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A theory is presented for the universal reduction of shot noise by Coulomb repulsion, which was observed in computer simulations of a disordered nondegenerate electron gas by Gonzalez *et al* [Phys Rev Lett **80**, 2901 (1998)] The universality of the reduction below the uncorrelated value is explained as a feature of the high-voltage regime of space-charge-limited conduction The reduction factor depends on the dimensionality d of the density of states, being close but not quite equal to $1/d$ in two and three dimensions [S0031-9007(99)08839-0]

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The motivation for this work comes from a remarkable recent Letter [1] by González, González, Mateos, Pardo, Reggiani, Bulashenko, and Rubí on the "universality of the $\frac{1}{3}$ shot-noise suppression factor in nondegenerate diffusive conductors" Shot noise is the time-dependent fluctuation in the electrical current caused by the discreteness of the charge In the last few years there has been a breakthrough in the use of shot-noise measurements to study correlation effects in diffusive conductors [2] In the absence of correlations between the electrons, the current fluctuations $\delta I(t)$ around the mean current $\bar{I}$ are described by a Poisson process, with a spectral density P at low frequencies equal to $P_{\text{Poisson}} = 2e\bar{I}$ Correlations reduce the noise below the Poisson value

The Pauli exclusion principle is one source of correlations, and Coulomb repulsion is another In a *degenerate* electron gas with elastic impurity scattering the reduction is a factor of $\frac{1}{3}$ [3,4] This reduction is due to the Pauli principle The remarkable finding of González *et al* was that Coulomb repulsion in a three-dimensional *nondegenerate* electron gas also gives $P/P_{\text{Poisson}} = \frac{1}{3}$ They argued for a universal physical principle behind the one-third reduction of the shot noise from elastic scattering, regardless of whether the origin of the electron correlations is quantum mechanical (Pauli principle) or classical (Coulomb repulsion)

The significance of Ref [1] has been assessed critically by Landauer [5] While in a degenerate electron gas the one-third reduction is independent of the number d of spatial dimensions, Ref [1] finds $P/P_{\text{Poisson}} = \frac{1}{2}$ for $d = 2$—spoiling the supposed universality of the reduction factor in nondegenerate conductors Still, it remains a remarkable finding that the ratio P/P_{Poisson} has no dependence on microscopic parameters (such as mean free path l or dielectric constant κ) or global parameters (such as temperature T, voltage V, or sample length L), as long as one stays in the high-voltage, diffusive regime ($eV \gg kT$ and $L \gg l$) The findings of Ref [1] are based on a computer simulation of the dynamics of an interacting electron gas Here we develop an analytical theory that explains these numerical results

We use the same theoretical framework as Nagaev [4] used for the degenerate electron gas, namely, the Boltzmann-Langevin equation [6] in the diffusion approximation (valid for $L \gg l$) A difference with Ref [4] is that the kinetic energy $\varepsilon = \frac{1}{2}mv^2$ now appears as an independent variable (In the degenerate case one may assume that all nonequilibrium electrons have velocity v equal to the Fermi velocity) The charge density $\rho(\mathbf{r},\varepsilon,t) = \bar{\rho} + \delta\rho$, current density $\mathbf{j}(\mathbf{r},\varepsilon,t) = \bar{\mathbf{j}} + \delta\mathbf{j}$, and electric field $\mathbf{E}(\mathbf{r},t) = \bar{\mathbf{E}} + \delta\mathbf{E}$ fluctuate in time around their time-averaged values (indicated by an overline) In the low-frequency regime of interest we may neglect the time derivative in the continuity equation,

$$\left(\frac{\partial}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial}{\partial \varepsilon}\right) \mathbf{j} = 0 \tag{1}$$

Current and charge density are related by the drift-diffusion equation

$$\mathbf{j} = -D\frac{\partial \rho}{\partial \mathbf{r}} - \sigma\frac{\partial f}{\partial \varepsilon}\mathbf{E} + \delta\mathbf{J}, \tag{2}$$

with $\sigma(\varepsilon) = e^2 D(\varepsilon)\nu(\varepsilon)$ the conductivity, $D(\varepsilon) = D_0\varepsilon$ the diffusion constant, and $\nu(\varepsilon) = \nu_0\varepsilon^{d/2-1}$ the density of states (The coefficients D_0 and ν_0 are ε independent, assuming an energy-independent effective mass and scattering rate) The function $f(\mathbf{r},\varepsilon,t) = \rho/e\nu = \bar{f} + \delta f$ is the occupation number of a quantum state (In equilibrium, the mean $\bar{f}$ is the Fermi-Dirac distribution function) The "Langevin current" $\delta\mathbf{J}(\mathbf{r},\varepsilon,t)$ is a stochastic source of current fluctuations from elastic scattering [6] Its first two moments are $\overline{\delta\mathbf{J}} = 0$ and

$$\overline{\delta J_i(\mathbf{r},\varepsilon,t)\delta J_j(\mathbf{r}',\varepsilon',t')} = 2\sigma(\varepsilon)\bar{f}(\mathbf{r},\varepsilon)[1 - \bar{f}(\mathbf{r},\varepsilon)] \times \delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\delta(\varepsilon-\varepsilon') \times \delta(t-t') \tag{3}$$

The definition of a nondegenerate electron gas is $\bar{f} \ll 1$, so that we may ignore the factor of $1 - \bar{f}$ in this correlator

We need one more equation to close the problem, namely, the Poisson equation

$$\kappa\frac{\partial}{\partial \mathbf{r}} \mathbf{E} = \int d\varepsilon(\rho - \rho_{\text{eq}}), \tag{4}$$

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with ρ_{eq} the compensating charge present in the semiconductor in equilibrium (equal to the charge density of the carriers prior to the injection from the contacts).

The geometry we are considering is a disordered semiconductor of uniform cross-sectional area A sandwiched between metal contacts at $x = 0$ and $x = L$. We denote by $\mathbf{r}_\perp$ the $d - 1$ dimensional vector of transverse coordinates. (Only $d = 3$ is physically relevant, but we consider arbitrary d for comparison with the computer simulations.) The mean values of ρ, $\mathbf{j}$, and $\mathbf{E}$ are independent of $\mathbf{r}_\perp$, but the fluctuations are not. We define the linear charge density $\rho(x,t) = \int d\mathbf{r}_\perp \int d\varepsilon\, \rho(\mathbf{r},\varepsilon,t)$, the electric field profile $E(x,t) = A^{-1}\int d\mathbf{r}_\perp\, E_x(\mathbf{r},t)$, and the currents $I(t) = \int d\mathbf{r}_\perp \int d\varepsilon\, j_x(\mathbf{r},\varepsilon,t)$ and $\delta J(x,t) = \int d\mathbf{r}_\perp \int d\varepsilon\, \delta J_x(\mathbf{r},\varepsilon,t)$. The total current I is independent of x at low frequencies because of the continuity equation, but the Langevin current δJ is not so restricted. In view of Eq. (2), the two currents are related by

$$I = -\int d\mathbf{r}_\perp \int d\varepsilon\, D\frac{\partial \rho}{\partial x} + \mu\rho E + \delta J, \tag{5}$$

with $\mu = \frac{1}{2}de\,D_0$ the mobility. (To write the drift term in the form $\mu\rho E$ we have made a partial integration over energy and linearized with respect to the fluctuations.)

A nonfluctuating voltage V is applied between the two metal contacts, with the current source at $x = 0$ and the current drain at $x = L$. (The charge e of the carriers is taken to be positive.) For high V the charge injected into the semiconductor by the current source will be much larger than the charge ρ_{eq} present in equilibrium. We will neglect ρ_{eq} altogether. For sufficiently high voltages, when all the surface charge at $x = 0$ has been injected into the semiconductor, the system enters the regime of space-charge-limited conduction, characterized by the boundary condition

$$\bar{E}(x) = 0 \quad \text{at } x = 0. \tag{6}$$

The mean charge and field distributions in this regime were studied extensively in the past [7], but apparently the shot-noise problem was not. We argue that the universality of the computer simulations [1] is a consequence of the homogeneity of the boundary condition (6). Indeed, if the boundary condition would have contained an external electric field, then the effect of Coulomb repulsion on the shot noise would have depended on the relative magnitude of the induced and external fields and hence on the value of κ. No universal reduction factor could have resulted. This scenario stands opposite to that in the degenerate case. There the reduction of shot noise occurs at low voltages, in the linear-response regime, when the induced electric field can be neglected relative to the external field [8].

The zero-frequency limit of the noise spectral density is given by

$$P = 2\int_{-\infty}^{\infty} dt\, \overline{\delta I(0)\delta I(t)}. \tag{7}$$

To compute P we need to relate the correlator of the total current δI to the correlator of the Langevin current δJ. At nonzero temperatures, δI contains also a contribution from the thermal fluctuations of the charge at the contacts (Johnson-Nyquist noise [6]). This source of noise may be neglected relative to the shot noise for $eV \gg kT$, and we will do so to simplify the problem. The most questionable simplification that we will make is to neglect the diffusion term ($\propto \partial\rho/\partial x$) relative to the drift term ($\propto E$) in the drift-diffusion equation (5). This approximation is customary in treatments of space-charge-limited conduction [7], but is only rigorously justified here in the formal limit $d \to \infty$ (when the ratio $\mu/D_0 \to \infty$).

We are now ready to proceed to a solution of the coupled kinetic and Poisson equations. We consider first the mean values and then the fluctuations. Combination of Eq. (4) (without the term ρ_{eq}) and Eq. (5) (without the diffusion term) gives for the mean electric field

$$\bar{E}\frac{d\bar{E}}{dx} = \frac{\bar{I}}{\mu\kappa A} \Rightarrow \bar{E}(x) = \left(\frac{2\bar{I}x}{\mu\kappa A}\right)^{1/2}, \tag{8}$$

where we have used the boundary condition (6). The $\sqrt{x}$ dependence of the electric field is the celebrated Mott-Gurney law [9]. The corresponding charge density has an inverse square root singularity at $x = 0$ [10]. The corresponding voltage $V = \int_0^L \bar{E}\,dx \propto \sqrt{\bar{I}}$, so that the current increases quadratically with the voltage. These are well-known results for space-charge-limited conduction [7].

Linearization of Eqs. (4) and (5) around the mean values gives for the fluctuations

$$\bar{E}\frac{\partial}{\partial x}\delta E + \frac{d\bar{E}}{dx}\delta E = \frac{\delta I - \delta J}{\mu\kappa A} \Rightarrow \delta E(x,t) = x^{-1/2}\int_0^x dx'\,\frac{\delta I(t) - \delta J(x',t)}{(2\bar{I}\mu\kappa A)^{1/2}}. \tag{9}$$

A nonfluctuating voltage requires $\int_0^L \delta E\,dx = 0$, hence

$$\delta I(t) = 3\int_0^L \frac{dx}{L}(1 - \sqrt{x/L})\delta J(x,t). \tag{10}$$

Combination of this relation between δI and δJ with Eqs. (3) and (7) gives an expression for the shot-noise power,

$$P = \frac{36A}{L}\int_0^L \frac{dx}{L}(1 - \sqrt{x/L})^2 \int d\varepsilon\, \sigma(\varepsilon)\bar{f}(x,\varepsilon). \tag{11}$$

To evaluate this expression we need to know the mean occupation number $\bar{f}$ out of equilibrium. For this purpose it is convenient to change variables from kinetic energy ε to total energy $u = \varepsilon + e\phi(x)$, with $\phi(x)$ the mean electrical potential. Since $\bar{E} = -d\phi/dx$, the derivative $\partial/\partial x + e\bar{E}\,\partial/\partial\varepsilon$ is equivalent to $\partial/\partial x$ at constant u. The kinetic equations (1) and (2) in the new variables x, u take the form

$$\frac{\partial \bar{j}}{\partial x} = 0, \qquad \bar{j} = -\frac{1}{e}\sigma[u - e\phi(x)]\frac{\partial \bar{f}}{\partial x}. \tag{12}$$

The solution is

$$\bar{f}(x,u) = \frac{\bar{f}(0,u)}{AR(u)} \int_x^L \frac{dx'}{\sigma[u - e\phi(x')]}, \quad (13)$$

where we have imposed the absorbing boundary condition $\bar{f}(L,u) = 0$ at the current drain. (At high voltages the charge injected into the semiconductor by the current drain can be neglected.) The factor $R(u) = A^{-1}\int_0^L dx/\sigma[u - e\phi(x)]$ is the resistance of the semiconductor. The mean current is related to R by $e\bar{I} = \int du\, \bar{f}(0,u)/R(u)$. The argument $u - e\phi(x)$ of σ may be replaced by $eV(x/L)^{3/2}$ in the high-V limit. Then $\sigma \propto x^{3d/4}$ and

$$\int du\, \sigma[u - e\phi(x)]\bar{f}(x,u) \to \frac{e\bar{I}}{A}\int_x^L dx' \left(\frac{x}{x'}\right)^{3d/4} = \frac{4e\bar{I}L}{(3d-4)A}\left[\frac{x}{L} - \left(\frac{x}{L}\right)^{3d/4}\right] \quad (14)$$

Substitution into Eq. (11) yields our final result

$$P = \frac{144e\bar{I}}{3d-4}\int_0^1 dx\,(1-\sqrt{x})^2(x - x^{3d/4})$$
$$= \frac{24e\bar{I}}{5}\frac{3d^2+22d+64}{(d+2)(3d+4)(3d+8)}. \quad (15)$$

The ratio $P/P_{\rm Poisson}$ equals 0.341 and 0.514 for $d = 3$ and $d = 2$, respectively, within error bars of the fractions $\frac{1}{3}$ and $\frac{1}{2}$ inferred by González *et al.* from their computer simulations [1]. The proximity of these numbers to d^{-1} appears to be accidental. Indeed, for large d we find that $P/P_{\rm Poisson} \to \frac{4}{3}d^{-1}$. The large-$d$ limit is a rigorous result, while the finite-d values are not because we have neglected the diffusion term in Eq. (5).

In closing, we comment on the universality of the results and on their experimental observability. Concerning the universality, the dimensionality dependence has already been noted [1]. For a given d there is no dependence on material parameters, however, the shot noise does depend on the model chosen for the energy dependence of the elastic scattering rate. We have followed the computer simulations [1] in assuming an ε-independent scattering rate. In a model of short-range impurity scattering one would have instead a rate proportional to the density of states. This would change the energy dependence of the diffusion constant from $D \propto \varepsilon$ to $D \propto \varepsilon^{2-d/2}$. The shot-noise power remains unaffected for $d = 2$, but for $d = 3$ one obtains [11] $P/P_{\rm Poisson} = 0.407$—some 20% above the value for an ε-independent scattering rate. Concerning the experimental observability, the main obstacle is the tendency of electron-phonon scattering to equilibrate the electron gas at the lattice temperature. Then, instead of shot noise one would measure thermal noise (modified for a non-Ohmic conductor [12]) that is not sensitive to correlation effects. Experiments in degenerate systems have succeeded recently in observing shot noise by reducing the sample dimensions to the mesoscopic scale [13]. The same approach may well be successful also in nondegenerate systems.

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