On the relation between period and form of the light-curve of variable stars of the δ Cephei type

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On the relation between period and form of the lightcurve of variable stars of the δ Cephei type, by *Ejnar Hertzsprung*.

In *B. A. N.* **52**, 83 the paper of LUDENDORFF on this subject has been mentioned and a few additional remarks were made. The increased number of lightcurves of δ Cephei-variables now available invites to a closer examination of the relation between period and form of the lightcurve for this class of stars. The relation in question proves to be still more intimate than would appear at first. To illustrate this the material for the accompanying diagrams has been collected from different sources as indicated in Table 1. I have selected 37 stars with fairly well known lightcurves and distributed over nearly the whole range of periods known for variables of the δ Cephei-type. The group of cluster-variables with periods of about half a day are not treated in the present note. I have tried to reduce all the lightcurves to the same scale, viz: that of steps used in my estimates on the Franklin-Adams plates, merely because about half of the lightcurves were already on that scale and the range is so different according to the effective wavelength of the light used, that the choice of scale is rather arbitrary. The following relations were adopted: 1 step = $1^{m}\cdot4$ photographic = $1^{m}\cdot4/1\cdot5$ visual = $1^{m}.4 \times 1\cdot1$ phgr. intrafocal U V Zeisstriplet, Potsdam. The phase is given in fractions of the period and the zero point of the abscissae is made to coincide with the minimum as indicated by the first Fourier-term, simply to have a fixed rule for comparison of the curves. Each diagram comprises $1\frac{1}{2}$ times the period to show the curve well both around the maximum and the minimum. The line of mean ordinates has been drawn at half height of each diagram. The normal height of a diagram is $1^{s}\cdot1$ or about 1^{m} on the visual scale. When a diagram is higher, the two lines $^{s}\cdot55$ above and below the middle line and corresponding to a height of $1^{s}\cdot1$ have been added for comparison.

A difficult question is, as to which degree the observed curves should be smoothed. Looking through literature, examples of observational errors taken for real irregularities are found more frequently than too strong smoothing, as most observers overrate the accuracy of their own observations. I hope that not too many mistakes of this kind have been made in the accompanying diagrams. The waves on the descending branch of δ Cephei suspected by STEBBINS have been omitted, as the lightcurve of GUTHNICK (*A. N. Jubil. Nr.*) does not show them. The slight hesitation near the top of the rising branch of the lightcurve of ζ Geminorum, observed by GUTHNICK, has been retained. These two examples of what I have smoothed out in doubtful cases and what not will give an idea of where I have put the limit in this respect.

The diagrams have been arranged according to the length of the period. A glance on the series shows that the curves are not distributed at random. Generally the great ranges are found among the longer periods, though the relation between period and range is otherwise not very marked. At the shorter periods many stars have small ranges *) but not all. Small ranges are again particularly frequent at periods of about 10^{d}.

Considering the form of the lightcurves the relation with the period is more definite. At the shorter periods up to about 6^{d} the curves show the characteristic regular δ Cephei form with quick rise and slow decrease, without additional peculiarities. Above 6^{d} a secondary wave on the descending branch of the lightcurve makes its appearance. This secondary wave is a very characteristic feature of the next following periods. It is well marked on the very accurate lightcurve of η Aquilae (period $7^{d}\cdot18$). On the lightcurve of S Sagittae ($8^{d}\cdot38$) the secondary wave appears shortly after maximum and has the character of a secondary maximum. For periods between 10^{d} and 13^{d} the secondary wave is, when present, situated on

*) Polaris has not been included, as its variability was found because it was a spectroscopic "binary" of known period.

top of the suppressed ordinary maximum, the form of the lightcurve being nearly symmetrical. Characteristic curves of this kind are those of AQ Carinae ($9^{d}\cdot77$) and Z Lacertae ($10^{d}\cdot89$) *). The secondary wave has thus superseded the maximum shown at shorter periods. At periods of about 14^{d} or 15^{d} the new superposed maximum occurs earlier, giving the lightcurve again an unsymmetrical shape with a hesitation in the increase of the brightness about midway between minimum and maximum. This hesitation is persistent at the periods mentioned and not found at any other period materially different from them. For still longer periods the number of well known curves is not sufficient to give a definite description of the changes in their form with the period. At periods longer than 16^{d} several curves are found rather similar to that of δ Cephei, showing quick rise and slow decrease apparently without complications. But even among these curves there seem to be characteristic changes depending on the period. The curve found for the star *B. A. N.* **95** *n* with a period of $23^{d}\cdot25$ shows an extraordinary quick rise to maximum and a rather flat minimum, somewhat similar to that of the cluster variable RR Lyrae. It is a striking fact that this peculiar form of the lightcurve was thereafter also found for WZ Carinae with a period of $23^{d}\cdot00$. An extended study of the lightcurves of δ Cephei-variables with such longer periods is much to be desired.

Disregarding the stars showing a very small range, the most striking general feature is perhaps the relatively great symmetry of the lightcurves at periods from about 9^{d} to 13^{d} as compared with both shorter and longer ones. To get this behaviour of the curves expressed in figures an appropriate measure of their skewness is wanted. Suppose a curve is drawn twice, counting the phase once from left to right and once from right to left. If then the two drawings are superposed in such a way, that maximum (and minimum) of the first Fourier term coincide, the two drawings, which are the catoptric images of each other, will differ the more, the skewer the curve is. The mean square of the difference between the ordinates on the two drawings in proportion to the square of the range of the first Fourier term is then a measure of the skewness. This proportion has been entered in Table 1, where it is seen, that all the values of that proportion are small between periods of 9^{d} and 13^{d}.

A physical explanation of the striking facts mentioned above has not been found as yet. As stated by HOPMANN (*A. N.* 5440, **227**, 260; 1926) in the case of ζ Geminorum there seems to be good reason to believe that these stars merely vary the quality and not the quantity of their radiation.

It should be remarked that it would possibly be more rational to start the series with the long periods instead of with the short ones as done above.

I am indepted to Dr. St. SZELIGOWSKI for providing me in advance of publication with the lightcurve of SV Velorum as determined with the Schilt microphotometer on the plates taken with the Franklin-Adams instrument of the η Carinae region.

I have been in doubt whether or not to include W Virginis of period $17^{d}\cdot27$, a good lightcurve of wich has been published in *Harv. Ann.* **80**, N°. 12, 225; 1917. The lightcurve of this variable does not fit into the series and therefore its exclusion could seem prejudiced, but the exceptional galactic latitude ($+ 57°$) of the object as compared with other variables of the δ Cephei type makes it possible that the star does not properly belong to the group treated in the present note. The star Y Ophiuchi with a period of $17^{d}\cdot12$ has a normal lightcurve.

While good lightcurves have only been determined for a small fraction of the variables classified as being of the δ Cephei-type, practically all the periods of these stars are well known. Excluding stars in the Magellanic clouds and including those published in *B. A. N.* **95** the 142 periods between 2^{d} and 30^{d} are distributed as follows:

period	2		3		4		5		6		7		8		9		10		11
number		4		10½		21½		17		16½		13½		3		5½		9½	

	11		12		13		14		15		16		17		18		19		20		21
		3		4		2		3		5		5		5½		3½		1		2	

	21		22		23		24		25		26		27		28		29		30^{d}
		1		½		1½		0		1		0		1		1½		½	

These numbers are shown graphically in Figure 1. The median period is 7^{d} and the mean deviations are represented by periods of about $4^{d}\cdot3$ and 18^{d}.

For intervals of ·1 in the logarithm of the period the numbers are as follows:

log. period	·2		·3		·4		·5		·6		·7		·8		·9		1·0		1·1		1·2		1·3		1·4		1·5
number		1		1		4		9		22		22		24		9½		14½		12		15		5		4	

there being added one star of period $1^{d}\cdot95$. As shown in Figure 2 these numbers are approximately represented by the Gaussian distribution log. period $=$ ·9 ± ·3 (m. e.), but the agreement is not very satisfactory. The value of this frequency curve is made rather problematical by the fact that the range, and therefore the probability of discovery, is systematically different for different periods.

*) The lightcurve of $\varkappa$ Pavonis with a period ($9^{d}\cdot09$), intermediate between those of S Sagittae and AQ Carinae, would be of special interest, but no extensive series of observations of this star has as yet been published.

Figure 1.

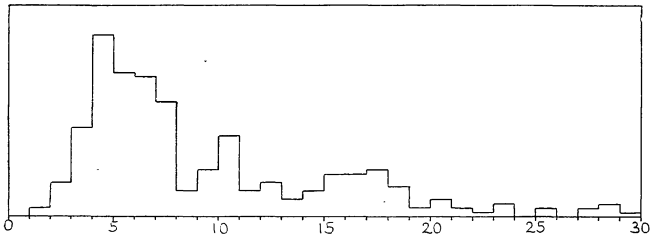


Figure 2.

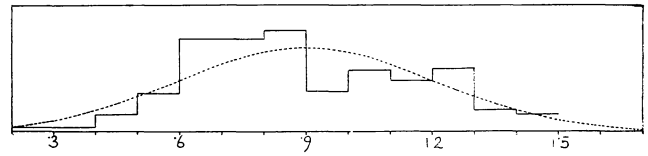
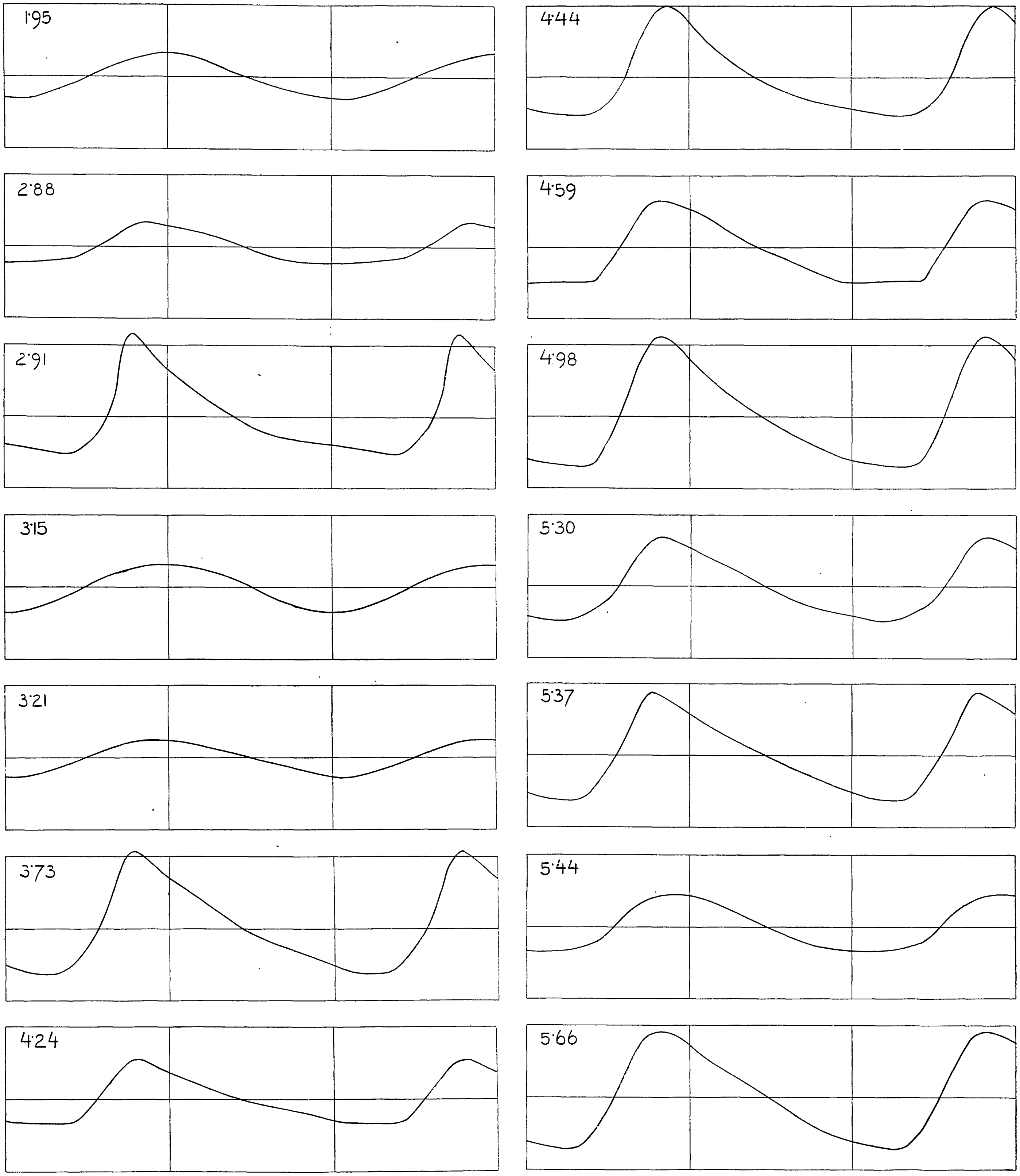
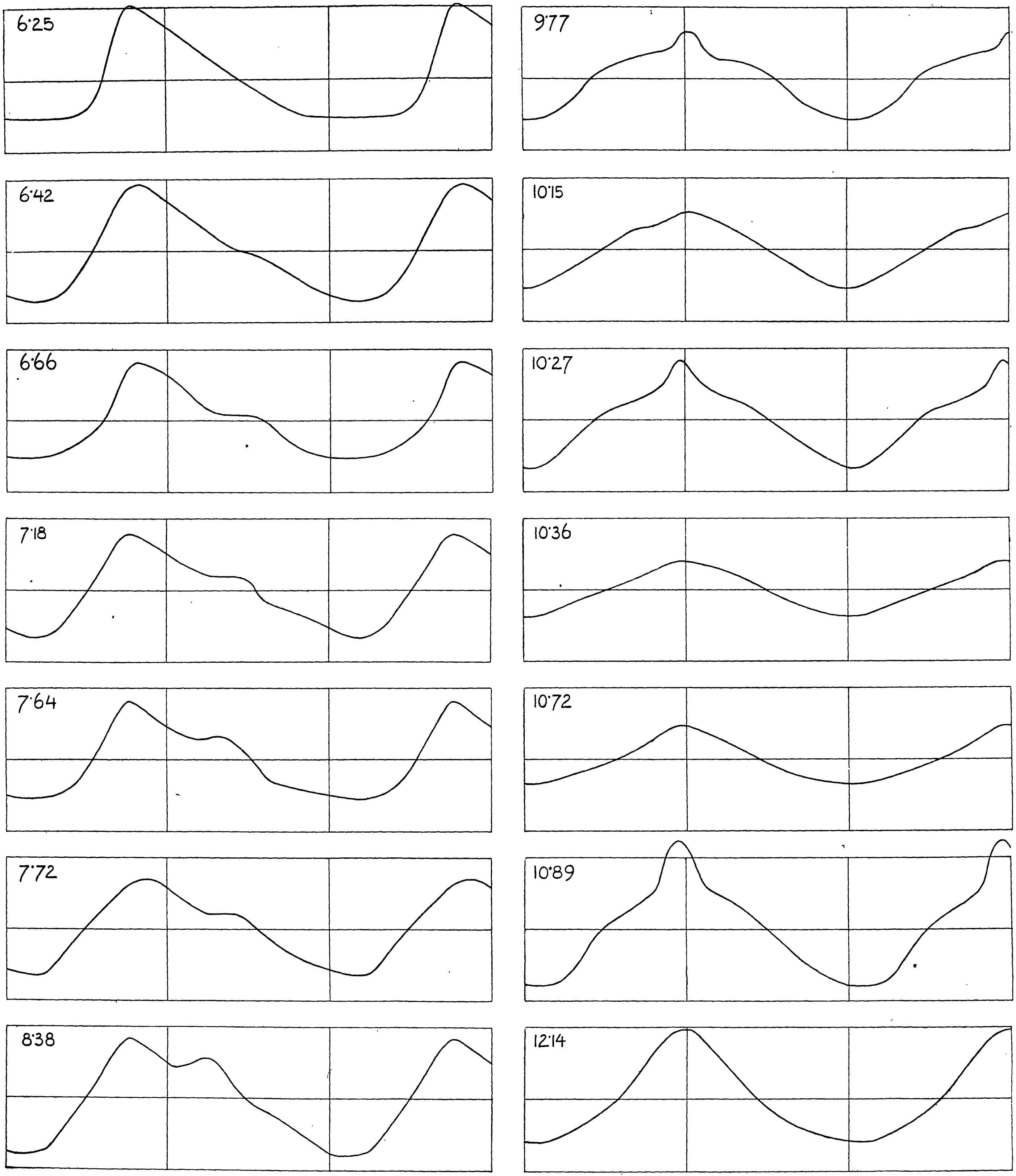
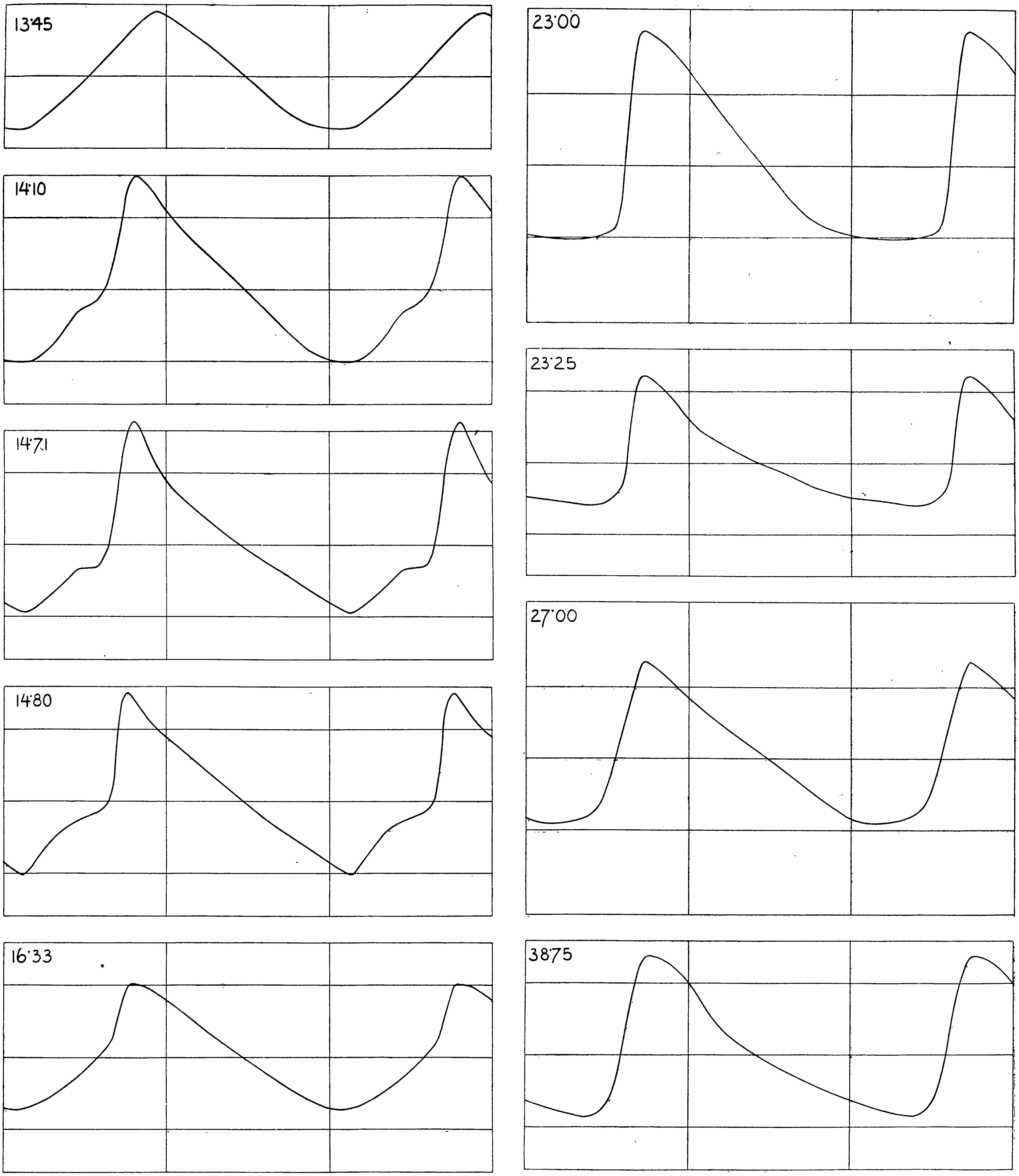


TABLE I.

star	period	total range	range of first Fourier-term	skewness of curve	reference
	d	s	s		
SU Cas	1·95	·35	·33	·04	PARKHURST, *Ap. J.* 28, 278; 1908
B. A. N. 95 *j*	2·88	·30	·30	·16	*B. A. N.* 95
B. A. N. 95 *g*	2·91	·93	·66	1·60	*B. A. N.* 95
SZ Tau	3·15	·36	·34	·01	SCHWARZSCHILD, *A. N.* 4532, 189, 345; 1911
B. A. N. 95 *u*	3·21	·30	·27	·13	*B. A. N.* 95
RT Aur	3·73	·94	·76	1·11	WENDELL, *H. A.* 69, 123; 1907
B. A. N. 95 *i*	4·24	·51	·41	1·23	*B. A. N.* 95
T Vul	4·44	·84	·69	·97	*A. N.* 4596
B. A. N. 95 *p*	4·59	·61	·60	·64	*B. A. N.* 95
V Lac	4·98	1·00	·85	1·05	*B. A. N.* 13
B. A. N. 95 *t*	5·30	·65	·57	·57	*B. A. N.* 95
δ Cep	5·37	·82	·68	1·13	STEBBINS, *Ap. J.* 27, 190; 1908
X Lac	5·44	·43	·44	·18	*B. A. N.* 13
B. A. N. 95 *l*	5·66	·89	·80	·78	*B. A. N.* 95
B. A. N. 95 *h*	6·25	·87	·76	·85	,, ,,
RR Lac	6·42	·90	·78	·94	*B. A. N.* 13
CS Car	6·66	·75	·55	1·42	*B. A. N.* 95
η Aql	7·18	·79	·65	1·08	WYLIE, *Ap. J.* 56, 217; 1922
B. A. N. 95 *o*	7·64	·73	·64	·67	*B. A. N.* 95
ER Car	7·72	·73	·66	·56	,, ,,
S Sge	8·38	·90	·81	·31	*A. N.* 4917
AQ Car	9·77	·68	·57	·04	*B. A. N.* 95
ζ Gem	10·15	·58	·54	·02	GUTHNICK, *A. N. Jubil. Nr.*
B. A. N. 95 *r*	10·27	·81	·66	·04	*B. A. N.* 95
B. A. N. 95 *q*	10·36	·43	·40	·02	,, ,,
B. A. N. 95 *s*	10·72	·44	·40	·01	,, ,,
Z Lac	10·89	1·11	·91	·16	*B. A. N.* 13
RY Cas	12·14	·86	·80	·01	LUIZET, *Bull. Astr.* 25, 250; 1908
B. A. N. 95 *m*	13·45	·86	·84	·04	*B. A. N.* 95
SV Vel	14·10	1·42	1·15	·59	SZELIGOWSKI, unpublished
TX Cyg	14·71	1·46	1·03	1·28	LEINER, *A. N.* 5363, 224, 181; 1925
RW Cas	14·80	1·37	1·01	·82	WHITTACKER and MARTIN, *M. N.* 71, 513; 1911
Harv. 1225	16·33	·99	·84	·44	*B. A. N.* 95
WZ Car	23·00	1·60	1·41	1·16	,, ,,
B. A. N. 95 *n*	23·25	·96	·72	2·36	,, ,,
T Mon	27·00	1·24	1·05	1·14	WENDELL, *H. A.* 69, 42; 1909
U Car	38·75	1·25	·98	1·77	*B. A. N.* 95


1·95
4·44
2·88
4·59
2·91
4·98
3·15
5·30
3·21
5·37
3·73
5·44
4·24
5·66


6·25
9·77
6·42
10·15
6·66
10·27
7·18
10·36
7·64
10·72
7·72
10·89
8·38
12·14


13·45
14·10
14·71
14·80
16·33
23·00
23·25
27·00
38·75