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Investigations concerning the rotational motion of the galactic system, together with new determinations of secular parallaxes, precession and motion of the equinox, by *J. H. Oort.*

1. *Introduction and summary.*

It has been shown in *B. A. N.* **120** that the radial velocities of nearly all distant stars give evidence of a rotation of the entire galactic system around a remote axis in the direction of the centre of the globular cluster system. The principal aim of the present article is to give the definitive discussion of the rotation shown by *proper motions*, some preliminary results of which have appeared in *B. A. N.* **120**. It is shown that the proper motions indicate a rotation in the same direction and of the same character as the radial velocities. From nearly 800 distant stars we find $B = -0.024$ *km/sec. parsec* $\pm$ 0.005 (m.e.) (for the meaning of A and B see section **2**). Combining this with the new value of A found in the 2nd section (viz. $A = +0.019$ *km/sec. parsec* $\pm$ 0.003 (m.e.)) preliminary results have been derived for the distance of the rotational axis and for the character of the gravitational force (see section **7**).

In the course of the investigation it was necessary to compute new values of the secular parallaxes of the distant stars. These values, which are independent of the rotation and of any uncertainty in the precessional constant, are shown in table 3. Especially those of the O stars and Md variables deviate strongly from formerly adopted results. It was also necessary to derive from the proper motions in galactic latitude an absolute value of the precessional constant, and in this solution a correction to NEWCOMB's motion of the equinox was introduced as an additional unknown. It was found to be very probable that NEWCOMB's motion of the equinox requires a correction of $+''.014 \pm ''.002$ (m. e.) (see page 85). The corresponding correction to NEWCOMB's precessional constant is then $+''.011 \pm ''.002$ (m.e.). If NEWCOMB's motion of the equinox is assumed to be correct, the correction to his precessional constant is found to amount to $+''.0029 \pm ''.001$ (m. e.). Two graphs showing how the observed proper motions in galactic longitude and latitude fit the computed curves are given in the sixth section, which discusses the systematic residuals.

In the second section some new rotation results are derived from radial velocities; from all the radial velocity results available a new value of A is computed, after application of tentative corrections to the average parallaxes in order to reduce them to reciprocals of average distances.

On page 80 I have collected all more or less reliable results for the galactic longitude of the centre of rotation. The final average is either 144° or 324° $\pm$ 2° (m. e.); the last value agrees closely with the centre of the system of globular clusters which is at 325° $\pm$ 3° (m. e.) galactic longitude.

At this place I want to thank Mr. PELS for the very efficient and helpful way in which he has carried out the greater part of the computations underlying the results of this paper.

2. *Additional results from radial velocities.*

I shall adopt the same notation as in *B. A. N.* **120**: the average uncorrected radial velocities, $\overline{\rho}$, will in general be represented by a formula of the following form

$$\overline{\rho} = V_0 \cos \overline{\lambda} + \overline{r} A \sin 2\,(\overline{l} - l_0) \cos^2 \overline{b}$$

and the average transverse velocities, after correction for solar motion and precessional errors, by *)

$$\overline{\mu}_l' = \frac{A}{4.74} \cos 2\,(\overline{l} - l_0) \cos \overline{b} + \frac{B}{4.74} \cos \overline{b}$$

The various letters represent the following quantities:

V_0 the solar velocity with respect to the stars considered

*) The formula may be transformed into equatorial components with the aid of the parallactic angle, η, between the great circle towards the pole of the milky way and that towards the pole of the equator. (This angle has been tabulated by INNES in *Union Observatory Circular* No. 29, 1915). We have $\mu_\alpha = \mu_l \cos\eta - \mu_b \sin\eta$ and $\mu_\delta = \mu_l \sin\eta + \mu_b \cos\eta$.

λ the distance to the solar apex
r the distance from the sun in parsecs
l and b the galactic co-ordinates of a star
l_0 the longitude of the centre, usually assumed to be 325°

$$A = \frac{V}{4R}\left(1 - \frac{R}{K}\frac{\partial K}{\partial R}\right) \text{ in } km/sec.\,parsec$$

$$B = A - \frac{V}{R} \text{ in } km/sec.\,parsec$$

R the distance from the sun to the centre
K the total gravitational force at this point
V the circular velocity in *km/sec.*

The coefficients $\cos b$ and $\cos^2 b$ were omitted when dealing with stars showing considerable galactic concentration and in some cases the solar motion was assumed to 20 *km/sec.*

It was shown in the paper quoted that not only did nearly all distant stars give positive values of $\overline{rA}$, thereby indicating a rotation in the direction expected, but also that these values of $\overline{rA}$ were roughly inversely proportional with the average parallaxes of the stars. Additional support was given to the rotational hypothesis by the fact that the empirically determined values of l_0 agreed very closely amongst each other, as well as with the longitude of the centre of the system of globular clusters.

In table 1 some results are presented which were not yet included in the discussion of *B. A. N.* **120.** They were derived from stars of spectral types later than B and with proper motions smaller than 0″.020 per annum. Most of the columns are self-explanatory; n represents the number of stars used in each subdivision, $\overline{\pi}_{v.Rh.}$ is the average parallax interpolated from VAN RHIJN's tables *) for a mean proper motion of 0″.014 and $\overline{\pi}_{M.W.}$ is the average spectroscopic parallax as determined at Mt Wilson. The mean errors of the various values of the solar velocity given in the last column of the table are very nearly the same as those of the $\overline{rA}$ values.

In addition to the solutions in table 1 we have also tried to compute a value of $\overline{rA}$ from the faint stars observed near the apex and anti-apex by VAN DE KAMP **), but it does not appear possible in this case to separate the rotation effect from that of a possible systematic error in the radial velocities.

The values of $\overline{rA}$ are all positive and therefore confirm the rotation hypothesis, showing that it holds also for stars with only moderate concentration towards the plane of the milky way. It is possible that the fact that the K and M type stars seem to give too low values of the rotation term has some connection with the negative results from the Md variables (see page 83).

*) *Groningen Publications*, 34, Tables 34—38, 1923.
**) *Lick Bulletin*, 12, 88, 1926.

TABLE 1.
Additional results from radial velocities.

Spectrum	$\overline{m}$	n	$\overline{\pi}_{v.Rh.}$	$\overline{\pi}_{M.W.}$	$\overline{rA}$	$m.e.$	V_0
A	5.3	64	″.007	″.015	+ 8 *km/sec.*	± 3	16 *km/sec.*
F—G	5.3	58	7	10	+ 6	± 3	17
K—M	5.3	125	6	10	+ 1	± 3	21
A	6.3	33	6	11	+ 8	± 4	17
F—G	6.5	55	6	09	+ 11	± 3	15
K—M	6.7	76	5	07	+ 2	± 4	16

I have also derived the longitude l_0 of the centre of rotation from two groups of the stars in table 1. The results are indicated below, where I have collected all the determinations of l_0 that can at present be computed from radial velocities with a reasonable accuracy. The groups have roughly been arranged in the order of their mean distance, which ranges from about 300 parsecs in the first line to about 1200 parsecs for the faintest c stars and still larger for the nebulae.

B3—B5	$l_0 = 322° \pm$	5° (*m. e.*)
A—G $\left\{\begin{matrix}\mu < ''.020 \\ 4.0 < m \leqq 5.8\end{matrix}\right\}$	345 ±	9
B0—B2	322 ±	5
c stars $m < 5.0$	330 ±	8
A—G $\left\{\begin{matrix}\mu < ''.020 \\ m > 5.8\end{matrix}\right\}$	337 ±	8
Oe5	308 ±	7
c stars $m \geqq 5.0$	321 ±	4
Planetary nebulae	333 ±	10

On the whole the agreement of the various values of l_0 is satisfactory. The average without weights is 327° ± 3° (mean error computed from the residuals), the weighted average is 324° ± 2° (m.e.).

SHAPLEY has estimated the longitude of the centre of the system of globular clusters as 325° *), exactly in the same direction. In order to get some notion about the accuracy with which this longitude can be obtained from the distribution of globular clusters I plotted the numbers of clusters within intervals of 10° in longitude against the average longitude and tried to draw the line about which the observed points were symmetrically situated. In this way the longitude l_0 was derived from 5 independent sets of points, the average being 325° with an estimated mean error of ± 3°. This mean error is admittedly uncertain, but it may help to give a better insight into the significance of the coincidence of the two values of l_0.

The second table is intended to show the possible use that can be made of the rotational effects for

*) *Astrophysical Journal*, 48, 170, 1918; *Mt Wilson Contr.* No. 152, p. 17.

determining relative distances. For various spectral types the stars have been subdivided into several groups of small range in apparent magnitude. The fainter *c* stars were also subdivided according to *spectral type*, but we found no difference of any significance in the size of the rotation term between the stars of spectrum B and those of later types, which is in accordance with the computation of SCHILT who found no difference in mean parallax between these subdivisions *). All known *c* stars were excluded from the various groups of B stars enumerated in the table.

In nearly all cases there is a well pronounced increase in $\overline{rA}$ with the apparent magnitude; the increase is even such as to make the values of $\overline{m} - 5 \log \overline{r}$ in the sixth column independent of the magnitude, indicating that the average apparent magnitude may be a good criterion to determine the average distance, on the assumption that the absolute magnitude is constant. The values of $\overline{r}$ were computed with the aid of the value of A derived below (+ 0.019). A colon has been added to indicate very uncertain results.

TABLE 2.

Relation between rotation effect and apparent magnitude and computation of the absolute value of A.

Spectrum	Magn.	$\overline{m}$	n	$\overline{rA}$ m. e.	$\overline{m} - 5 \log \overline{r}$	$\overline{\pi}$ m. e.	A m. e.
Bo — B2	3.5 — 4.9	4.5	23	+ 3 ± 3	— 6.5:	.0042 ± .0006	+ .010 ± .010
„	5.0 — 5.8	5.4	17	+ 13 ± 3	— 8.8	20 ± 5	+ 21 ± 7
„	5.9 — 6.9	6.1	7	+ 15 ± 6	— 8.4	17 ± 8	+ 21 ± 13
B3 — B5	3.5 — 4.9	4.5	75	+ 4 ± 2	— 7.1:	66 ± 5	+ 21 ± 10
„	5.0 — 5.8	5.4	74	+ 1 ± 2	— 3.1:	48 ± 3	+ 4 ± 8
„	5.9 — 6.9	6.3	10	+ 16 ± 5	— 8.3	30 ± 15	+ 39 ± 23
c stars	< 5.0	3.8	44	+ 9 ± 3	— 9.6	30 ± 5	+ 23 ± 9
„	5.0 — 5.8	5.4	26	+ 14 ± 4	— 9.0	19 ± 6	+ 23 ± 10
„	≧ 5.9	6.8	23	+ 35 ± 3	— 9.5	9 ± 4	+ 28 ± 14
Oe5	2.5 — 4.9	4.1	7	+ 2 ± 8	— 6.1:	—	—
„	5.0 — 6.2	5.8	9	+ 19 ± 9	— 9.2	—	—
„	6.3 — 7.3	6.9	5	+ 23 ± 9	— 8.5	—	—
„	7.4 — 8.0	7.6	6	+ 31 ± 8	— 8.4	—	—
δ Cep var.'s	—	5.4	23	+ 8 ± 4	— 7.7:	39 ± 10	+ 30 ± 17
A — M	4.0 — 5.8	5.3	247	+ 5 ± 2	— 6.7	66 ± 33	+ 26 ± 17
„	≧ 5.9	6.5	164	+ 7 ± 2	— 6.3	55 ± 28	+ 30 ± 17

The last three lines of the table were added only to be used for the determination of A, which we shall now discuss.

In order to compute A we have to know the average distances of the stars considered. For this purpose we use the average parallaxes derived in the third section of this paper. The values in the last column but one of table 2 have either been taken directly from table 3 or have been deduced from it by interpolation. Only for the groups of stars fainter than $5^m.8$ an extrapolation was necessary. The average parallaxes of these groups have therefore been given a mean error equal to one half of the parallax. Knowledge of the average parallaxes is not sufficient for the computation of A. In general the average distance will be larger than the reciprocal of the average parallax, the difference depending upon the distribution in distance of the stars considered. If the star density is assumed to be approximately constant this space distribution can be computed from the distribution of the absolute magnitudes. Let us suppose that the latter distribution is of the Gaussian type with a mean square deviation ε, then it is easily seen that average distance and parallax are connected by the following formula:

$$\overline{r}\,.\,\overline{\pi} = e^{\left(\frac{0.2\,\varepsilon}{\text{Mod}}\right)^2} = e^{(0.461\,\varepsilon)^2}$$

We have adopted the following values of ε:
For each subdivision of the B stars $\pm\, 1^m.0$ ($\overline{r}\,.\,\overline{\pi} = 1\cdot24$). From KAPTEYN's studies on the individual parallaxes of these stars*) we compute a somewhat larger value

*) *B. A. N.* 48 (Vol II), p. 49, 1924.

*) *Astrophysical Journal*, 40, 43, 1924 and 47, 104, 146 and 255, 1918; *Mt Wilson Contr.* Nos. 82 and 147.

(± $1^m.4$) for the dispersion of stars of a given apparent magnitude. This would raise the product $\overline{r}.\,\overline{\pi}$ from 1.24 to 1.50 and lower the computed values of A by one sixth of their amount. We adopted the lower value of ± $1^m.0$ because from the B stars in open clusters one finds a dispersion of only ± $0^m.9$. It does not appear impossible that KAPTEYN's dispersion is a little too large through the influence of systematic and accidental motions of the stars.

For the c stars ± $0^m.8$ *); $\overline{r}.\,\overline{\pi} = 1.15$.

For the Oe5 stars no trustworthy data are available.

For the δ Cephei variables ± $0^m.29$ **); $\overline{r}.\,\overline{\pi} = 1.02$.

In the case of the stars with proper motions smaller than 0".020 per annum, exhibited in the last two lines of table 2, we have computed the distribution of the parallaxes from the numbers given by VAN RHIJN***). From this distribution we find $\overline{r}.\,\overline{\pi} = 1.27$.

The computed values of A are given in the last column of the table. The mean errors shown in the same column were computed from the mean errors of the average parallax and of $\overline{rA}$. No allowance was made for the uncertainty in the adopted reduction factor $\overline{r}.\,\overline{\pi}$. For the stars with proper motions smaller than ".020 we adopted a mean error in the parallax equal to 50°/₀ of the parallax itself, in order to take account of the possibility of systematic errors. There is a considerable systematic difference between VAN RHIJN's parallaxes and the spectroscopic ones for these stars (see table 1). The former were used, as they are probably the most fundamental parallaxes; they appear to yield values of A which agree well with those found from the distant stars in general.

On the whole the agreement between the various results for A is not unsatisfactory. The weighted average is found to be + **0.019** km/sec. parsec ± **0.003** (m. e.), whereas the average without weights is + 0.023 ± .003 (m. e.) The former value will be adopted. In *B.A.N.* **120** a larger result was derived (viz: $A = + 0.031$); the difference is partly due to the fact that in the provisional solution of *B.A.N.* **120** no reduction factors were applied to reduce $1/\pi$ to $\overline{r}$.

3. *Secular parallaxes of distant stars.*

Although the rotation cannot have influenced the solution of the secular parallax for stars evenly distributed over the sky it may have had considerable influence in some cases where the distribution was rather uneven. For this reason we have thought it worth while to communicate in table 3 the average secular parallaxes derived in the course of our work. They were found in the following way; the stars were divided over a number of areas, from 0° to 30° galactic longitude, from 30° to 60°, etc., and, unless mentioned otherwise, those with galactic latitudes in excess of 19° were excluded; in each area the average proper motion in right ascension and in declination was computed, weights being used roughly in accordance with the probable error and the average amount of the residual proper motion. From these mean proper motions we determined the average value of υ, the component in the direction of the solar antapex, and combined the $\overline{\upsilon}$'s from diametrically opposite areas with equal weights. In this way all systematic errors arising fromr otation or from errors in the precessional constant are eliminated. From these final values of $\overline{\upsilon}$ the average secular parallax was then determined by the method of least squares.

The above computations were performed, assuming that all stars of a certain area could be considered to lie at the centre of each area. As a check on the validity of this assumption a recomputation was made for one of the most unfavourable groups, using the average position of the stars in each area instead of the centre of the area. The results computed in this more exact way were found to agree quite satisfactorily with the results from the approximate method.

TABLE 3.

Mean secular parallaxes of distant stars.

Type.		$\overline{m}$	n	υ_0	$\overline{\pi}$	m.e.	Other authorities.
$B3 - B5$	$3^m.0 - 4^m.9$	4.3	124	+ ".0309	".0073 ±	.0005	".0063 [5]
»	5.0 — 5.8	5.4	157	+ 203	48 ±	3	46 [5]
$B0 - B2$	3.0 — 4.9	4.3	47	+ 199	47 ±	6	20 [5]
»	5.0 — 5.8	5.4	39	+ 86	20 ±	5	19 [5]
δ Cep var.'s [1]	π_{Sh} = ".0015	—	48	+ 86	20 ±	5	15 [6]
Md variables [2]	< 5.0	3.6	6	+ 689	59 ±	8	—
»	5.0 — 5.9	5.4	7	+ 407	35 ±	8	—
»	≧ 6.0	7.4	88	+ 130	11 ±	2	29 [7]
c stars	< 5.0	4.0	52	+ 112	27 ±	5	36 [8]
»	≧ 5.0	5.7	63	+ 73	17 ±	7	17 [8]
$Oa - Oe$ [3]		6.2	39	+ 120	28 ±	25	} 13 [3]
$Oe5$ [3]		5.6	43	+ 84	20 ±	6	}
N [4]		8. :	85	+ 70	17 ±	8	14 [9]
$A0 - A9$	$\pi_{M.W.}$ < ".010	—	85	+ 155	37 ±	4	—

1) *Astronomical Journal*, 35, 35, 1923.
2) » » 34, 183, 1923. (Stars with Md spectrum).
3) *Astronomical Journal*, 36, 4, 1924.
4) » » 34, 183, 1923.
5) GERASMOVIČ, *V. J. S. Astr. Ges.*, 61, 223, 1926.
6) SHAPLEY, *Astrophysical Journal*, 48, 282, 1918; *Mt Wilson Contr.* No. 153.
7) R. E. WILSON, *Astronomical Journal*, 35, 129, 1923.
8) SCHILT, *B.A.N.* 48, Vol. II, 49, 1924.
9) R. E. WILSON, *Astronomical Journal*, 34, 191, 1923.

*) J. SCHILT, *B.A.N.* 48, p. 50, 1924.
**) *Harvard Circulars* No. 280, p. 4, 1925.
***) *Groningen Publications*, 34, 1923.

Remarks to the table.

The successive columns of the table give the subdivisions of spectral type and magnitude of the stars used, their average apparent magnitude*), their number, the average secular parallax υ_0, the average annual parallax which except for the Md variables was derived from the preceding column by multiplication by a factor of 0.237 (corresponding to a solar velocity of 20 *km/sec*), and its mean error (in the derivation of which we took account of the mean peculiar motions as well as of the mean errors of the proper motions). The last column shows the average parallaxes as interpolated for the same average magnitudes from other sources, of which as a rule only the most recent ones were considered.

Most of the proper motions were taken from BOSS, for those taken from R. E. WILSON's special lists references are given in footnotes. All of them have been corrected for RAYMOND's values of the systematic errors in μ_δ**), but the correction to the precessional constant given in the introduction to BOSS' catalogue has not been applied.

The proper motions of the *δ Cephei variables* were all reduced to a parallax of 0″.0015, SHAPLEY's values being used for this purpose. The average parallax should therefore have come out equal to 0″.0015 if it were in exact agreement with SHAPLEY's results. The new parallax is a little larger, but the difference does not exceed its mean error. A solar velocity of 20 *km/sec* was used; with the lower value 13 *km/sec* of found from the radial velocities of these stars, the average parallax would be increased to ″.0031.

The *Md stars* were treated a little differently from the others. Because of their considerable peculiar motions they were not included in the groups of stars used for the determination of the precessional constant and the rotation. Mean parallaxes were derived also from the τ components of the proper motions. From the stars whose proper motions had a probable error smaller than ± ″.010 the following parallaxes were found on the assumption that the average peculiar

TABLE 4.

Mean parallax of Md variables.

$\overline{m}$	n	$\overline{\pi}_\tau$	$\overline{\pi}_\upsilon$	$\overline{\pi}$	$m+5\log\overline{\pi}$
3.6	5	″.0170	″.0059	″.0114	−6.2
5.4	6	33	35	34	−6.9
7.4	41	14	11	13	−7.0

*) For the Md stars the magnitude at maximum, taken from *Harvard Annals*, 56, 197.

**) *Astronomical Journal*, 36, 136, 1925.

velocity in the τ direction was equal to that in radial direction. The mean peculiar velocity was assumed to be ± 33 *km/sec* *) and the τ components were corrected for probable error.

The last column but one gives the simple mean of $\overline{\pi}_\tau$ and the $\overline{\pi}_\upsilon$ given in table 3. The latter values were got from υ_0 by a reduction factor of 0.086, corresponding to a solar velocity of 55 *km/sec* *). The stars were not subdivided according to their spectrum, as it had previously been shown that there is little or no difference in absolute magnitude between the different subclasses. The present division into magnitude classes indicates a fairly strong decrease of parallax with increasing apparent magnitude. The disagreement between my value of the parallax for $m=7.4$ and the value of ″.0026 found by R. E. WILSON is mainly caused by the fact that WILSON used a solar velocity of only 20.7 *km/sec* in stead of the 55 *km/sec* deduced from the radial velocities of these stars. From the present computations the Md variables are found to belong among the most distant, and at maximum among the brightest, objects known to us, their average absolute magnitude being about $-7^{M}.0$ (unit of distance 1 parsec). There is a difference of as much as 2 magnitudes between this result and the mean absolute magnitude derived by MERRILL and STRÖMBERG **). We do not know what the explanation of this difference is; it may partly have been caused by different weighting.

The large distances found above are somewhat alarming in view of the fact that the radial velocities of these variables do not show any signs of the rotation effects which have been found to exist for all other distant stars (see Table 2, *B.A.N.* **120**). With the parallaxes just determined one should expect a rotation term with a semi-amplitude of about 20 *km/sec*. It is true that there are among these variables a large percentage of high velocity stars, but as the total rotational velocity is of the order of 300 *km/sec* the relative decrease of the rotational velocity cannot be important. As one might nevertheless fear a systematic influence of the stars of high velocity we have made a new solution in which all stars whose radial velocities after correction for a solar motion of 20 *km/sec* were larger than 50 *km/sec* were excluded. Moreover some southern stars not used in the former solution were included, and opposite areas of the sky were combined with equal weights. The result was

$\overline{r}A = +2.4$ *km/sec* ± 6 (*m.e.*), $K = -0.9$ *km/sec* ± 4 (*m.e.*), still much too small.

*) MERRILL, *Astrophysical Journal*, 58, 252, 1923; *Mt Wilson Contr*. No. 264.

**) *Astrophysical Journal*, 59, 97, 1924; *Mt Wilson Contr*. No. 267.

Personally I do not believe, however, that these apparently discordant results should be considered as an insurmountable difficulty in the way of the theory of a general galactic rotation. It seems more probable that the absence of a rotation effect in the radial velocities of these stars should yet be ascribed to the large percentage of high velocities. There is nothing to guarantee that this percentage will be the same at different distances from the galactic centre. The zero result from the radial velocities might be an indication that the number of high velocities increases with decreasing distance to the centre in such a way that the angular velocity of rotation becomes approximately constant for these stars. In this connection it may be remarked that the proper motions of the Md variables indicate a rotation of the same amount as that found for the other stars, but these data are not strong (see page 87).

If the above interpretation is right, it makes the distances of the planetary nebulae derived in *B. A. N.* **120** rather uncertain, because these nebulae also contain a very considerable number of high velocities.

From the group of fainter *c stars* two stars with proper motions in excess of 0".100 per annum (Boss 2513 and 5838) were excluded. Their proper motions stand clearly out from the rest and it is almost certain that these stars were erroneously classified as *c* stars.

The average parallax found for the *O stars* is somewhat larger than the parallax computed by R. E. WILSON. The difference is partly due to a difference in the assumed velocity of the sun. We adopted 20 *km/sec* (see *B. A. N.* **120**, p. 279) whereas WILSON used 31.7 *km/sec.*

In the determination of the results for the *N type*, stars with latitudes up to 30° were included.

The *A stars* with small spectroscopic parallaxes, in the last line of the table, were taken from *Mt Wilson Contribution* No. 244. They are rather unevenly distributed over the sky, so that the solution for the various unknowns had to be made in a different way. They were kept separate from the other stars in the final determination of the rotation.

4. *The constant of precession and the equinox correction.*

The constant of precession has usually been derived on the assumption that the stars of a certain catalogue possess on the whole no rotational motion around the sun. It is possible, however, to derive an absolute value of the precessional constant without making any such doubtful assumption, if we determine it from the proper motion components in galactic latitude of stars which are strongly concentrated towards the milky way. For in this case it is extremely probable that the systematic motions in a direction perpendicular to the galaxy will be much smaller than those in the galactic plane, and therefore negligible. *)

In the present section we have made a solution according to this principle from various groups of stars with small peculiar motions; the results were afterwards supplemented and checked by a solution from all the stars of BOSS' catalogue. A correction to the motion of the equinox was introduced as a second unknown, but an additional solution for the precessional constant has been made on the assumption that NEWCOMB'S equinox as used by BOSS is right.

The equations of condition take the following form:

$$\overline{\mu'_b} = \Delta p \sin \varepsilon_1 \cos (l - l_1) + (-\Delta e - \Delta\lambda) \sin \varepsilon_2 \cos (l - l_2)$$

where $\overline{\mu'_b}$ represents the average proper motion in galactic latitude, corrected for solar motion, for stars in an area at galactic longitude l, Δp the correction to the annual precession on the ecliptic used by BOSS in his *Preliminary General Catalogue*, ε_1 the inclination of the ecliptic on the plane of the galaxy and l_1 the galactic longitude of the ascending node of the ecliptic on the galaxy; ε_2 and l_2 are the corresponding quantities for the equator on the milky way, while Δe represents a correction to NEWCOMB'S motion of the equinox (which was used by BOSS) and $\Delta\lambda$ a correction to his planetary precession **). Thus $-\Delta e - \Delta\lambda$ is the constant error of the proper motions in right ascension, remaining after the correction Δp has been applied, and reckoned in seconds of arc per annum measured on the equator.

Adopting the co-ordinates of the galactic pole found by NEWCOMB we have

$$\varepsilon_1 = 61°.2 \qquad \varepsilon_2 = 63°.2$$
$$l_1 = 153°.5 \qquad l_2 = 180°.0$$

Before making the solution, the proper motions of each group of which the secular parallax was determined in the previous section were in each area corrected for the solar motion as found in table 3 and decomposed into galactic co-ordinates, $\overline{\mu'_b}$ and $\overline{\mu'_l}$. These average proper motions were then with proper weights (depending upon the size of the peculiar motions and of the accidental errors) combined into the following more comprehensive groups:

*) It may also be noted that in such a solution differences in mixture of the star streams in different parts of the sky, as suggested by HOUGH and HALM in order to explain a discrepancy in the determination of the precessional constant, cannot have any influence.

**) The notation is the same as that used by LEWIS BOSS in his articles on "Precession and solar motion" (*Astronomical Journal*, 26, 95, 1910). We have $-\Delta e - \Delta\lambda = \Delta k - \Delta' m$, where $\Delta' m = p \cos \varepsilon$. In the introduction to his *Preliminary General Catalogue* BOSS counts Δk in the opposite direction.

group I containing all B0 — B2 stars
group II containing all B3 — B5 stars
group III containing the *c*, O and N stars and the δ Cephei variables.

The values of Δp and of $(-\Delta e - \Delta\lambda)$ resulting from least squares solutions for these three groups and their average, are represented in the first four lines of table 5. Units are seconds of arc per annum.

TABLE 5.

Corrections to the motion of the equinox and to the precessional constant.

Stars used	Number	Δp	$-\Delta e - \Delta\lambda$	m. e.	$\Delta p'$ $(\Delta e + \Delta\lambda = 0)$
Group I	105	+ .014	— .012	± .004	+ .002
„ II	336	+ 13	— 12	„	+ 2
„ III	330	+ 14	— 11	„	+ 4
I + II + III	771	+ 13	— 11	± 3	+ 3
Boss, b 0°— 7°	1263	+ 17	— 17	± 4	—
„ „ 8 —19	1127	+ 10	— 11	„	—
„ „ 20—42	1560	+ 10	— 9	„	—
„ „ 43—90	1463	+ 8	— 12	„	—
Boss, b 0 — 90	5413	+ 10	— 12	± 2	—

A rather large positive correction is found to the constant of precession and an almost equally large correction to the motion of the equinox. The three values are beautifully consistent and the total average exceeds its mean error several times. Figure 2 gives a graphical representation of the way in which the observed mean motions fit these constants (full curve). The dotted curve is the best we can draw if we leave the equinox unchanged.

The mean errors in table 5 were estimated from the residual motions left in the twelve galactic areas; they can, therefore, be said to include the effects of systematic errors in the proper motions as well as of non-eliminated systematic motions. This can be further substantiated by dividing the galactic zone into two parts, one comprising the northern milky way (l between 0° and 180°), the other the southern half (l between 180° and 360°) and by solving for the two corrections in each of these parts separately. The two solutions are based on stars in quite different regions of the sky, with different systematic errors, and are therefore entirely independent of one another. We find from the three groups togeher:

$$\text{for } l \;\; 0^\circ - 180^\circ: \Delta p = +''.017\,; -\Delta e - \Delta\lambda = -''.012$$
$$\text{for } l \;\; 180^\circ - 360^\circ: \Delta p = +''.008\,; -\Delta e - \Delta\lambda = -''.010$$

agreeing as well as might be expected. Nevertheless, in view of the difficulty of accepting such a large equinox error, a corroboration from proper motions in still other parts of the sky, outside the galactic zone, appeared to be desirable. Such a corroboration is furnished in the second half of table 5, showing an exactly similar solution in four different galactic zones from 5413 stars of BOSS' catalogue. The stars were used by BOSS in his determination of the precessional constant. He divided the sky into 108 areas and computed the average proper motions in declination and right ascension for each area. Although some very large proper motions wore rejected *) the moderately large proper motions must have had a greater influence on the averages than the very small proper motions used in the preceding solution, so that the two solutions may safely be considered as independent of each other (except of course for possible systematic errors). The solution from BOSS' catalogue has been greatly facilitated by the average residuals published by BOSS **). These were transformed into galactic co-ordinates and used for determining the precession as well as the galactic rotation. The Δp and $(-\Delta e - \Delta\lambda)$ given in the table have been reduced to apply to the constants used in the *P.G.C.* and not to the residuals just mentioned. The mean errors have been estimated as much as possible in the same manner as those for the stars of small proper motions. They are considerably higher than those given by BOSS, but they are intended to include the effects of systematic errors.

The good agreement of the corrections found entirely independently from several parts of the sky makes it very probable indeed that the large corrections are real and not due to errors in the proper motions. One might also express it in this way, that if the discrepancies found had to be explained in terms of a systematic error in the proper motions, this error must very nearly have the form of a constant error of all the proper motions in right ascension.

The final average adopted is

$$\Delta e + \Delta\lambda = +''.0117 \pm .0020 \text{ (m. e.)}$$

(half weight being given to each of the groups I, II, III, and to the two BOSS zones nearest to the galaxy, in order to reduce the effect of possible systematic errors in the galactic regions). Modern determinations indicate that the mass of Venus should be somewhat larger than the value which NEWCOMB used; this would bring about a correction of about $-''.0020$ to NEWCOMB'S planetary precession. We are thus led to believe that NEWCOMB'S motion of the equinox requires a correction of $+''.0137 \pm ''.0020$ (m. e.) annually. The correction is large, but not a priori impossible. There also remains the possibility suggested by

*) There may be considerable danger in such a rejection. See also p. 87 of the present article.

**) *Astronomical Journal*, 26, 97, 1910.

FOTHERINGHAM *) of a motion of the earth's orbit, due to factors not considered in the computation of the planetary precession.

From table 5 we find as the best correction to NEWCOMB's lunisolar precession $\Delta p = +''.0113 \pm ''.0020$ (m. e.), the annual lunisolar precession becoming $50''.3821$ (1900). Of this $0''.0191$ is due to the "geodetic precession", leaving $50''.3630$ for the true lunisolar precession in longitude for 1900.

It may be seen from the table that the two corrections Δp and $(-\Delta e - \Delta\lambda)$ nearly cancel, so that the total corrections never become very large. Because of the small inclination of the ecliptic the weight of the determination of Δp has been decreased more than four times by the introduction of an unknown equinox correction. The last column of the table exhibits the corrections $\Delta p'$ to the precessional constant, computed on the assumption that $\Delta e + \Delta\lambda = 0$ (see also the dotted curve in fig. 2). The residuals are larger than with the former solution, so that the mean error has been decreased only about 1.5 times.

The correction to be applied to the proper motions in longitude is practically independent of the assumed value of $\Delta e + \Delta\lambda$, at least in the zones near the galaxy. As, moreover, the corrections enter into the longitude components with a factor of about one half, not much uncertainty will be introduced by these corrections.

It is interesting to make a comparison between the relative accuracy of the solution from BOSS' catalogue without weights and that of a weighted solution from the most distant stars only. In BOSS' solution the average residual proper motion is about $\pm ''.030$ in each component. In our case it is estimated between $\pm ''.010$ and $\pm ''.015$, and an accuracy equal to that obtained by BOSS should be reached with $^1/_6$ th of the number of stars. This is corroborated by the mean errors in table 5 which were estimated in an independent way. The comparison still gives too favourable an impression of the accuracy obtained from BOSS' catalogue, because his residuals have been artificially reduced by the exclusion of a considerable number of stars with large proper motions.

The present results from proper motions in galactic latitude agree very nearly with those obtained by BOSS from both components of the proper motions. He finds $\Delta p = +''.0085, -\Delta e - \Delta\lambda = -''.0115$ **). BOSS' investigation has recently been extended by FOTHERINGHAM who introduced a uniform rotation around the galactic axis as an additional unknown into BOSS' equations *). He finds a total correction of $+''.0093$ to NEWCOMB's lunisolar precession (not including the correction for "geodetic precession") and a correction of $+''.0137$ to the motion of the equinox, in close agreement with the values derived above.

*) *Monthly Notices R. A. S.*, 86, 424, 1926.

**) These have been computed from the results given on p. 115 of the quoted article. On p. 118 BOSS gives final corrections of $+''.0058$ and $-''.0085$ as a compromise between the results derived from proper motions and those of direct observations of the equinox.

5. *Determination of the rotation from the proper motions in galactic longitude.*

The proper motions in galactic longitude, corrected for the effects of solar motion and for the precession derived above **) were now used to determine the constants A, B and l_0. Of these B is the most important as the two others can also be derived from radial velocities, and that with a considerably greater accuracy. From the equations of condition for each area:

$$\bar{\mu}_l = \frac{A}{4.74} \cos 2\,(\bar{l} - l_0) \cos \bar{b} + \frac{B}{4.74} \cos \bar{b}$$

we derive the results shown in table 6. A graphical representation is given in figure 1.

TABLE 6.

Rotation from proper motions.

Stars	$\frac{A}{4.74}$ m. e.	$\frac{B}{4.74}$ m. e.	l_0 m. e.
Group I	— —	$-''.0075 \pm ''.0014$	— —
» II	— —	$-$ 42 ± 14	— —
» III	— —	$-$ 42 ± 14	— —
I + II + III	$''.0024 \pm ''.0016$	$-$ 50 ± 11	$326° \pm 17°$
BOSS, b 0°— 7°	11 ± 16	$-$ 14 ± 11	310 ± 28
» » 8 —19	20 ± 16	$-$ 22 ± 11	332 ± 19
» » 20 —42	38 + 18	$-$ 30 ± 13	324 + 13
» » 43 —90	45 ± 32	$-$ 30 ± 22	27 ± 18
BOSS, b 0 —90	18 ± 10	$-$ 23 ± 7	333 ± 15

The mean errors have been computed on the same plan as in table 5, they are supposed to include the effects of systematic errors.

The values of B in the first three lines were determined on the assumption that $A = +.019$ and $l_0 = 325°$. The result for B is practically independent of these constants, however.

When examining the table two things will appear to need an explanation. Firstly the smallness of the values of A. From radial velocities we found $\frac{A}{4.74} = +''.0040$, whereas both the stars of small proper motion and BOSS' catalogue seem to give smaller values. It is

*) *Monthly Notices R.A.S.*, 86, 414—426, 1926.

**) The galactic stars of small proper motion were corrected with their own precessional constant, and so were the Boss stars.

rather probable that the discrepancy is caused by errors in the proper motions, especially by those in the southern part of the milky way. On the whole the residuals are not sensibly diminished by introducing the smaller value of A. The second point concerns the considerable difference in the value of B found from the stars of small proper motion on one side and from all the Boss stars on the other side. Though we have not made any numerical calculation about it, it does not seem unlikely that great part of the difference may be accounted for by BOSS' exclusion of large proper motions. The way in which this exclusion was made *) would inevitably tend to diminish any such effects as rotation, but we are not certain that the diminution would be sufficiently great. An investigation by CHARLIER **) may support this explanation, as without any exclusion of large proper motions he finds from 3617 stars of the 4th and 5th magnitudes $\frac{B}{4.74} = -''.0035 \pm ''.0016$ (m. e.).

For the present it seemed best to adopt the value derived from the three groups of distant stars, viz: $\frac{B}{4.74} = -''.0050$ per annum or

$$B = -0.024 \text{ km/sec. parsec} \pm 0.005 \text{ (m. e.)}.$$

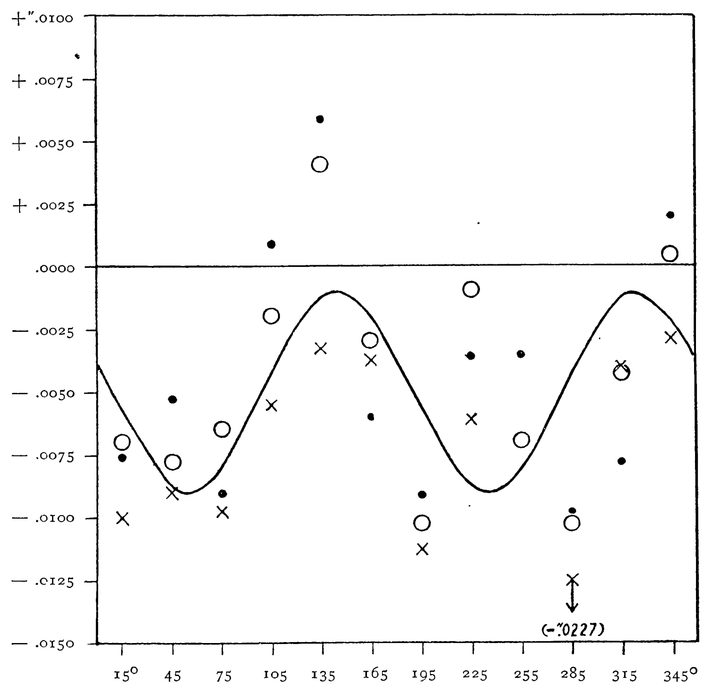
It may be of interest to compare this result to that obtained from the Md variables which were treated in a somewhat different manner, giving $B = -0.026$ *km/sec. parsec*. Also the A stars with small spectroscopic parallaxes, mentioned in the last line of table 3, have been used for a determination of B, with the result $B = -0.027$ *km/sec. parsec*. Both results depend on stars distributed over the whole sky; in the latter case the correction to the precessional constant was derived from the stars themselves, so that this value is in all respects independent of the result derived above. The agreement of the various results is quite satisfactory.

The existence of a systematic term in the proper motions, explicable as a uniform rotation in the plane of the galaxy, has been suspected by several authors. In 1887 LUDWIG STRUVE introduced such a rotation into his equation for the determination of the constant of precession ***). As far as I know the first result of real significance was found by CHARLIER in the study cited above ****). The possibility of a galactic rotation has also been considered by INNES *) from a small number of stars near the apex and antapex. FOTHERINGHAM **) derived a uniform rotation of $-''.0015$ per annum from the residuals published by BOSS.

The various determinations of l_0 shown in the last column of table 6 agree well with the value found from the radial velocities ($l_0 = 324°$), but on account of the large mean errors attached to the former values this corroboration of the radial velocity results is not as significant as we might have hoped. It is probable that more accurate results can only be obtained after the systematic errors in our fundamental catalogues have been considerably reduced.

The negative sign of B is, of course, also in agreement whith the rotation indicated by the radial velocities.

FIGURE 1.
Proper motions in galactic longitude.



6. *The residuals.*

The average proper motions in galactic longitude and latitude have been plotted in figures 1 and 2 respectively. The crosses refer to group I, the dots to group II and the small open circles to group III. Abscissae are average galactic longitudes, ordinates are mean proper motions. The proper motions have been corrected for the effects of solar motion and for the systematic errors in the proper motions of declination as determined by RAYMOND. The proper motions in longitude have also been reduced with

*) *Astronomical Journal*, 26, 98.

**) *Lund Meddelanden*, Serie II, No. 9, p. 78. 1913.

***) *Mémoires de l'Académie impériale de St. Pétersbourg*, 7ième Série, XXXV, No. 3.

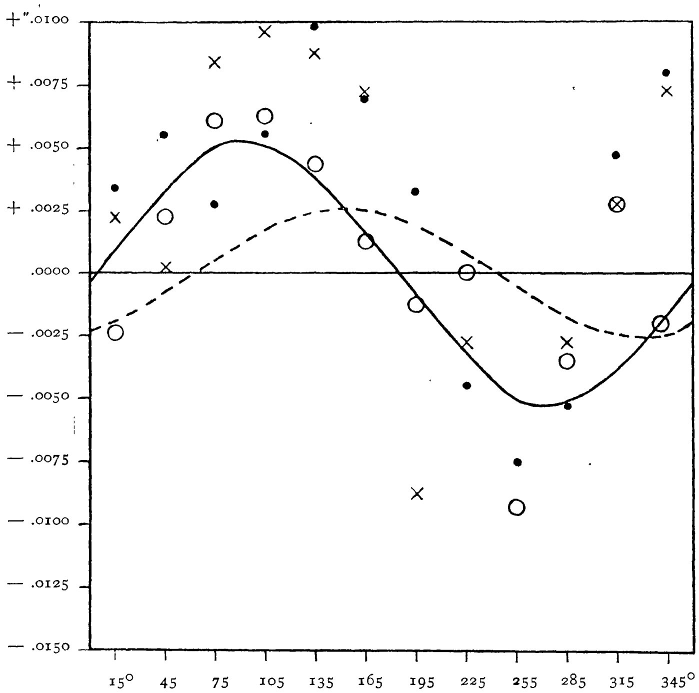
****) A later result may be found on page 32 of CHARLIER's memoir "The motion and the distribution of the stars" (*Mem. of the Univ. of California*, 7, 32, 1926).

*) *Union Observatory Circular* No. 17, p. 119, 1914.

**) *L. c.*, p. 418.

the correction to the precessional constant found in this paper (viz: $\Delta p' = +''.0029$). One observation has been omitted on account of very small weight. The

FIGURE 2.
Proper motions in galactic latitude.



drawn curves have been computed with the results derived in sections 4 and 5 from the groups of distant stars, viz: $\mu_l = +''.0040 \cos (l - 323°) - ''.0050$ and $\mu_b = +''.0117 \sin (l - 63°.5) + ''.0100 \cos l$. The dotted curve is the one best fitting the observations if we suppose that NEWCOMB's motion of the equinox is right. It is derived from the formula $\mu_b = +''.0025 \sin (l - 63°.5)$. It is evident that it gives a very much poorer representation of the observations than the full curve.

On the whole the observed proper motions appear to be satisfactorily represented by the curves, but in a few intervals the deviations appear to be somewhat too large and systematic. In the case of the proper motions in longitude this seems to be especially marked in the southern regions between 190° and 320° galactic longitude. The proper motions in galactic latitude show a tendency to be systematically too large in the interval from 90° to 170° longitude, and also near 320° longitude.

That the deviations are probably due to systematic errors in the proper motions, and not to chance coincidences, is shown by a computation of the residuals of the various groups from the mean of the three groups in each interval. From these the mean error of the average proper motion in one interval is found to be $\pm''.0024$ for the motions in longitude and $\pm''.0032$ for those in latitude. The mean errors computed from the deviations from the theoretical curve are considerably higher, viz. $\pm''.0050$ and $\pm''.0045$ respectively.

7. *Dynamical consequences.*

If we make the supposition that the average motions are circular (from the agreement of the longitude of the centre derived from these motions with that determined from the globular clusters this supposition appears not to be unreasonable), we can determine the character of the general gravitational force, as well as the distance of the rotation axis, from the values of A and B found in the preceding sections. We find from the formulae given on page 80:

$$V/R = A - B = +0.043 \ km/sec.parsec \pm .006 \text{ (m. e.)}$$

and

$$\partial K/\partial R = (3A + B)(A - B) = +0.0014 g.\, km^2/sec.^2 parsec^2$$
$$= +1.5 \times 10^{-30} g.\, sec^{-2}$$

If, as an illustration, we suppose that the total force can be considered as the sum of two forces K_1 and K_2, the first of which varies as the inverse square of R and the other proportional to R, we find:

$$K_1/K = 4A/3(A - B) = 0.59 \pm 0.12 \text{ (m. e.)}.$$

The data would therefore imply a very strong concentration of mass towards the centre, three fifths of the total gravitational force being found to arise from the mass concentrated near this centre.

Assuming $V = 272$ *km/sec* (*c.f. B.A.N.* **120**, p. 281) we can derive absolute values of R and of the gravitational force. Thus we get

$$R = 6300 \text{ parsecs} \pm 2000 \text{ (m. e.)}$$
$$K = -(A - B)V = -12 g.\, km^2/sec.^2 parsec.$$
$$= -3.8 \times 10^{-8} \text{ dynes}$$

Mass concentrated near the centre $= 6 \times 10^{10} \times$ sun's mass $= 1.2 \times 10^{44}$ grammes.

It may again be remarked that these results are more or less hypothetical; however, it will be possible to test the hypotheses and to derive much more trustworthy results for K and R if the radial velocities of only a small number of 9th and 10th magnitude Cepheids can be determined.

Remarks.

a) It may be worth while to draw attention to the fact that if the radial velocities of the Magellanic clouds are corrected for our motion relative to the centre of the globular clusters, the resulting velocities are found to be so small as to make it possible that the clouds are satellites of the galactic system. The uncorrected radial velocity of the larger cloud is + 276 *km/sec* (from 17 nebulae) and of the smaller cloud + 163 *km/sec* (from 2 nebulae). With a velocity of the sun amounting to 286 *km/sec* towards 55° galactic longitude and 0° latitude (corresponding with the value of V adopted above) the corrected radial velocities become + 41 *km/sec* and —9 *km/sec* respectively.

b) There is one more dynamical phenomenon connected with the centre near 325°, a fact which so far as I know has never been stated explicitly. It is well known that the planetary nebulae are quite asymmetrically distributed in the sky. This distribution exhibits a strong preference for a region in the neighbourhood of the direction to the above centre, though not quite as strong as that shown by the globular clusters.

If we plot the average peculiar velocity of the planetary nebulae against the galactic longitude it appears that there is also a pronounced increase in average velocity as we approach the direction to the centre. The increase cannot be the consequence of a preferential motion of the same kind as that exhibited by the stars in general, for the peculiar velocities are found to differ in diametrically opposite points of the sky: The average velocity is lowest in a direction opposite the centre.

The data are shown below *). The average peculiar velocities have in each area been obtained by subtracting from the individual velocities the algebraic mean of the radial velocities in that area.

Gal. long.	10°— 40°	± 25 *km/sec*	(17)
	40 — 70	± 34	(11)
	70 — 100	± 13	(7)
	100 — 130	± 27	(4)
	130 — 160	± 25	(3)
	160 — 190	± 27	(6)
	190 — 220	± 10	(4)
	220 — 250	± 11	(5)
	250 — 280	± 20	(10)
	280 — 310	± 44	(7)
	310 — 340	± 52	(27)
	340 — 10	± 51	(10)

The numbers of velocities have been added between parentheses. The total average velocity in the quadrant from 280° to 10° galactic longitude is ± 50 *km/sec* ± 6 (m. e.) as against ± 23 *km/sec* ± 2 (m. e.) for the rest of the sky.

Attempts to find a similar effect from the radial velocities of other distant objects have not given positive results.

*) The radial velocities have been taken from *Lick Publications*, 13, 168, 1918, supplemented by a few Mt Wilson results (*Publ. A. S. P.*, 37, 225).