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ABSTRACT

Wind induced circulation of suspended matter plays an important role in shallow lake ecosystems. A 'sink' designed for 'trapping' particulate phosphate may be located on the site of the natural maxima of one of the circulation components, i.e. the gross sedimentation. Periodical dredging of such a sink provides a permanent in-lake support for eutrophication control measurements. This paper illustrates a procedure which has been developed to locate these gross sedimentation maxima, characterized by being "too shallow" compared to the depth predicted by the lake's 'gross erosion pattern' (GEP). It is shown that these sites are not from the shallowest places, i.e. at the lee shores. The 'gross sedimentation pattern' (GSP) thus found may be explained by analysing the wind driven currents. Data used for testing and illustration are from Lake Westeinder, The Netherlands.

INTRODUCTION: THE FUNCTION OF TWO DREDGING STRATEGIES

In deep lakes, the lowermost water layers and the bottom act as a permanent sink of particulate matter, including phosphate-rich material, originating from the inflow or from in-lake biological production. Often, the bottom may also act as a phosphate source. The release of dissolved phosphates by lake sediments has been well documented in the literature (e.g., 22 and 10). The release of particulate phosphates is known to a much lesser extent. This particulate phosphate release, caused by erosion of phosphate-rich sediments, may play an important role in the phosphate budget of shallow lakes, increasing their vulnerability to eutrophication and hampering their recovery after eutrophication control measures have been taken.

Particulate phosphate release (or delayed particulate phosphate sedimentation) is especially salient in shallow lakes with a relatively uniform depth and a soft, loose sediment. In Europe, the Puszta lakes and the Dutch polder lakes are typical examples. The sediments in these lakes are constantly moved around by turbulent forces, eroding the bottom and mixing the water body. Yousef et al. (30) have shown that turbulence, caused by motor boats may be a generating factor in the resuspension of sediments. In most shallow lakes the wind, by generating waves and currents, will be the dominant driving force in the recirculation process. The wind's influence on the particulate phosphate concentration usually will be measured as increased total phosphate concentrations at times of increased wind strength. Jónasson and Lindegaard (12), Ryding and Forsberg (22) and De Haan (9) have qualitatively documented this relation. In a previous study, De Groot (7) developed a statistical regression formula predicting the total phosphate concentrations in a shallow Dutch lake, in which the wind is the dominant factor. Although not as reactive as dissolved phosphate, particulate phosphate is ecologically relevant, whether through mineralisation, chemical release or through direct bio-availability. For instance, Golterman et al. (11) have shown that the alga *Scenedesmus* thrives well on particulate phosphate as its only phosphate source.

Therefore, dredging of sediments has been proposed and applied as a phosphate eutrophication control measurement, which is particularly feasible as a supplementary action, to reinforce the effect of control of the external phosphate loading. Dunst et al. (10) provide a general summary, Peterson (20) reviews dredging techniques, while Cronberg (4) has documented the successful dredging of the shallow Lake Trummen, Sweden. In all these cases, the dredging strategy is aimed to break the chain of phosphate recycling between sediment and overlying water in one concentrated effort. However, also an alternative aim may be pursued by dredging, i.e. to create a permanent drain in the phosphate recycling process in shallow lakes. Dunst et al. (10) mention this strategy as an "interesting possibility", based upon the reversal of Lee's (16) suggestion that the loss of the thermocline, due to sediment infilling, results in rapid acceleration of eutrophication. In this paper, Dunst's "interesting possibility" will be made operational.

In order to get a clear picture of the difference between the two dredging strategies, it is relevant to distinguish between the gross sedimentation, the gross erosion and their resultant, the net transport, which is either positive (net accretion) or negative (net erosion). They are related as:

$$nT = gS - gE \quad (1)$$

in which nT = net (vertical) transport (erosion or accretion); gS = gross sedimentation; gE = gross erosion. As will be shown elsewhere in this paper, gross sedimentation and gross erosion may vary considerably over the lake and over time, depending on the velocity and direction of the wind. The annual averages of nT , gS and gE form definite patterns over the lake. Locations of a positive average net transport become visible as accretion sites of sediment material. The gross sedimentation is only equal to this accretion at places where no gross erosion takes place, e.g., in the hypolimnion of deep reservoirs or very sheltered bays of shallow lakes.

The two dredging strategies may be defined in the above terms. The typical situation in which the first strategy is to be applied may be described as follows. Some control of the external phosphate loading is essentially sufficient to restore the lake to a less eutro-

phic state, but the internal phosphate release will delay the lake's response for an unacceptable period of time. This situation calls for a one-off removal of the phosphate-rich sediments. The effort may involve the whole lake. If, for financial reasons, dredging can not take place over the whole lake, it should concentrate on places where the maximum thickness of recent sediments has accumulated, i.e., on those locations where nT , the net transport, is maximum. Most commonly, in shallow lakes, these are to be found at the lee (upwind) shore of the lake, relative to the dominant wind, and other quiet parts.

The typical situation in which the alternative strategy is to be applied may be described as follows. Some control of the external phosphate loading has been reached but cannot be pushed to the point where the lake will show enough response. For shallow lakes, this situation calls for a permanent in-lake phosphate drain, a "sink" or "trap". This drain could be established in a crude fashion by deepening the whole lake or an annual dredging of its whole area. Or, as in the first strategy, the locations with maximum accretion (net transport) could be selected for permanent or annual dredging. But would that be the most efficient method? Could not the dredging equipment be used to create a place where the accumulation is made higher than the existing net transport maxima? Eq. (1) shows that this is indeed possible. In some places in the lake, the net transport (nT) may be low although the two counterbalancing components may be high: a high gross sedimentation (gS) at some wind velocities and directions and a high gross erosion (gE) at other wind velocities and directions. Such locations may be dredged to such a depth that the sediments are below the reach of erosive forces, reducing the gross erosion (gE) to practically zero and increasing the net transport (nT) to the level of the gross sedimentation (gS). If this is carried out on a location of maximum gross sedimentation, a net transport is created which fundamentally exceeds the net transport on all other locations. Hence, in order to create the most efficient permanent drain on the phosphate recycling in a shallow lake, the following procedure may be adopted:

- determine the location(s) in the lake where the natural gross sedimentation is maximum,
- dredge the location to a depth at which the gross erosion is greatly reduced,

- and keep the location at that depth, either by annual dredging ("harvesting") of the accumulated material, or by continuous dredging with smaller equipment.

This is the concept of the permanent phosphate sink. If a lake shows clearly defined gross sedimentation maxima, a large percentage of the annual input and in-lake production of particulate phosphate may be trapped and collected at locations, which form only a small percentage of the total lake area. The wind induced circulation will transport the sediments directly to that location. This paper is dedicated to a procedure by which the most effective phosphate sink locations, i.e. the gross sedimentation maxima are determined.

The two dredging strategies may be combined if the situation after external phosphate loading control so requires. Sometimes both strategies are even partly interchangeable. Deepening of the whole lake, designed primarily to decrease the lake's response period, will automatically move more sediments out of reach of the erosive forces, thus increasing the net sedimentation. Dredging of a sink on a gross sedimentation maximum will also help catching sediments accumulated in other places, provided they are resuspended from time to time by wind forces. This may be particularly useful in situations where phosphate-enriched sediments are distributed as a thin, loose but active layer over the whole lake. In such a case, a one-off removal may be deemed too crude and costly. However, since the layer will be resuspended frequently, it may be trapped rapidly in a properly located sink.

This paper is aimed to show a method by which the most effective location for a phosphate sink can be found. In the following paragraphs, a conceptual basis will be laid first, showing that basically only two parameters are needed, the Yearly Erosion Time (YET) and the lake depth. In order to determine YET, the influences of the wind induced short waves, currents and set-up are quantified as far as necessary for the purpose. Then, the procedures are outlined to determine the intermediary "gross erosion pattern", resulting finally in the "gross sedimentation pattern", which defines the optimal sink location. This is followed by a qualitative analysis which serves as a retrospective explanation.

Throughout the paper, data from Lake Westeinder, situated near Amsterdam, The Netherlands, are used as a test and illustration. The

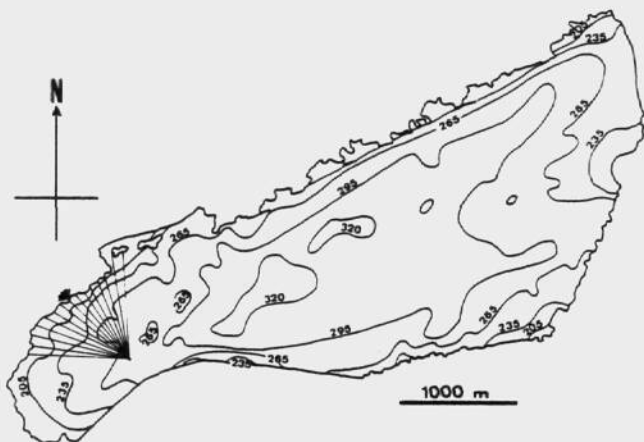


Fig. 1. Morphology of Lake Westeinder and example of effective fetch distance determination radials.

lake's morphology is shown in Fig. 1. It is further characterized by an average Chlorophyll-a concentration of 60 $\mu\text{g/l}$, total phosphate levels around 0.30 mgP/l, an average water retention time of 10.7 months and a soft, peaty sediment. More information is given by De Groot (7).

APPROACH OF THE LOCATION PROBLEM

As specified above, the crux of the phosphate sink strategy is the determination of the area in the lake where the gross sedimentation has its maximum.

One method could be the installation of a large number of sediment traps all over the lake and measuring their yields over the year. In a sediment trap, the gross erosion is zero. Hence, its yield represents the gross sedimentation (ref. Eq. 1). This direct approach may be considered too cumbersome and costly. Moreover, trap measure-

ments are untrustworthy in turbulent water, which makes them less suitable for the lakes under consideration.

Another method could be the computation of the gross sedimentation through hydrodynamic theory and modelling. According to Smith (24), the gross sedimentation depends primarily on the turbulence of the water, the concentration of suspended matter and its fall velocity. The turbulence is influenced by all water movements which, in their turn, are complex functions of the wind, the water throughflow and other driving forces. Many water movement types can be distinguished, e.g., wind driven short waves, seiches and drift currents, Coriolis and Langmuir currents and many others, reviewed by Sly (23), Csanady (6) and Allen (1). In any place of the lake, the concentration of suspended matter not only depends on the above water movements, but also on the resuspension, inflow and sedimentation that has taken place upstream, relative to the currents. The fall velocity, the third factor in the gross sedimentation, varies considerably. The fall velocity distribution in a lake will depend on the gross erosion, algal species, flocculation etc. (14). In most cases, a direct theoretical approach of the gross sedimentation will require a massive modelling effort. With regard to lakes and estuaries for which a hydrodynamic model is already in existence, it may be feasible to add a sedimentation mechanism to the model equations. Cole and Miles (5) and Uchirin and Weber (27) have followed this line, focussing on the net sedimentation (accretion), which is easier to be used in model calibrations than our variable of interest, the gross sedimentation.

The approach expounded in this paper uses hydrodynamic theory and empirical data while circumventing the above problems. Basically, it uses the fact that in many cases the net sedimentation (nS) and the gross erosion (gE) may be estimated with less effort than the gross sedimentation (gS). Eq. 1 shows that the spatial pattern of the relevant variable gS can be found if the patterns of nS and gE are known separately. Both may be computed manually, starting from a depth contour map and a yearly wind distribution as the only data sources. These are available for almost any lake in the world. The complete line of reasoning is as follows.

A variable YET, the Yearly Erosion Time, is defined as the number of days in an average year that gross erosion takes place (disregarding possible concurrent gross sedimentation). The variable YET will show a certain pattern over the lake. Our analysis will be

confined to lakes (or part of lakes) that satisfy the following conditions:

- (1) the depth is not a relic and is not maintained artificially
- (2) $YET > 0$.

Areas ruled out by the first condition most often will be known beforehand. If not, they will appear as inexplicable, "unnatural" spots in the course of the analysis and may be omitted at that moment. Areas excluded by the second condition may be found retrospectively, after YET is calculated. Fig. 7 shows that Lake Westeinder satisfies this condition over the full surface, as will most shallow lakes. On places where $YET > 0$ any decrease in depth will increase the bottom's exposure to wind driven waves and currents and generate its own counterbalancing process of erosion. Hence, in all (parts of) lakes that satisfy the above condition *the spatial depth pattern expresses the spatial and temporal pattern of net sediment transport, nT .*

The relation between depth and net transport can be stated as:

$$\text{depth} :: - nT \quad (2)$$

introducing the following notation: $y :: x$ means that y is some linear function of x ($y = a \cdot x + b$), with a positive slope a . Combined with Eq. 1 it follows that:

$$\text{depth} :: (gE - gS) \quad (3)$$

The depth is the integrated effect of gross sedimentation and gross erosion taking place during the full meteorological year. Hence, gE and gS are to be conceived as yearly totals. Conceptually, the gross sedimentation may be split into a spatial average $\overline{gS}$ and an additional, spatially differentiating component, ΔgS , variable over the lake:

$$\text{depth} :: (gE - \overline{gS} - \Delta gS) \quad (4)$$

In this equation, $\overline{gS}$ may be regarded as the evenly distributed gross sedimentation which would occur if all sedimentation determining factors (sediment inflow, currents, vortices etc.) would be distributed evenly over the lake, taken over the year. Now, it may be temporarily assumed that these factors indeed have this character of even distribution, implying:

$$\Delta gS = 0 \quad (5)$$

$$\text{depth} :: (gE - \overline{gS}) \quad (6)$$

which, since $\overline{gS}$ is a constant, equals:

$$\text{depth} :: gE \quad (7)$$

In a subsequent paragraph, it will be shown that it is possible to calculate a parameter YET, the Yearly Erosion Time, which is related to gE by:

$$gE :: YET \quad (8)$$

Hence,

$$\text{depth} :: YET \quad (9)$$

$$\text{depth} = a.YET + b \quad (10)$$

Maintaining the above assumption of even distribution of gS , a regression analysis can now be set up. The lake depth may be measured on a number of points (i) in the lake and the corresponding YETs may be computed. Combining these two series, a best fitting a and b in Eq. 10 can be found for the lake under consideration. Then, at any place i of the lake the depth can be predicted by the regression formula and compared with the actual depth. Their differences are the prediction errors:

$$\text{error}_i = (a.YET_i + b) - (\text{actual depth})_i \quad (11)$$

These errors can be regarded as the influence of the assumption of evenly distributed gross sedimentation ($\Delta gS_i = 0$). Hence:

$$\text{error}_i :: \Delta gS_i \quad (12)$$

Consequently, without knowing the actual values of the gross sedimentation, the Gross Sedimentation Pattern (GSP, ref. Fig. 7) is found as the pattern of depth prediction errors. In places where the lake is

"too shallow" in terms of the depth predicted through the assumption of even sedimentation distribution, additional gross sedimentation is apparent. Hence, without reference to the complex and ever shifting sedimentation phenomena themselves, the most effective location for a phosphate sink is found. The example of Lake Westeinder will show that these "too shallow" places are located well distinct from the shallowest places. In other words, the gross sedimentation pattern does not equal the net sedimentation (accretion) pattern.

THE EROSIVE INFLUENCE OF THE WAVES

As seen above, quantifying the gross erosion is the first step leading to the gross sedimentation pattern. The actual driving force for erosion is the shear stress, exerted on the bottom particles by water movements. These movements are of two types, operating complementary: oscillations resulting from wind induced short waves and movements of a more continuous character, the currents. In this paragraph, attention is paid to the former.

For any place on the lake, the waves' characteristics are determined by the effective fetch distance and the velocity of the wind. The wind velocity is measured at a standard elevation of 10 m. For lakes situated in flat landscapes, as shallow lakes usually are, the average annual distribution of wind directions and velocities, measured at a nearby meteorological station, may be used as the basis for the erosion analysis. (If wind obstacles exist, see Stefan and Anderson, 25.) The effective fetch distance can be determined by means of a procedure, given by the Coastal Engineering Research Center, CERC (3):

- (1) from the point where the effective fetch is to be known, draw a "central radial" upwind, until it intersects the shoreline;
- (2) with steps of 6 degrees, draw 7 more radials on each side of the central radial;
- (3) measure x_k (k from 1 to 15), the lengths of the radials.

Then:

$$F = \sum_{k=1}^{15} (x_k \cdot \cos^2 \beta_k) / 13.5 \quad (13)$$

in which F = the effective fetch distance; x_i = length of the k -th radial; β_k = angle between the k -th radial and the central radial. In Fig. 1, an example is shown.

In order to express the waves' characteristics in terms of F and wind velocity, several semi-empirical formulae are available, among which those of Wu (28) and the revised Sverdrup-Munk-Bretschneider relationships. For Lake Westeinder case the latter appeared to be the most accurate. They are given by CERC (3) as:

$$H = \frac{0.283}{g} \cdot U^2 \cdot \tanh \{0.0125 (gF/U^2)^{0.42}\} \quad (14)$$

$$T = \frac{2.40 \pi}{g} \cdot U \cdot \tanh \{0.077 (gF/U^2)^{0.25}\} \quad (15)$$

$$L = gT^2/2\pi \quad (16)$$

in which H = significant wave height (corresponding with the average height in a visual field measurement of a wave spectrum) (m); T = wave period (s); L = wave length (distance between crests) (m); g = acceleration of gravity (ca. 10 m/s^2); U = wind velocity at elevation of 10 m (m/s); F = effective fetch distance (m). These equations are only applicable for so-called "deep" water, by which it is meant (17) that:

$$d > 0.25 L \quad (17)$$

in which d is the depth (m). Assuming a certain fetch distance and applying Eqs. 15 and 16, the length (L) of the wind driven waves can be calculated as a function of the wind velocity. Substituting this result in Eq. 17 it appears that most shallow lakes are in fact "deep" in the above sense up to rather high wind velocities, e.g., for Lake Westeinder, up to $U = 27 \text{ m/s}$. For cases with $d < 0.25 L$, the formulae become more complicated (3).

In the waves, water particles have an almost orbital motion, whereby the velocity and amplitude rapidly decrease in downward direction. At a very short distance from the bottom, the vertical velocity component has vanished, leaving an oscillating horizontal motion (17), described by:

$$\dot{u}_b = \pi H / \{T \cdot \sinh (2\pi d/L)\} \quad (18)$$

$$a_b = H / \{2 \sinh (2\pi d/L)\} \quad (19)$$

in which $\hat{u}_b$ = the maximum velocity (m/s) in the oscillating motion and a_b = its amplitude (m), with the other parameters defined previously. It may be noted here that the depth does not appear in Eqs. 14, 15 and 16, indicating that the waves are not influenced by the bottom (in "deep" water), while yet, according to Eqs. 18 and 19, the bottom is influenced by the waves. This fact, which is crucial for the understanding of the sediment circulation in shallow lakes, goes surprisingly unnoticed in hydrodynamic literature.

The velocity $\hat{u}_b$ gives rise to a shear stress close to the bottom. The extent to which this stress is transferred to the bottom particles proper depends on the "relative smoothness" of the sediment surface, the ratio between the amplitude a_b and the bottom roughness, r . If r is small relative to a_b , the shear stresses are partly taken up by a laminar sublayer at the bottom. Above a certain threshold, the shear stresses are fully exerted on the bottom particles. Quantitatively, from Swart (26) it can be found that:

$$p = \text{MIN} \{1, 1.155(a_b/r)^{-0.375}\} \quad (20)$$

in which p is the effective fraction of $\hat{u}_b$. For soft sediments, the magnitude of r is still a matter of debate. By analysing a measured current profile (see Appendix I) it has been estimated that the Lake Westeinder bottom roughness has the order of magnitude of $r = 0.01$ m. This figure may be indicative for many soft sediments in dynamic circumstances. For our purpose, this approximation is sufficiently accurate. For the resulting bottom shear stress, boundary layer theory formulates:

$$\hat{\tau}_b = p \cdot \kappa^2 (p \cdot \hat{u}_b)^2 \quad (21)$$

in which the new symbols denote: $\hat{\tau}_b$ = bottom shear stress maximum due to the waves ($\text{kg/s}^2 \cdot \text{m} = \text{N/m}^2$); ρ = density of water (1000 kg/m^3); κ = Von Kármán's constant (0.41).

Erosion will take place if the shear stress exceeds a critical value τ_{cr} :

$$\hat{\tau}_b > \tau_{cr} \quad (22)$$

This implies that, given a rising wind, a sharp jump from stable to

erosive circumstances will occur. By means of Eqs. 13 through 22, for any place on the lake it may be computed how many days a year the shear stress is above the critical value, given a full spectrum of wind directions and velocities over the year. In order to establish the gross erosion pattern it is not necessary to gather data on the actual resuspension rates (ref. Eq. 8).

THE EROSION INFLUENCE OF THE CURRENTS; THE WIND SET-UP AND THE CRITICAL SHEAR STRESS

Besides short waves, the wind induces a drift current. This current transports resuspended material and contributes to the water turbulence, thereby being an important factor in the gross sedimentation. As to the gross erosion, however, its influence is negligible, as shown below.

In Appendix I, hydrodynamic theory is combined with water velocity measurements in Lake Westeinder. This leads to an approximate quantification of the shear stress transfer from the air to the water surface and from the water surface to the bottom, resulting in a relation between bottom shear stress and wind velocity:

$$\tau_b = 5 \cdot 10^{-5} \cdot \rho_a \cdot U^2 \quad (23)$$

in which τ_b = the bottom shear stress created by the wind induced drift, and ρ_a = the density of air (1.2 kg/m^3). In order of magnitude, this relation will hold for most shallow lakes.

Now, using Eqs. 14 through 21 and Eq. 23, both the bottom shear stress due to the waves and to the current may be computed as a function of the wind velocity only. In Fig. 2, the result is shown, for the average fetch distance and depth of Lake Westeinder (1500 m and 2.70 m, respectively). In the Figure, a critical shear stress is indicated, which will be empirically grounded below. It appears that the current influence predominates at low velocities, but that by the time the critical shear stress is reached, the waves' influence is already more than one order of magnitude higher. This will be true for almost any shallow lake. (In any case it may be checked using the above equations.) Hence, for the computation of the gross erosion pattern of a lake, the wind induced drift currents may be neglected.

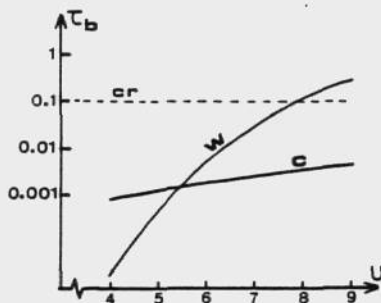


Fig. 2. Bottom shear stress (N/m^2) by waves (w) and currents (c) as a function of wind velocity (m/s), with depth = 2.70 m and effective fetch = 1500 m

On top of the waves and current, the wind causes a slope of the average water level. Slopes of 10^{-5} are a typical order of magnitude for winds around 12 m/s. Stefan and Anderson (25) give a useful introduction. Although the slope as such does not influence the gross erosion or sedimentation, two secondary effects may arise:

- (1) If the wind shifts or drops, the water level will move to a new equilibrium position, causing currents, which may slowly oscillate due to the inertia of the water masses (seiches).
- (2) If basins are connected by narrow channels and the wind is directed along the lake chain axis, a water level difference will develop over those channels, causing compensating currents.

Extensive literature describing the seiche phenomenon is available. It can be calculated that in almost any lake its erosive influence is negligible compared to the waves'. Stefan and Anderson (25) quantitatively deal with the channel effect, while Jónasson and Lindgaard (12) give data on the wind induced currents between the two bassins of Lake Mývatn, Iceland. For lake chains and double-basin lakes, an examination of possible channel effects may be necessary. In order to avoid this kind of complications, the next paragraphs will be confined to single-basin lakes.

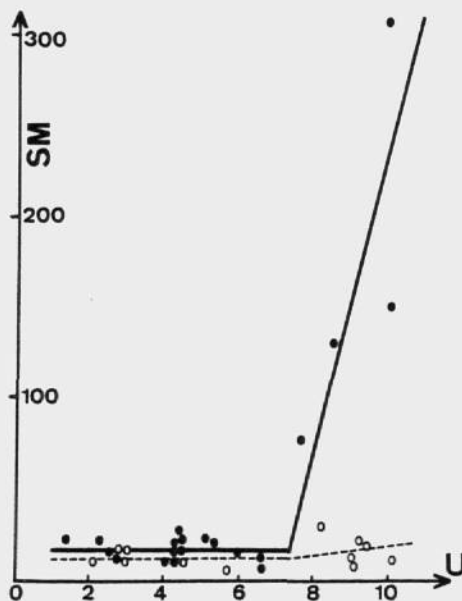


Fig. 3. Suspended matter concentration (SM, mg dry weight/l) for a Lake Westeinder shore location, as a function of wind velocity (m/s). Open dots: offshore winds; closed dots: onshore winds. Incidental measurements from 1941 (8).

It now becomes relevant to examine Fig. 3 (derived from De Gruyter and Molt, 8), which represent the relation between the wind velocity and the concentration of suspended matter, at a location on a Lake Westeinder shore. The strong effect of the wind is clearly visible, indicating that wind is indeed the main erosion driving force. Additionally, it appears that offshore winds do not affect the suspended matter concentrations at a shoreline location. In such a situation, the waves are low due to the small fetch distance, independent of the wind velocity. The currents will not exhibit such a strong onshore-offshore dependency, because they circulate in the lake. The onshore-offshore dichotomy in the figure supports the conclusion that the currents' erosive influence is indeed negligible.

In the wind effect, a clear-cut jump from stable to erosive circumstances is apparent at a wind velocity of ca. 7 m/s. Using this figure and the average fetch distance, the critical bottom shear stress for Lake Westeinder can be evaluated by means of Eqs. 14-22, resulting in $\tau_{cr} = 0.10 \text{ N/m}^2$ (1 dyne/cm²). This complies well with experimental data for soft muds given by Krone (15), Metha and Parthenaides (18), Kelly and Guarte (13) and Cole and Miles (5). The value of 0.10 N/m² will be used to establish the gross erosion pattern, below.

THE GROSS EROSION PATTERN (GEP)

As stated in the second paragraph, determining the gross erosion pattern (GEP) is the first step towards the most effective phosphate sink location. The previous paragraphs indicate that only the waves need to be taken into account.

For almost any place in the world, data are available concerning the average yearly number of hours or days that the wind blows in a certain direction with a certain velocity. This yearly wind spectrum can be condensed into a number of characteristic wind groups, each with its own number of days of occurrence. Table I shows the wind spectrum of Lake Westeinder, condensed into 26 wind groups by defining six classes of average direction and five classes of average wind velocity, in such a way that not too large a number of days appears for winds with a velocity around the critical one (7 m/s for Lake Westeinder).

Now, the following general procedure may be applied:

- (1) Divide the lake into a number of grid points (100 or more).
- (2) Assign a depth and a critical bottom shear stress to each point.
- (3) Determine the bottom shear stress $\bar{\tau}_b$ for each point and each wind group by means of Eqs. 13-21.
- (4) Assume a function of the form $gE :: \bar{\tau}_b$ for each point.
- (5) Calculate the average yearly gross erosion for each point, weighting for the number of days in each wind group.

These gE 's will form a pattern over the lake (GEP), a spatial function of the wind distribution and the lake morphology.

Below, a simplified procedure, adapted to manual computation, shows good results. It consists of the following steps:

- (1) Determine the critical fetch distance (F_{cr}) for each wind direction, based on the average depth and an approximate value of the critical bottom shear stress (τ_{cr}), through Eqs. 14-22.
- (2) For each wind group, determine the area of the lake where the critical fetch distance is exceeded. This may be done either by computer (using grid points), or manually (on the map). Assign the number of days of each wind group to the area where F_{cr} is exceeded.
- (3) Now, add up all these "gross erosion days" for each area in the lake. In formula:

$$YET_i = \sum_{j=1}^2 (M_{i,j} \cdot N_j) \quad (24)$$

in which YET_i = Yearly Erosion Time on lake point i (days/year); j = wind group number; N_j = yearly number of days with winds in group j ; and M = a Heaviside operator; $M_{i,j} = 0$ if $F_{i,j} < F_{cr}$; $M_{i,j} = 1$ if $F_{i,j} \geq F_{cr}$.
The yearly erosion times will appear as a pattern over the lake.

In Table I, the critical fetch distances for Lake Westeinder are shown ($d = 2.70$ m, $\tau_{cr} = 0.10$ N/m²). Fig. 4 illustrates the second step in the procedure, for Lake Westeinder's wind groups 15 and 19 only. Fig. 5 shows the complete result: the distribution of Yearly Erosion Times over Lake Westeinder. In this figure, lee shores (small YETs) are clearly visible.

It may be noted that a different (albeit not too far-fetched) choice of the critical bottom shear stress will change all calculated YETs but leave the pattern intact. Between the general and simplified procedures many compromises can be made. For example, in determining critical fetch distances for different depths, depth differences may be accounted for in step 2 of the simplified procedure.

The yearly gross erosion is assumed as linearly dependent on YET (Eq. 8). Then, Fig. 5 can be defined as the gross erosion pattern (GEP), with YET as its parameter. As shown by Eqs. 1-12, no further specification is necessary for the sink location problem.

		----- Average Wind Velocity (m/s) -----				
		1.5	4.0	6.5	9.0	13
Average Wind Direction (degrees)	60	10 (1)	29 (5)	27 (9)	11 (15)	3 (21)
	150	12 (2)	29 (6)	23 (10)	11 (16)	3 (22)
	210	} 11 (3)	} 28 (7)	12 (11)	9 (17)	8 (23)
	240			14 (12)	14 (18)	15 (24)
	270			12 (13)	10 (19)	7 (25)
	330	10 (4)	18 (8)	20 (14)	13 (20)	6 (26)
Critical fetch		»3500	>3500	2400	1100	580

Table I.— Number of days per year in 26 Lake Westeinder wind groups. Between brackets: wind group number

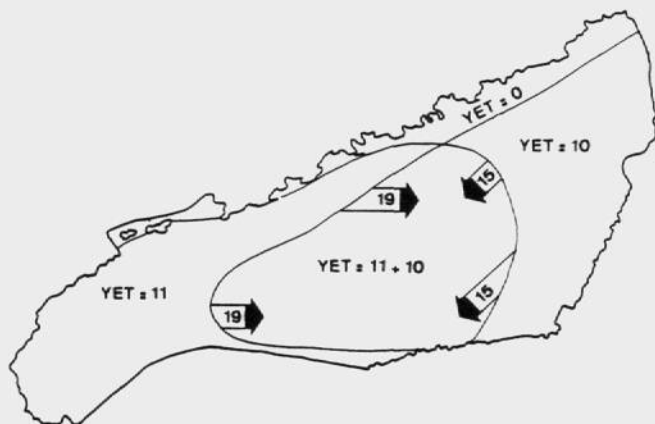


Fig. 4. Example of determination of Yearly Erosion Time (YET) for Lake Westeinder wind groups 15 and 19.

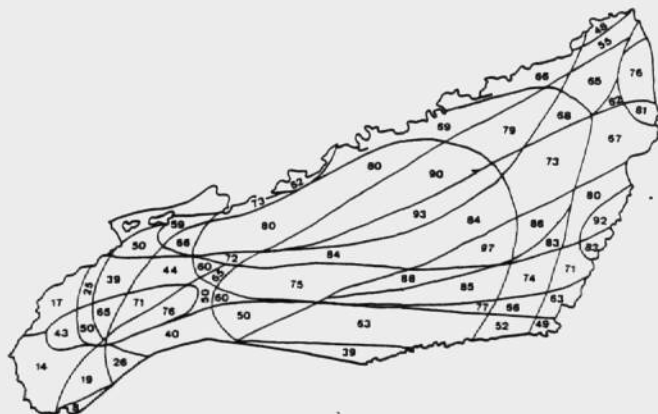


Fig. 5. Gross Erosion Pattern (GEP) of Lake Westeinder, with YET (days per year) as parameter.

THE GROSS SEDIMENTATION PATTERN (GSP)

In the paragraph concerning the problem approach, two conditions have been formulated which must be satisfied if the gross sedimentation pattern (GSP) is to be derived from the patterns of gross erosion (GEP) and depth. The first condition ($YET > 0$) is satisfied over the whole of Lake Westeinder, as can be seen in Fig. 5. Concerning the second condition (no artificial depth maintenance), it is noteworthy that some Lake Westeinder sediments are dredged for use in agriculture. From what is known about this activity, it may be assumed that its overall effect will not be of significant importance. Below, this subject will be returned to.

Following the line of reasoning set out in the second paragraph, it is now relevant to investigate how the depth is expressed in terms of the Yearly Erosion Time (Eq. 10). For that purpose, a random sample of YET-depth pairs must be drawn from the lake map. For Lake Westeinder a grid with 200 by 500 m intervals was used, yielding 99 pairs. The regression equation was calculated as:

$$\text{depth} = 1.07 \text{ YET} + 198 \text{ (cm)} \quad (25)$$

while the correlation coefficient between YET and depth was $R = 0.69$, significant on the 0.0005 level. This regression is an implicit check on the second condition; a low R would indicate that it is not satisfied. On the other hand, a very high R would imply that the YET-pattern explains the depth pattern completely, which would mean that no part of the lake is especially suitable for a phosphate sink ($\text{error}_i = 0$; ref. Eqs. 11 and 12).

Fig. 6 shows the YET-depth scattergram, visualizing which points deviate the strongest from the regression line. These errors indicate the places where the gross sedimentation unevenness concentrates (ref. Eq. 12). They have been divided into six classes. "EE", "E" and "e" for the places where the lake is "too deep" to a greater or lesser extent, and "SS", "S" and "s" for the "too shallow" places. Transferring these points back to the map reveals the Gross Sedimentation Pattern, GSP. As shown in Fig. 7, the GSP of Lake Westeinder has a well-defined structure. The "S"-clusters are the locations of gross sedimentation maxima, the most effective sites for the dredging of a

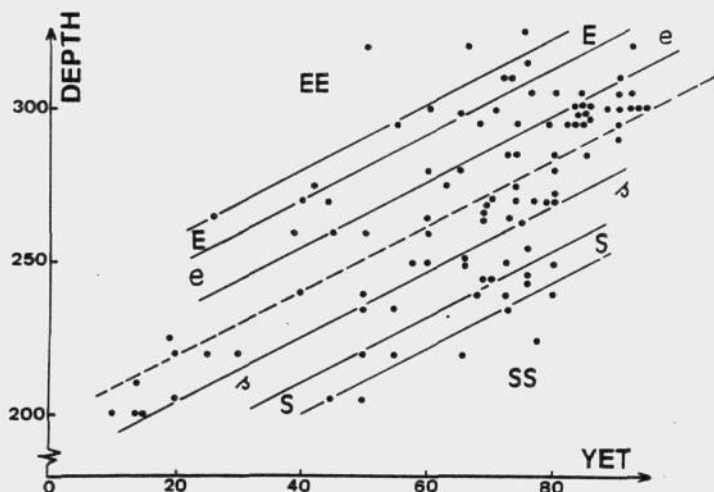


Fig. 6. Lake Westeinder YET-Depth Scattergram. Dashed: regression line. Indicated are the E- and S-classes of deviation.

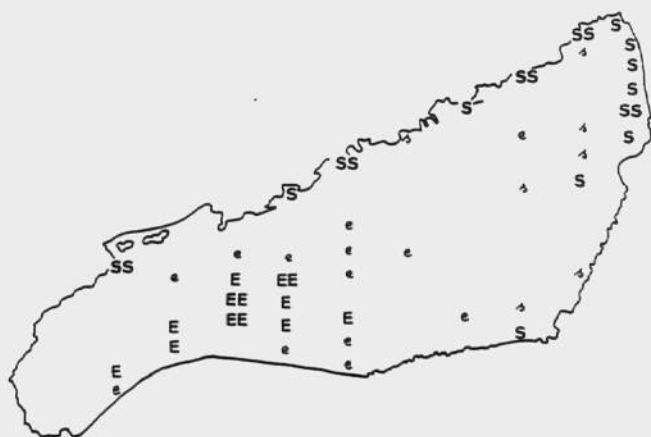


Fig. 7. Lake Westeinder Gross Sedimentation Pattern (GSP). EE, E and e: subnormal gross sedimentation. SS, S and s: gross sedimentation maxima.

phosphate sink. Applying Eqs. 13-22 on the north-eastern corner of the lake it appears that the YET can be artificially reduced to 20 days by a sink depth of approximately 5 m. This indicates that a large part of the material settling in this maximum gross sedimentation area can be prevented from resuspending and thus be removed by dredging.

In the Lake Westeinder GSP it appears that the gross sedimentation does not concentrate on all shallow parts of the lake, nor does it on all lee shores. With reference to the dominant wind directions, the strongest S-cluster is not even a lee shore at all. Gross sedimentation maxima are indicated not by the shallowest places but by places that are too shallow relative to their being exposed to erosion (YET).

The depth contours of Fig. 1 are from 1974 (19). Comparing these with a map from 1954 (21), it is possible to evaluate the possible effect of the commercial dredging. It appears that no significant differences are visible, even with respect to the two protruding parts on the eastern side of the 295 cm depth contour, which might seem unnatural at first sight. In fact, this feature of the depth pattern is a reflection of the GEP (Fig. 5), which also shows a "fork" in the east.

DRIFT CURRENTS IN A QUALITATIVE EXPLANATION

The drift currents, though not necessary in the derivation of the GSP, may be used in a qualitative explanation of that pattern. Such a retrospective exercise will also yield a general rule concerning the GSP.

Some notion about the drift current circulations is required as a starting point. Pons (21) has drawn a circulation pattern on Lake Westeinder for a western wind, given in Fig. 8 (top). The pattern's intuitive basis is that the wind driven current, on approaching the downwind shore, will be deviated towards a lee corner on that shore, because in that area the wind set-up will tend to be less. Following the same principle, the circulation pattern of a south-western wind could also be drawn tentatively, as shown in Fig. 8 (bottom). These currents will transport suspended matter, but not contribute to the gross erosion, as shown before.

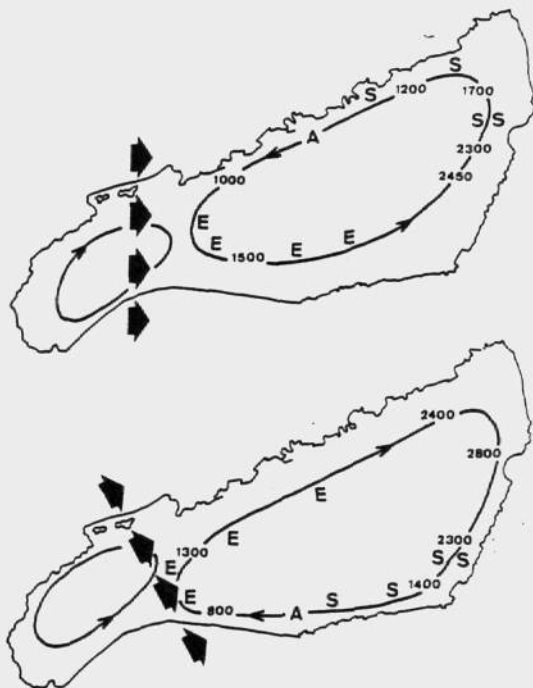


Fig. 8. Tentative Lake Westeinder circulation pattern for a western and a south-western wind. Indicated are the fetch distances and the resulting minima (E) and maxima (S) of the gross sedimentation.

The gross sedimentation of suspended matter may be conceptualized as follows. Primary factors are the suspended matter concentration [SM], and the vertical diffusion coefficient D . A high [SM] and a low D will yield a high gross sedimentation. In its turn, D is a function of the turbulence induced by the waves' height and the current's velocity. Along a drift current stream line, this velocity will be approximately constant. Subsequently, the diffusion coefficient will vary with the wave height only, and, hence, with the fetch distance

only. Thus, following a particular stream line (and assuming a constant wind velocity, depth and suspended matter fall velocity) the gross sedimentation will increase with a decreasing fetch distance and an increasing [SM].

Now, let A in Fig. 8 be the point of departure and let it be assumed that [SM] has a low value, so that, in spite of the small fetch distance present at A, the gross sedimentation rate is normal. Following the streamline, the water will move away from the lee shore at a certain point. From there on, the fetch distance will grow rapidly (from 1000 to 1500 m in the top figure and from 800 to 1300 m in the bottom figure). Depending on the wind velocity, there will exist some distance along this stretch where the critical shear stress is not yet exceeded, so that [SM] will not increase, but where D will be raised, due to the increased fetch. Hence, the gross sedimentation will be below normal ("E" in Fig. 3). When the critical shear stress is reached, conditions will change. Due to the gross erosion, [SM] will rise rapidly, bringing the gross sedimentation back to normal. Up to the fetch distance maximum (2450 and 2800 m in the figures), the current's [SM] will steadily grow, but since the fetch distance grows with it, the gross sedimentation will not be high. This changes drastically when the current has passed the fetch distance maximum. The high SM-load combined with the decreasing fetch will enhance the gross sedimentation. This will be the most salient at the steepest fetch distance drop (from 2300 to 1700 and from 2300 to 1400 m, respectively), the places indicated by SS in the figure. The increased gross sedimentation will slowly diminish when the current loses its excess suspended matter. Returning to A, the cycle starts again.

Superimposing the top and bottom halves of Fig. 8 reveals the basic elements of the GSP already derived. As a general result, it is seen that the gross sedimentation centres on places where "loaded" currents penetrate low fetch distance areas.

DISCUSSION AND CONCLUSIONS

The experiments of Krone (15) and Metha and Partheniades (18) have contributed greatly to the understanding of gross sedimentation. It should be noted that their experimental setting cannot incorporate erosion phenomena and that the clay sediments used are strongly cohesive, so that they may become practically erosion resistant once settled. The Metha and Partheniades model states that sedimentation

may be slow or incomplete at very turbulent flow conditions, but will always be irreversible. This is contradicted by the sharp, wind related increases in suspended matter concentration, particulate phosphate and turbidity in shallow lakes of which Fig. 3 is an example, supplementary to the cases reported by the authors mentioned in the Introduction. For these increases, erosion of sediments is the only possible source. Through its influence of the suspended matter concentration, the erosion is closely connected with the sedimentation in these lakes.

The impact of wind induced short waves and currents on the erosion and sedimentation of particulate matter in shallow lakes is a neglected aspect of limnology. Indicative for this neglect is the common notion that in hydraulically deep water, the bottom and the waves do not significantly influence each other. As for the influence of the bottom on the waves this is true; the reverse certainly is not, as shown in this paper both theoretically and empirically. More attention to the wind induced sediment phenomena is justified if a comprehensive understanding and conservation of the shallow lake ecosystem is desired.

The sink concept is not only applicable for the control of phosphate. Also other sediment associated substances like heavy metals and organic toxins may be trapped and removed in this way.

In this paper it has been explained how the establishment of a dredged phosphate sink can play a role in the control of eutrophication. Emphasis has been put on the procedure through which to locate the gross sedimentation maxima, being the most effective sites for creation of such a sink. Determination and comparison of the pattern of wind induced gross erosion and the depth pattern form the core of this procedure. The example of Lake Westeinder shows that the underlying model is not only conceptually but also empirically consistent. This result has a twofold relevance. First, it may support the development of more quantitative and comprehensive theory concerning the wind influences on shallow lake ecosystems. Secondly, the localizing procedure may be put to a direct practical use in environmental engineering.

ACKNOWLEDGEMENTS

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APPENDIX 1 - QUANTIFYING THE CONTRIBUTION OF THE WIND DRIVEN CURRENTS

The drift current is the effect of the shear stress exerted on the water surface by the wind, given by Wu (28) as:

$$\tau_s = 0.8 \cdot C_z \cdot \rho_a \cdot U^2 \quad (26)$$

in which the new symbols denote: τ_s = shear stress at the water surface (N/m^2); C_z = wind drag coefficient; ρ_a = density of air (1.2 kg/m^3). The drag coefficient depends mainly on the wave height and, through that, on the wind velocity and fetch distance. Wu's review (29) of empirical field data can be summarized as:

$$C_z = (0.08 U + 0.7 \pm 0.6) 10^{-3} \quad (27)$$

The shear stress τ_s is transferred to the bottom through the moving water. The bottom shear stress can not be measured directly. It can be inferred by measuring a water velocity profile and applying a mathematical equation in which the bottom shear stress appears in relation with these water velocities. It is commonly accepted that the velocity profile is a logarithmic function of the vertical (depth) coordinate. Traditionally, the attention is focussed on the velocities near the bottom in relation to the bottom shear stress and on the velocities near the surface in relation to the surface shear stress, respectively. This gives rise to problems when both bottom and surface shear stresses are present. Some arbitrary decision seems to be necessary as to over which part of the depth the one or the other equation is applicable. However, a bottom shear stress exerts an influence up to the surface, while a surface shear stress exerts influence down to the bottom. Therefore, the "surface" and "bottom" equations can be simply

super-imposed over the full water depth. This gives rise to the equation blow, in which the traditional "surface" and "bottom" equations appear as the two terms on the right hand side:

$$u(z_1) - u(z_2) = \pm \sqrt{\frac{\tau_s}{\rho K^2}} \cdot \ln(z_1/z_2) \pm \sqrt{\frac{\tau_b}{\rho K^2}} \cdot \ln\left(\frac{d - z_1}{d - z_2}\right) \quad (28)$$

in which: z_1 and z_2 = measuring depths ($z_2 > z_1 > 0$); $u(z_1) - u(z_2)$ = difference of water velocities on depths z_1 and z_2 (m/s); τ_s = surface shear stress ($z = 0$); τ_b = bottom shear stress ($z = d$), with the other symbols already defined. It can be seen that the influence of the bottom shear stress approaches is small in the surface region ($z_1, z_2 \leq d$), and vice versa.

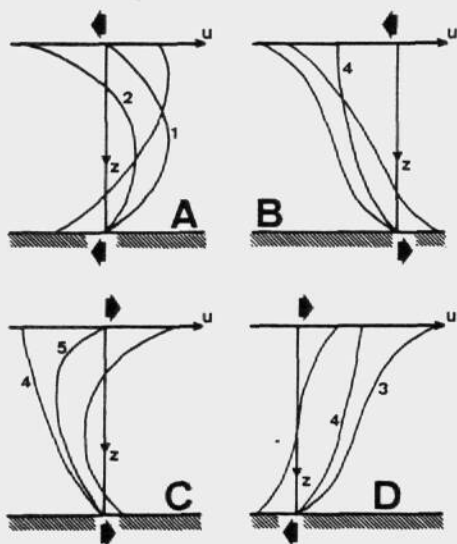


Fig. 9. Examples of velocity profiles described by Eq. 25. Broad arrows indicate direction of shear stress on the water. Profile 1: flow between two planes of equal resistance. Profile 2: wind driven current in a long narrow basin. Profile 3: wind driven current encountered in this study. Profile 4: normal open channel flow. Profile 5: typical ice-covered channel flow.

Eq. 28 allows for a simultaneous and direct estimate of the surface and bottom shear stress from measured water velocity differences. As for the velocities, the formula does not hold over very short distances near the surface and the bottom (the boundary layers). However, the shear stresses are those at the surface and the bottom proper. In Fig. 9 it is shown which profile types can be described by Eq. 28. The choice of the signs in this equation depends on the profile type encountered. Profiles of type A are described by choosing a negative and a positive sign, while the types B, C and D need $(-, -)$, $(+, -)$ and $(+, +)$, respectively.

Theoretically it follows from Eq. 28 that two velocity differences suffice to write out the equation twice and solve for the two unknowns, τ_s and τ_b . In the field, more data are needed to allow for the large velocity variations and insecurities (ref. Fig. 10).

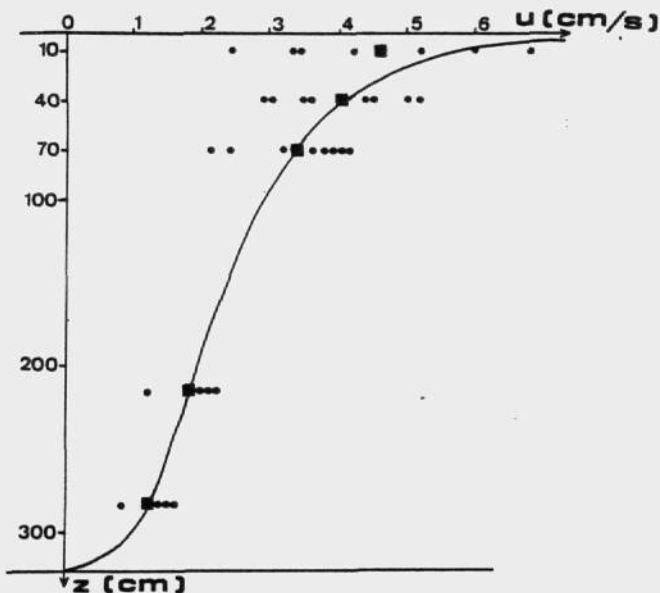


Fig. 10 Flow measured on Lake Westeinder. Round dots: measured values. Square dots: averages. Line: profile inferred through Eq. 28.

On a day with a steady northern wind with a velocity of $U = 4.4$ m/s, velocities were measured at a Westeinder mid-lake station. This location assured that the velocities were indicative for the average drift. Local velocity increases, possibly present at the shores, are not described. The measurements took place by noting down the travel times and directions of drogues, suspended on depths of 10, 40, 70, 215 and 283 cm, within a quadrangle of 5×5 m, erected by stretching ropes just above the water table between poles driven in the bottom. The results are shown in Fig. 10. Using $u(70)$, $u(215)$ and $u(283)$, it is found that $\tau_s = 27$ and $\tau_b = 1.05 \text{ N/m}^2$, resulting in the calculated profile drawn in Fig. 10. Using $u(40)$ instead of $u(70)$ makes only a minor difference. Using $u(10)$ would influence the results considerably, but this deviation has been ignored, reasoning that it may be ascribed to the drogue's relatively large size compared to that depth and to artefact turbulence, caused by the drogue's inertia in the waves. Substituting U and τ_s in Eq. 26, it is found that $C_z = 1.45 \times 10^{-3}$, which falls within the range of Eq. 27. Also it appears that:

$$\tau_b / \tau_s = 0.04 \quad (29)$$

With this ratio and the C_z -value substituted in Eq. 27, the bottom shear stress can be calculated as a function of the wind velocity:

$$\tau_b = 5 \times 10^{-5} \cdot \rho_a \cdot U^2 \quad (30)$$

In order of magnitude this value will hold for most shallow lakes.

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APPENDIX 111 - NOTATION

The following symbols are used in this paper:

a	=	amplitude of wave induced water motion
b	=	subscript, bottom
C_2	=	wind drag coefficient
d^2	=	lake depth
D	=	vertical diffusion coefficient
F	=	fetch distance
F_{cr}	=	critical fetch distance
g	=	acceleration of gravity
g_E	=	gross erosion
g_S	=	gross sedimentation
$\bar{g}_S$	=	gross sedimentation, averaged over the lake
Δg_S	=	gross sedimentation deviations from g_S
GEP	=	Gross Erosion Pattern
GSP	=	Gross Sedimentation Pattern
H	=	wave height
i	=	subscript, lake location number
j	=	subscript, wind group number
k	=	subscript, radial number
L	=	wave length
M	=	Heaviside operator
N	=	number of days in wind group
nT	=	net (vertical) transport of sediment between water and bottom over a given period
p	=	effective fraction of $\bar{u}$
r	=	bottom roughness
s	=	subscript, surface
[SM]	=	suspended matter concentration
T	=	wave period
U	=	wind velocity
u	=	horizontal water velocity
$\bar{u}$	=	maximum horizontal velocity of wave induced water motion
x	=	length of radial
YET	=	Yearly Erosion Time
z	=	distance from water surface
β	=	angle between radial and centre radial
κ	=	Von Kármán constant
ρ	=	density of water
ρ_a	=	density of air
τ	=	shear stress
$\bar{\tau}$	=	maximum shear stress induced by wave induced water motion
τ_{cr}	=	critical shear stress