THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME 112 JULY 1950 NUMBER 1

THE AMOUNT OF POLARIZATION BY INTERSTELLAR GRAINS

H. C. VAN DE HULST
Leiden Observatory

Received April 21, 1950

ABSTRACT

The extinction of light by long dielectric cylinders is computed for different directions of polarization. Cylinders with diameters of the order of the wave length give the correct dependence of interstellar polarization on wave length, whereas cylinders or ellipsoids small compared to the wave length do not. The computed amount of polarization is barely sufficient to explain the observations, even if the grains are strongly elongated and fully aligned.

The important discovery by Hiltner[1] and its confirmation by Hall,[2] that the light of many distant stars is polarized, has been followed by several explanations. These suggestions contain many speculative points, but, unless some quite different possibility is overlooked, we may assert these facts: (1) The phenomenon is interstellar rather than stellar, for it appears in stars of diverse types and only at large distances. (2) The phenomenon has the character of a differential extinction, light vibrating in one direction (electric vector roughly parallel to galactic plane) being less strongly attenuated in interstellar space than light vibrating in the perpendicular direction. Hall has found for one star that the transmitted light is indeed linearly, not elliptically, polarized. (3) This extinction is caused by the same solid grains that are believed to cause the ordinary interstellar extinction; for no other constituent of interstellar matter is known to be able to give even the minimum extinction needed for one polarized component if the other component is assumed to suffer no extinction at all. Hiltner[3] has shown that this holds true also for free electrons.

Admitting that interstellar grains are responsible, we may estimate the ratio of the extinction values of the two polarized components. Let these values, expressed in magnitudes, be A_1 and A_2, where $A_1 > A_2$. The magnitude difference, $A_1 - A_2$, is the polarization measured by Hiltner. The photographic extinction for unpolarized light follows from

$$\frac{1}{2}(A_1 + A_2) = 9E_1 ,$$

where E_1 is the color excess on Stebbins' scale. Several stars have a well-observed value of $(A_1 - A_2)/E_1$ as high as 0.50. This gives

$$\frac{A_1 - A_2}{A_1 + A_2} = 0.03 ; \quad \text{so} \quad \frac{A_1}{A_2} = 1.06$$

[1] *Ap. J.*, **109**, 471, 1949.

[2] J. S. Hall and A. H. Mikesell, *A.J.*, **54**, 187, 1949.

[3] *Phys. Rev.*, **78**, 170, 1950.

for blue light. Hiltner's observations[3] show that $A_1 - A_2$ is nearly independent of λ, but, since $A_1 + A_2$ decreases by a factor of 2 if the wave length increases from λ 4400 to λ 8000, the ratio A_1/A_2 increases with increasing λ, reaching the value 1.12 for strongly polarized stars in the infrared.

All previous explanations suggest that the grains are elongated and aligned by magnetic forces.[4] Apart from the physical difficulties in this suggestion, one simple question has to be answered: Do such particles give a sufficiently different extinction for different directions of polarization of the incident light? I started some calculations on this question soon after the discovery of interstellar polarization was announced; the preliminary results are reported below.

Scattering problems are simple applications of Maxwell's theory, in principle, but few of them have solutions that admit of a simple numerical evaluation. Such solutions are obtained, for instance, if the scattering particles are (*a*) homogeneous ellipsoids that are small compared to the wave length of the incident light and (*b*) long, circular cylinders with arbitrary radii, with light incident in a direction perpendicular to the cylinder axis. The formal solution for ellipsoids of arbitrary size has also been obtained,[5] but its evaluation would require very lengthy computations; *a* and *b* are special cases of this more general problem.

Problem *a* is the only one that I have seen referred to in the papers on interstellar polarization; it leads to simple formulae, first derived by Gans.[6] The authors[4] who suggest that these formulae are applicable make the implicit assumption that the grains causing interstellar polarization are small compared with the wave length. These grains would thus form a class separate from the grains causing ordinary extinction, which must have sizes of the order of the wave length in order to explain the observed reddening law. Such a superposition of two classes of interstellar grains would seem artificial, but not impossible. For instance, if the polarizing grains have the extinction values $A_1' > A_2'$ and the ordinary grains have the values $A_1'' = A_2''$, the observed ratio is

$$\frac{A_1' + A_1''}{A_2' + A_2''} = 1.06 .$$

We may estimate that the wave-length dependence of interstellar reddening is not affected if $A_1' + A_2' \leqslant \frac{1}{10}(A_1'' + A_2'')$. Combining these equations, we find that A_1'/A_2' has to be about 2, or larger. Such a large ratio requires prolate spheroids with the axial ratios 2/1 or 6/1, for refractive indices 2 or 1.4, respectively. This makes this explanation very improbable. An even stronger argument against it is found from the dependence of polarization on wave length. The difference, $A_1 - A_2 = A_1' - A_2'$, which is a constant according to observation, would have to be proportional to λ^{-4} (nonabsorbing grains), or λ^{-1} (absorbing grains), according to the theory for small particles. It thus seems fairly certain that a solution based on the formulae of Gans must be rejected.

Problem *b* leads to more complex formulae,[7] analogous in many details to Mie's formulae for scattering by spheres. For instance, if the electric vector of the incident light is parallel to the cylinder axis, which gives the strongest effects of scattering and extinction, the principal quantities to be computed are the complex coefficients

$$b_n(x, m) = \frac{mJ_n'(y)\,J_n(x) - J_n(y)\,J_n'(x)}{mJ_n'(y)\,H_n(x) - J_n(y)\,H_n'(x)} . \tag{1}$$

[4] L. Spitzer and J. W. Tukey, *Science*, **109**, 461, 1949; L. Davis and J. L. Greenstein, *Phys. Rev.*, **75**, 1605, 1949.

[5] F. Möglich, *Ann. d. Phys.*, **83**, 609, 1927.

[6] *Ann. d. Phys.*, **37**, 881, 1912.

[7] Cl. Schaeffer and F. Grossman, *Ann. d. Phys.*, **31**, 455, 1910.

Here n is an integer, m is the refractive index, $y = mx$, and x is the ratio $2\pi r/\lambda$, where r is the radius of the cylinder and λ is the wave length. Further, $J_n(x)$ are Bessel functions, $H_n(x)$ are Hankel functions of the second kind, and primes denote derivatives. The efficiency factor for extinction, defined as the ratio between the extinction cross-section and the geometrical cross-section, is found from

$$Q_1(x, m) = \frac{2}{x}\sum_{n=-\infty}^{\infty} \Re\, b_n\,, \tag{2}$$

where $b_n = b_{-n}$ and the symbol $\Re$ means that the real part is taken. If the magnetic vector of the incident light is parallel to the cylinder axis, the solution is found from

$$Q_2(x, m) = \frac{2}{x}\sum_{n=-\infty}^{\infty} \Re\, a_n\,, \tag{3}$$

where the coefficients $a_n(x,m)$ are obtained by removing the factors m in equation (1) from the first terms of the numerator and denominator and placing them with the second terms.

The results of computations for refractive indices 1.25 and 1.50 are shown in Figure 1. The actual procedure was to write b_n and a_n in the form

$$b_n = \tfrac{1}{2}\left(1 - e^{-i\beta_n}\right), \qquad a_n = \tfrac{1}{2}\left(1 - e^{-i\alpha_n}\right),$$

and to compute the real angles β_n and α_n. The *Tables of Amplitudes and Phase Angles*,[8] a copy of which was very kindly made available to me by Dr. A. N. Lowan, were of great help in this computation. Very effective use was also made of graphs of the angles β_n and α_n, as suggested earlier for spheres.[9] The "nodes," where $\beta_n = \alpha_n$, have some very curious properties that afford checks in so many ways that errors are detected easily from these graphs and a full check is obtained at the stage of the computation where it is most needed. Additional values were found by interpolation from these graphs. The largest number of terms needed in the series for Q_1 and Q_2 was from $n = 0$ to $n = 10$ for $m = 1.25$, $x = 8.0$.

The bottom curve of Figure 1 refers to refractive indices close to 1. A derivation similar to the one made for spheres[10] leads to the formula

$$Q(x, m) = \pi \mathfrak{S}_1(\rho)$$

for both directions of polarization alike. Here $\mathfrak{S}_1$ is Struve's function tabulated by Jahnke and Emde[11] and $\rho = 2x(m - 1)$. The curves of Figure 1 have been drawn to a common scale of ρ, which equals x for $m = 1.50$ and $\frac{1}{2}x$ for $m = 1.25$. The values corresponding to the first terms of the series expansion for small ρ, or x, are indicated by dotted curves. The formulae are:

$$\text{For } m = 1.50: \quad Q_1 = 1.93x^3, \quad Q_2 = 0.364x^3, \quad \frac{Q_1}{Q_2} = 5.30\,;$$

$$\text{For } m = 1.25: \quad Q_1 = 0.39x^3, \quad Q_2 = 0.128x^3, \quad \frac{Q_1}{Q_2} = 3.03\,;$$

[8] A. N. Lowan, P. M. Morse, H. Feshbach, and M. Lax, *Scattering and Radiation from Circular Cylinders and Spheres: Tables of Amplitudes and Phase Angles* (U.S. Navy Department, 1946).

[9] H. C. van de Hulst, *Rech. Astr. Obs. Utrecht*, Vol. **11**, Part I, chap. viii, 1946.

[10] *Ibid.*, chap. vii.

[11] Jahnke and Emde, *Tables of Functions* ("Dover Publications" [New York, 1945]), esp. Fig. 117.

and for any real value of m:

$$Q_1 = \frac{\pi^2 (m^2 - 1)^2}{8} x^3 , \qquad Q_2 = \frac{\pi^2 (m^2 - 1)^2}{4 (m^2 + 1)^2} x^3 , \qquad \frac{Q_1}{Q_2} = \frac{(m^2 + 1)^2}{2} .$$

The further discussion follows closely the pattern set by previous discussions of interstellar reddening.[12] The argument, ρ, is proportional to λ^{-1}; the ordinates, Q_1 and Q_2, are

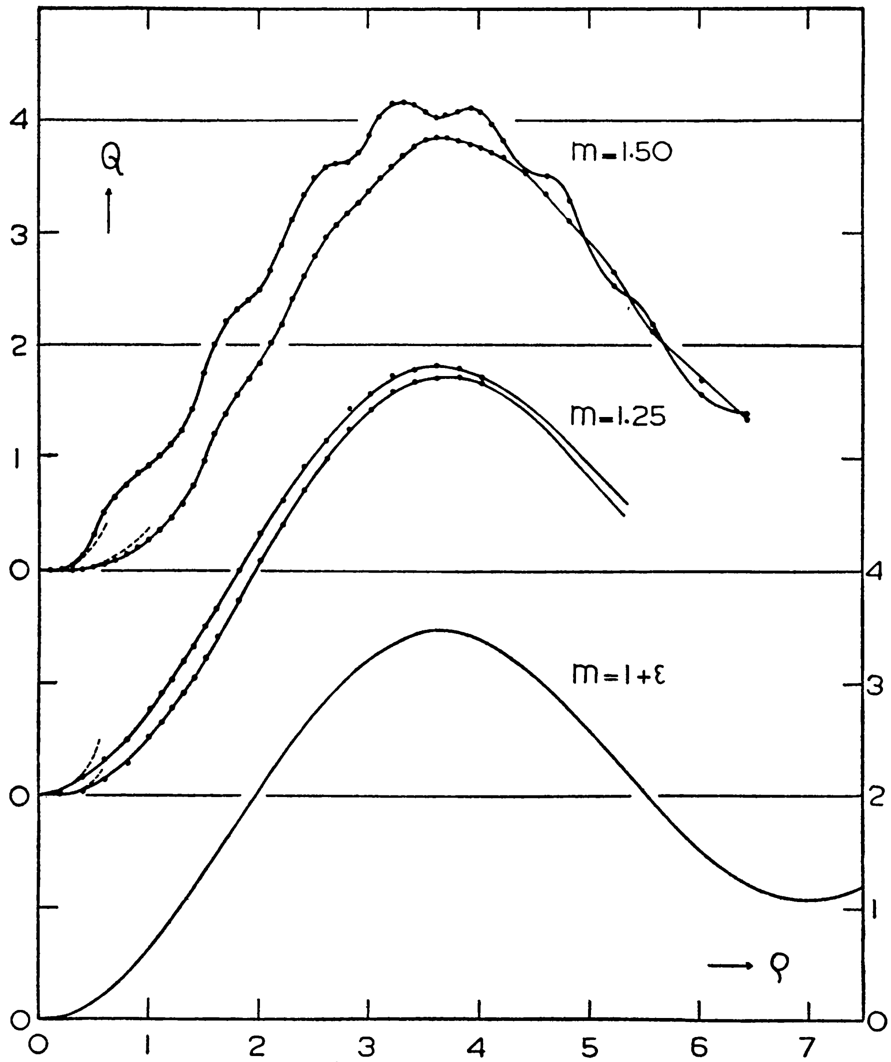

FIG. 1.—Extinction by long, circular cylinders for different values of the refractive index m, as a function of their size with respect to the wave length (perpendicular incidence). Q is the efficiency factor; the upper curve of each set refers to linear polarization with the electric vector parallel to the cylinder axis (Q_1), the lower curve to electric vector perpendicular to the cylinder axis (Q_2). The scale of abscissae: $\rho = 2x(m - 1)$, where $x = 2\pi r/\lambda$, is common to all curves.

proportional to the extinction values, A_1 and A_2, at a given wave length. The observed reddening law is well represented by cylinders of a single size if λ^{-1} is about equal to ρ, the wave length being expressed in microns. So blue light, $\lambda = 0.41\mu$, corresponds to $\rho = 2.44$; yellow light, $\lambda = 0.55\ \mu$, to $\rho = 1.82$; and the near infrared, $\lambda = 0.80\ \mu$, to

[12] H. C. van de Hulst, *op. cit.*, Part II, chap. iii, 1949.

$\rho = 1.25$. All these values lie outside the region where the theory for thin rods holds, so that the ratio $A_1/A_2 = Q_1/Q_2$ is notably smaller than the values 5.3 and 3.0 mentioned above. In fact, we read from the curves that $Q_1/Q_2 = 1.22$ for $m = 1.50$ and $Q_1/Q_2 = 1.07$ for $m = 1.25$ in the region corresponding to blue light. Since the strongest observed value is $A_1/A_2 = 1.06$ for blue light, we may infer that ice needles ($m = 1.25$) can just explain the observed amount of polarization if they are very long and perfectly aligned. No margin at all is left for the effects of imperfect alignment and of reduced length, both of which tend to reduce the expected polarization. For $m = 1.50$, which is already an improbably high value, the margin is still small. Rough estimates, that cannot be explained in the present context, show that the amount of polarization, $Q_1 - Q_2$, is reduced by a factor of 2 if long cylinders are replaced by prolate spheroids with the axial ratio 2 to 1. It is reduced by a factor of 4 if the spheroids have the axial ratio 1.3 to 1. So glass spheroids ($m = 1.50$) with the axial ratio 1.5 to 1, all perfectly aligned, would also be capable of explaining the observed ratio $A_1/A_2 = 1.06$ for blue light.

The dependence of $Q_1 - Q_2$ on ρ is very interesting. Apart from the irregular bumps, that will be smoothed out if particles of various sizes are mixed, we see that $Q_1 - Q_2$ is almost independent of ρ in the entire range of interest. So, while Q_1 and Q_2 increase rapidly with increasing ρ, their difference is virtually constant, so that the ratio Q_1/Q_2 decreases. If Q_1 and Q_2 are replaced by A_1 and A_2, and ρ by λ^{-1}, this is exactly what Hiltner's recent observations show. The sign is such that the long axes of the grains have to be perpendicular to the galactic plane. It would seem premature to make any more detailed computations at this stage.

The conclusion is that interstellar grains of the ordinary size, as indicated by reddening measurements, may give a barely sufficient amount of polarization, which, however, has the correct dependence on wave length. We hope that partly absorbing grains will leave a better margin by giving a stronger polarization effect. Computations on such particles are in preparation.