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## On the stationary Calcium lines

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$$(18) \quad \text{number of iron atoms} = 1.1 \times 10^{56}.$$

The energy required for the removal of one  $K$ -electron corresponds to 7100 volts, equalling

$$(19) \quad 1.1 \times 10^{-8} \text{ erg.}$$

So the energy required to rob  $\frac{1}{6}$  of the atoms of a  $K$ -electron is:

$$(20) \quad \frac{1}{6} \times 1.1 \times 10^{56} \times 1.1 \times 10^{-8} = 0.12 \times 10^{48} \text{ erg.}$$

6. It is clear from the details of the preceding considerations that the amount (20) is only a rather rough estimate of the energy involved in the change of ionisation degree corresponding to the transition from the effective temperature  $3000^\circ$  to  $6000^\circ$ . However let us compare it with the energy available for radiation if the only source of stellar energy were derived from contraction.

The available contraction energy is equal to the difference of

$$(21) \quad f \frac{M^2}{r_0} \left\{ \frac{3}{2} - \frac{3}{4} \beta - \frac{3}{2} (1 - \beta) \right\} = \frac{3}{4} \beta f \frac{M^2}{r_0}$$

in the two states corresponding to the two effective temperatures.

The numerical value of this difference is equal to

$$(22) \quad 3.6 \times 10^{48} \text{ erg.}$$

As (20) is of the same order of magnitude as (22), and under appropriate circumstances might become equal to this value, we reach the following conclusion: the energy of a star gained by contraction is not only used to augment the molecular and the radiant energy content of the star, but also to furnish the energy required for the progressing ionisation to such a degree that possibly nothing remains available for radiation.

### On the stationary Calcium lines, by *J. Woltjer Jr.*

This note contains some numerical tests of the hypothesis \*) of an interstellar Calcium cloud which, being ionised by the radiation of hot stars, would produce the stationary  $H$ - and  $K$ -lines by selective absorption.

I. The most important factor for the determination of the value of the density required to account for the appearance of the Calcium lines is the mass-absorption coefficient. This coefficient may attain enormous values for monochromatic absorption. MILNE \*\*) has adduced theoretical and experimental evidence to show that values of the order of  $10^9$  C. G. S. units are quite common. \*\*\*)

If we assume that a star is able to ionise the calcium atoms over a distance equal to  $\alpha r_0$  ( $r_0$  being the radius of the star), then the Calcium density  $\rho$ , required to produce a diminution of intensity of the continuous spectrum in the ratio  $1/e^x$ , follows from the relation:

$$(1) \quad x = \alpha r_0 \rho \times 10^9.$$

Here the mass-absorption coefficient has been put equal to  $10^9$ .

I consider a star of mass 50 (sun's mass as unit) and effective temperature  $30000^\circ$ . From EDDINGTON'S

\*) J. S. PLASKETT, The O-type stars, *Publ. of the Dom. Astroph. Obs. Victoria B. C.* II nr. 16.

\*\*) *Phil. Mag.* 47.

\*\*\*) This value corresponds to a diminution of intensity in the ratio  $1/e$  by passing through a screen that contains  $10^{-9}$  gram per  $\text{cm}^2$ .

theory \*) I take the corresponding absolute magnitude equal to  $-7.2$  corresponding to a total emission of energy of

$$2.6 \times 10^{38} \frac{\text{erg}}{\text{sec.}}$$

Hence the radius is equal to

$$0.7 \times 10^{12} \text{ cm.}$$

Substituting in (1) we get:

$$(2) \quad \rho = 1.4 \frac{x}{\alpha} 10^{-21} \frac{\text{gram}}{\text{cm}^3},$$

corresponding to

$$(3) \quad 21 \frac{x}{\alpha} \text{ Ca atoms per cm}^3;$$

$x$  is a not very large number; the value of  $\alpha$  will be discussed under V.

II. We will now compare gravitation and light-pressure for these atoms. I consider a volume of  $1 \text{ cm}^3$  at a distance  $r$  from the centre of the star. The force of gravitation is equal to:

$$(4) \quad f \frac{M \rho}{r^2},$$

$f$  being the gravitation constant, and  $M$  the mass of the star. If we place this volume-element so that it exhibits a surface of  $1 \text{ cm}^2$  to the star, then it receives a radiant energy per second equal to:

\*) *M. N.* 84, p. 310.

$$(5) \quad 2\pi \frac{r_0^2}{r^2} \frac{h\nu^3}{c^2} \frac{d\nu}{e^{\frac{h\nu}{kT_e}-1}}$$

$h$  and  $k$  being the usual constants,  $T_e$  the effective temperature,  $\nu$  the frequency of the  $Ca$ -line,  $d\nu$  the interval in  $\nu$  corresponding to the breadth of the line. In this formula I have neglected the circumstance that not all parts of the stellar photosphere are equally distant from the volume-element under consideration. The momentum absorbed by the volume element per second is equal to the product of (5), the mass-absorption coefficient,  $\rho$  and  $c^{-1}$ ,  $c$  being the velocity of light. In the ratio of light-pressure to gravitational force the quantities  $\rho$  and  $r$  cancel out; substituting numerical values (wave-length of the  $Ca$ -lines approximately  $4000 \text{ \AA}$ ) we find for the ratio of light-pressure to gravitation

$$(6) \quad 1.0 \times 10^3 \times (d\lambda \text{ in } \text{\AA}).$$

If we take \*) e. g.  $d\lambda = 0.01 \text{ \AA}$ , and keep in mind possible uncertainties in the value of the mass-absorption coefficient, (6) is seen to be of the order of magnitude of unity.

III. A rough estimate of the force of resistance offered to the star on its passage through the calcium cloud may be obtained in the following way. Let  $V$  be the stellar velocity. The volume of space swept through by the star in one second is  $\pi r_0^2 V$ .

Suppose all  $Ca$ -atoms in this volume to be captured by the star. Then the maximum possible contribution to the momentum of the star is equal to:

$$(7) \quad \rho \pi r_0^2 V^2,$$

corresponding to a negative acceleration of

$$(8) \quad \pi \rho r_0^2 \frac{V^2}{M}.$$

Substituting  $V = 50 \frac{\text{km}}{\text{sec}}$ , the resulting diminution of the velocity is equal to

$$(9) \quad 1.1 \times 10^5 \rho \frac{\text{km}}{\text{sec}} \text{ in a year.}$$

IV. The distance of the O stars is of the order of magnitude \*\*) of 1000 parsecs. As these stars are strongly condensed towards the galactic plane (the same being true of the B stars) I take \*\*\*) the volume of the cloud equal to a cylinder with a height of 100 parsecs and a volume of  $\pi \times 10^3 \times 10^2$  cubic parsecs,

\*) MILNE, *l. c.* p. 217.

\*\*) J. S. PLASKETT, *l. c.*

\*\*\*) Only for the sake of numerical definition.

equalling  $0.92 \times 10^{64} \text{ cm}^3$ . Taking the density according to (2) the resulting mass is equal to:

$$(10) \quad 6 \times 10^9 \frac{x}{\alpha} (\text{sun's mass as unit}).$$

V. From the preceding formulas it appears that an estimate of the value of  $\rho$  is only possible if a value of  $\alpha$  has been found. MILNE \*) has derived a formula for the rate of photo-electric ionisation in thermodynamic equilibrium. According to his results the number of photo-electric ionisations of a single atom in one second is equal to:

$$(11) \quad 8 \frac{\pi^2 m_e^2 \sigma}{q h^3} \mu k T e^{-\frac{\chi}{kT}},$$

$m_e$  being the mass of an electron,  $\sigma$  and  $q$  the symmetry number and the "statistical weight" of the atom,  $\chi$  corresponds to the ionisation potential;  $\mu$  is a constant that we may take equal to

$$(12) \quad 0.83 \times 10^{-4} \times \text{atomic number.}$$

Substituting  $T = 30000^\circ$ ,  $\sigma = 2$ ,  $q = 1$ ,  $\chi = 6.0$  volt, we get

$$(13) \quad 0.3 \times 10^{12}.$$

Here the atom is supposed to be subjected to isotropic radiation; the volume element however only receives light from the stellar disc; the corresponding diminution is effected by multiplication by

$$(14) \quad \frac{r_0^2}{4 r^2}.$$

The difficulty however is that we do not know the number of electron-captures per atom per second; this number will depend on the concentration of the electrons and the electronic velocities, quantities about which it is rather difficult to make an estimate.

As (13) is rather large, it is possible that the fraction (14) may be small and yet all the atoms at this distance are ionised. Thus presumably  $\alpha$  may take large values.

VI. Recapitulating we can state that, as far as regards the points of view under consideration, the hypothesis that the stationary  $H$  and  $K$  lines of Calcium are due to an interstellar cloud which, being ionised by the presence of very hot stars, absorbs these lines vigorously, presents nothing improbable. Light pressure and gravitation nearly balance, the resistance offered to the motion of the stars is negligible, the total mass is not extravagant.

\*) *l. c.*