per cubic parsec and per interval of $0^{M}\cdot2$			
			Ko—K2	K3—K9	M	Ko—K2	K3—K9	Ko—K9	M
— 5·0	2740·× 10^6	16700·× 10^6	0·	2·	2·1	0·000 × 10^{-6}	0·001 × 10^{-6}	0·001 × 10^{-6}	0·001 × 10^{-6}
4·0	1240·	4190·	3·5	2·	2·6	·003	·002	·005	·002
3·0	503·	1050·	10·5	1·5	0·	·021	·003	·024	·000
2·0	180·	264·	10·5	5·5	2·7	·058	·031	·089	·015
1·5	102·	132·	6·5	8·	9·9	·064	·078	·142	·097
1·0	55·9	66·4	5·5	13·5	15·3	·10	·24	·34	·274
— 0·5	29·7	33·3	31·	25·5	17·6	1·04	·86	1·90	·59
0·0	15·4	16·7	111·5	30·	17·9	17·2	1·9	9·1	1·16
+ 0·2	11·9	12·7	125·5	24·5		10·6	2·1	12·7	
0·4	9·13	9·60	125·5	18·		13·7	2·0	15·7	
0·5	7.99	8·38	121·	15·5	14·0	15·1	1·9	17·0	1 75
0·6	7·01	7·29	105·	13·		5·0	1·9	16·9	
0·8	5·30	5·53	44·	9·5		8·3	1·8	10·1	
1·0	4·07	4·19	16·5	7·	3·9	4·1	1·7	5·8	0·96
1·5	2·08	2·10	8·	4·5	0·9	3·8	2·2	6·0	·43
2·0	1·04	1·05	6·5	2·5		6·2	2·4	8·6	
2·5	0·528	0·528	7·5	1·		14·	2·	:6·	
3·0	·264	·264	8·	0·		30·	0·	30·	
3·5	·132	·132	2·5	0·		20·	0·	20·	
+ 4·0	·066	·066	0·	0·		0·	0·	0·	

A minimum in the frequency of Ko—K2 stars near $+ 1^{M}\cdot2$ seems to be fairly well established by these numbers (compare Figure 1). For the K3—K9 stars there is no indication of such a minimum. For the M stars a decrease in the luminosity curve beyond $+ 0^{M}\cdot5$ is indicated but the data are far too scanty to determine the minimum.

In the first figure the two K sub-groups have been combined; this curve practically represents the luminosity distribution of the Ko—K5 stars of the Draper Catalogue. It has been extended in the direction of the dwarf branch with the aid of data derived from trigonometric parallaxes of the nearest stars. These were taken or interpolated from Table 46 of *Groningen Publications*, No. 38 (5th column) and have been entered in the figure as asterisks. The positions of the three lowest asterisks are especially uncertain, each resting on less than 10 stars. These asterisks as well as other especially uncertain points have been marked with a colon. In the region between $+ 1^{M}\cdot5$ and $+ 3^{M}\cdot0$ the luminosity curve has been drawn so as to correspond to the values derived from the smooth dotted curve and has further been extended to the asterisk at $+ 4^{M}.2$ near the top of the figure. It is well known that for fainter absolute magnitudes the luminosity curve rises to still much higher values reaching about 500.10^{-6} near $+ 8^{M}$. The number of K dwarfs between $+ 2^{M}\cdot0$ and $+ 9^{M}\cdot0$ may be estimated as 8000.10^{-6} and there may be as many again fainter than $+ 9^{M}\cdot0$. The total number of K type giants brighter than $+ 2^{M}\cdot0$, as inferred from the new luminosity curve, is 116.10^{-6} per cubic parsec, so that K giants are from 100 to 150 times less numerous in space than K dwarfs. There are about 15.10^{-6} M type giants brighter than $+ 2^{M}\cdot0$ per cubic parsec. Here the ratio between the numbers of dwarfs and giants is at least 2000 and probably considerably larger.

The sharp giant maximum shown in Figure 1 is in general accordance with the phenomenon shown in the Hyades and Praesepe [1]) both of which contain four giants of spectrum Ko (one star has probably a G5 spectrum). In each case these four stars are practically of the same magnitude, the average deviation being only $\pm {}^{m}\cdot14$. The average visual magnitude of the Hyades giants is $3^{m}\cdot86$; the parallax is probably between $''\cdot024$ and $''\cdot030$, the corresponding limits for the average absolute magnitude are $+ \cdot8$ and $+ 1\cdot2$, in sufficient agreement with the first maximum in Figure 1. For Praesepe the Harvard visual magnitude averages 6·64, the average Groningen photographic magnitude is 7·29, corresponding with visual absolute

[1]) It is well known that many open clusters contain yellow giants. For M 37 the measures by VON ZEIPEL and LINDGREN show that the cluster contains about 25 yellow giants. The average deviation from the mean magnitude, 12·6 (pg), is between ± 0·2 and ± 0·3

magnitude + ·8, photographic + 1·5, if a parallax of ″·007 is assumed.

The luminosity curve of K giants as found from STRÖMBERG's results deviates very considerably from the luminosity curve adopted in *Groningen Publications*, No. 38. The latter is indicated by the black squares in Figure 1. For the M stars there is no important systematic difference between the new curve and that of *Groningen Publications*, No. 38 (Figure 3). In a comparison of the two curves it should be kept in mind that the Groningen values have been computed from intervals of $1^{M}\cdot0$. If we integrate the new curve over these intervals the agreement for the M stars becomes quite good. A complete luminosity curve for the K stars giving logarithms of the numbers of stars in intervals of 1·0 magnitude may be found under $\log \Phi_3$ in Table 16, *B. A. N.* No. 238. The first part of these curves was derived from the results of the present paper; the extension toward the fainter absolute magnitudes rests on trigonometric parallaxes [1]).

Though I do not know the origin of the discrepancy for the K stars, I think it most likely that VAN RHIJN's curve is in error. For one thing STRÖMBERG's curve was derived much more directly, avoiding the detour through mean parallaxes of stars of given magnitude and proper motion, as well as the still longer detour through spectroscopic absolute magnitudes.

In the preceding article it has been remarked that VAN RHIJN's luminosity curve of the K stars, if used to compute the numbers of stars of various apparent magnitudes near the galactic poles, yields numbers which are about two and a half times larger than the direct counts [2]) (compare the 3rd column of Table 3, headed Comp. I). If for the giants we use the luminosity curve as found in the present note and for the dwarfs the frequencies given by trigonometric parallaxes of the nearest stars [1]), the differences are diminished so much as to become insignificant (column headed Comp. II).

TABLE 3.

Zone at ± 80° galactic latitude				Zone from 0° to ± 20° gal. lat.			
m vis.	Counted	Comp. I	Comp. II	*m* vis.	Counted	Comp. I	Comp. II
4·0 — 5·9	88	212	100	4	38	80	33
6·0 — 7·9	773	2018	946	5	133	300	120
8·0 — 8·2	380	768	350	6	414	1000	460
8·3 — 8·7	680	1880	860	7	1320	3500	1550
				8	4160	11000	5200

[1]) *Groningen Publications*, No. 38, Table 46, 5th column.

[2]) Compare *B. A. N.* No. 238, p. 264. The numbers given cover an area of 3940 square degrees and have been counted from the Draper Catalogue.

It may be remarked that it is not only in high galactic latitudes that VAN RHIJN's curve yields too high a number of stars; the same thing is shown if, using the preliminary density law given by KAPTEYN and VAN RHIJN (2nd column of Table 1 in the present note), we compute the number of K stars of various apparent magnitudes per 1000 square degrees in the zone from 0° to ± 20° galactic latitude. Again the numbers obtained with the curve of *Groningen Publications*, No. 38, as shown under Comp. I in the 7th column of Table 3 [1]), are very much higher than those observed [2]) whereas with the new curve the differences have almost disappeared (compare the last column). Thus the evidence available appears to be favourable for the luminosity curve as found in the present article.

Note concerning the influence of the star-streams on the distribution of peculiar velocities in the directions of τ, υ and the radius vector.

As mentioned on an earlier page VAN RHIJN has called attention to the fact that the distribution of the linear velocities in the direction of the τ components is noticeably different from that of the radial velocities; on account of the ellipsoidal velocity distribution the radial velocities will on the average be a little larger. It is easy to see that this is the case if we take averages over the whole sky. If the moduli of the SCHWARZSCHILD velocity distribution are called h and k respectively, h corresponding to the major axis of the velocity ellipsoid, I find

$$\overline{V_\tau^2} = \frac{1}{2h^2}\left\{\frac{\sin^2\alpha}{2} + \frac{h^2}{k^2}\left(1 - \frac{\sin^2\alpha}{2}\right)\right\}$$

$$\overline{V_\rho^2} = \frac{1}{2h^2}\left(\frac{1}{3} + \frac{2h^2}{3k^2}\right)$$

where $\overline{V_\tau^2}$ and $\overline{V_\rho^2}$ represent the average over the whole sky of the square linear velocities in the direction of the τ component and of the radius vector respectively. α Is the distance between the apex and the true vertex. Putting $\alpha = 42°$ and $h/k = 0{\cdot}60$ (which is the average value found for K and M stars) we obtain $\overline{V_\tau^2} = 0{\cdot}503/2h^2$, $\overline{V_\rho^2} = 0{\cdot}573/2h^2$, indicating an excess of the average radial velocity over the average velocity in the τ direction of 7%. It is quite conceivable that this amount might be of consequence in the derivation of the distribution of absolute magnitudes by a comparison of these two distributions. It would certainly have been easy to avoid the difficulty by assigning different weights to different areas in the computation of the distribution of the radial velocities.

[1]) In the computation of the first three figures some extrapolation had to be used.

[2]) Taken from *Groningen Publications*, No. 30, Table 20.

However, I do not believe that this error caused by the star-streams can be very serious, for we may note that in the case of the υ components it must work in the opposite sense. Denoting by V_υ the peculiar velocity in the υ direction we find

$$\overline{V_\upsilon^2} = \frac{1}{2h^2}\left\{\frac{2}{3} - \frac{\sin^2\alpha}{2} + \frac{h^2}{k^2}\left(\frac{1}{3} + \frac{\sin^2\alpha}{2}\right)\right\} = \frac{0{\cdot}644}{2h^2}$$

(averaging over the whole sky). In discussing the υ components STRÖMBERG has excluded the stars within 30° of the apex and anti-apex. Averaging over the part of the sky actually used the result becomes somewhat larger:

$$\overline{V_\upsilon^2} = \frac{0{\cdot}661}{2h^2}.$$

In deriving the distribution of the absolute magnitudes from the υ components the effect will to some extent be masked by the superposed motion of the sun. Therefore, the error may not have disappeared completely from the combined result from υ and τ components.