RADIUS- AND GRAVITY VARIATIONS OF AI VELORUM

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ABSTRACT

R, B and U photometry is used to study the variation of radius, temperature and gravity of AI Velorum. These are compared with radial velocity variations observed by Gratton and Lavagnino. It appears that all observed quantities can be derived from a single sum of two sinusoidal variations and are interrelated by the process of differentiation or integration in time.
Equilibrium values are found for the radius $r_o = 2 \times 10^6$km, for $T_{eff} = 7450°$ K and log $g_{eff} = 3.75$.

INTRODUCTION

AI Velorum is a double-period pulsating variable with periods $P_0 = 0.111574$ and $P_1 = 0.086208$ day. The strong modulation of the amplitude and phase of the lightcurves show that two interfering oscillations are present which have comparable amplitude.

The star was observed photo-electrically in B only at the Leiden Southern Observatory, in 1951, 1952 and 1953 (Walraven, 1955). An extensive series of observations with the same instrument was also made in 1979.

At our personal observatory we obtained multicolour RVBLUW observations in 1987 and 1989. The channels R,B and U could be used to derive radius- and gravity variations by means of a modified grid of Kurucz's flux calculations (Kurucz, 1979)

STUDY OF THE R,B AND U OBSERVATIONS

A systematic study of a double-period variable is possible only by finding the average of magnitude (colours) at fixed values of the two phases $\phi_0 = t/P_0$ and $\phi_1 = t/P_1$, i.e. we must determine a mean light surface (colour surface) rather than a mean light curve (colour curve). It is advantageous to describe the surface by means of a set of cross sections parallel to the line $\phi_1 = \phi_0$. This means we study the mean magnitude or colour as a function of

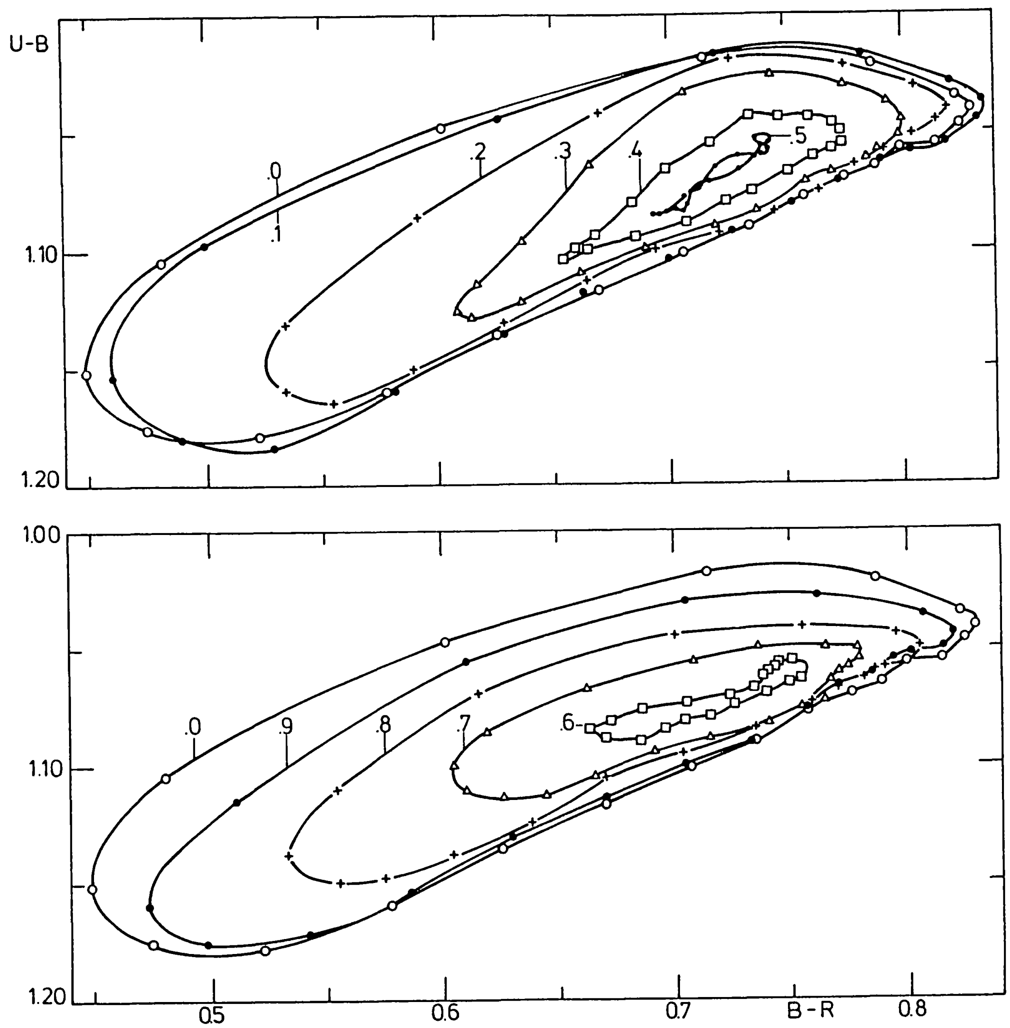

Figure 1 AI Velorum describes loops in anti-clockwise direction as a function of χ, for fixed values of ψ which are shown as labels. The markers are at intervals of 0.05 in χ.

$\chi = (\phi_1 + \phi_0)/2$ and for constant values of $\psi = \phi_1 - \phi_0$.
In Figure 1 the path of AI Velorum in the two-colour diagram U-B versus B-R is shown. If plotted as a function of time the lines become hopelessly entangled. But, if plotted for fixed values of ψ, we obtain separate closed loops. The upper half shows the largest loop of ψ = 0 surrounding the successively smaller loops of ψ = 0.1, 0.2, 0.3, 0.4 and 0.5. The lower half shows the loop for ψ = 0.6 surrounded by the successively larger loops op ψ = 0.7, 0.8, 0.9 and ψ = 0.
The horizontal scale, B-R, depends mainly on temperature and the

righthand tip of the loops corresponds to minimum light or temperature. The loops are all described in anti-clockwise direction. The upper part of a loop is passed quickly from right to left and corresponds to the rising branch of the lightcurve. The lower part is described slowly from left to right and represents the descending branch of the lightcurve. The variation in speed can be judged from the markers, which indicate intervals of 0.05 in χ. The vertical scale, U-B, depends mainly on the effective gravity and the fact that loops are described indicates that the gravity is higher on the rising branch.
A remarkable effect is the systematic difference in shape between the upper half and the lower half of Figure 1. This proves that the maxima of gravity occur at an earlier phase in the upper loops and at a later phase in the lower loops relative to the variations in light and temperature. This can be explained as follows. With a high degree of accuracy the light variations can be reproduced by a distortion of the linear sum of two sinusoidal terms (Walraven, 1955). The undistorted sum is written as

$$u = a \sin 2\pi t/P_0 + b \sin 2\pi t/P_1 \qquad (1)$$

The distortion, which depends in the value of u only, consists of a displacement in time, proportional to u, called S-distortion, followed by a non-linear displacement in ordinate, the M-distortion.
Gratton (1953) showed that the radial velocity observed by him and Lavagnino in 1950 could also be described by formula (1) but subject to S-distortion only.
His phases and ratio of the amplitudes are the same as for the light curves of 1951 (and also of 1989). This ratio is b/a = 0.76. The acceleration, which gives the main part of the variation in effective gravity, is obtained by differentiating formula (1). This gives the sum of two cosine waves but the ratio of the amplitudes is increased from b/a = 0.76 to $bP_0/aP_1 = 0.98$.
The transition from sine to cosine produces the loops. The increased ratio of amplitudes produces an increased phase modulation which is responsible for the difference in shape between the upper and lower loops in Figure 1.
The variations of the radius, in their undistorted shape, can be obtained by integrating (1). This gives the sum of two cosines but with the ratio of amplitudes decreased from b/a = 0.76 to $bP_1/aP_0 = 0.58$.
For a direct comparison of the observed variation of radius and gravity with the integrated respectively differentiated velocity the S-distortion must be included. This can be achieved by using the implicit formula for the velocity dr/dt or $\dot{r}$.

$$\dot{r} = a \sin 2\pi(t+S\dot{r})/P_0 + b \sin 2\pi(t+S\dot{r})/P_1 \qquad (2)$$

We use Gratton's amplitudes multiplied by -24/17, i.e. a = 16.24 km/sec and b = 12.28 km/sec and the coefficient of S-distortion: S = 30 sec per km/sec, if the periods P_0 and P_1 are in seconds.

THE RADIUS VARIATION

Using a modified Kurucz-grid the quantities 5log r/r_0, log g_{eff} and T_{eff} were derived. The modification, necessary because the original Kurucz-fluxes (Kurucz, 1979) produce unrealistic radius variations, is applied in R only and is, at most, 0.03 magnitude. Most probably the cause of the descrepancy is that in AI Velorum the effect of convective energy transport is less than in the Kurucz models. The observed variations of 5log r/r_0 as a function of χ and for ten values of ψ is shown in Figure 2a.
The equilibrium value is indicated by the horizontal lines. The curves are vertically displaced by 0.05 magnitude. The larger scatter of the points is due to photometric errors in R. The curves through the points are hand drawn estimates. In Figure 2b 5log r/r_0 is shown as obtained by integrating $\dot{r}$ from (2)

$$r = -\frac{P_0 a}{2\pi} \cos 2\pi\ (t+S\dot{r})/P_0 - \frac{P_1 b}{2\pi} \cos 2\pi\ (t+S\dot{r})/P_1 - \tfrac{1}{2}\, S\dot{r}^2 + r_0 \quad (3)$$

From the comparison the equilibrium radius of AI Velorum is found as $r_0 = 2 \times 10^6$ km ± 10%.
The largest total amplitude of the relative radius variation is 4%.

THE EFFECTIVE GRAVITY

In Figure 2c the observed log g_{eff} values are shown. The horizontal lines indicate equilibrium gravity : log g_0 = 3.75. The curves are displaced vertically by 0.25 in log. In Figure 2d the computed values of log g_{eff} are shown as found from:

$$g_{eff} = g_0\ (r_0/r)^2 + \ddot{r} \quad (4)$$

and using $\ddot{r}$ = W/1-SW where

$$W = \frac{2\pi a}{P_0} \cos 2\pi\ (t+S\dot{r})/P_0 + \frac{2\pi b}{P_1} \cos 2\pi\ (t+S\dot{r})/P_1 \quad (5)$$

With regard to shape, amplitude and variation of phase χ as a function of ψ there is a general qualitative agreement. The major discordance is a systematic shift in phase χ of the observed curves. Pel observed a similar shift in classical Cepheids (Pel, 1978).

CONCLUSION

The observations show that the variation of radius r, velocity $\dot{r}$ and acceleration $\ddot{r}$ are related via the process of differentiation. This is as expected for the relation between r and $\dot{r}$ because the total displacement of the observed layers is more than a hundred times larger than the pressure scale height. However, the connection of $\ddot{r}$ and gravity, through (4) is more

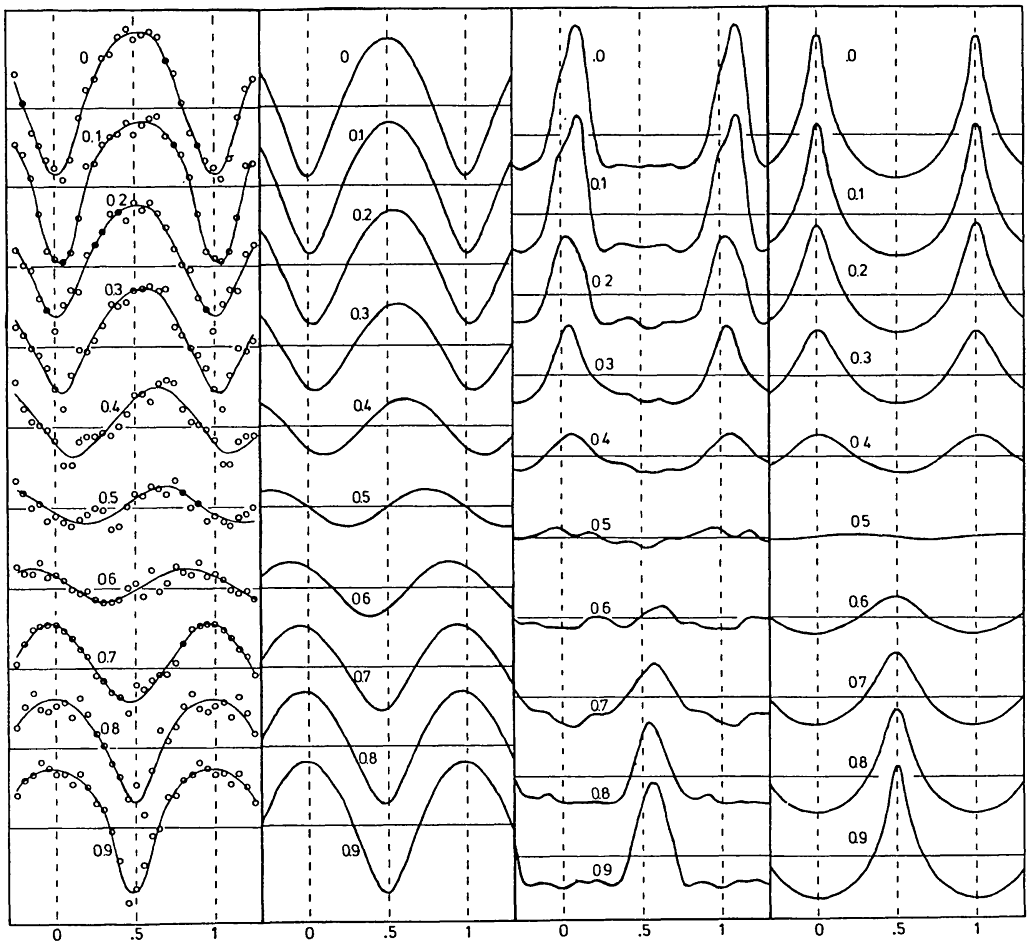

Figure 2a Observed $5 \log r/r_0$ as a function of χ for fixed values of ψ. Horizontal lines, indicating equilibrium value, are displaced vertically by 0.05 magnitude.
Figure 2b $5 \log r/r_0$ calculated from integrated radial velocity using $r_0 = 2 \times 10^6$ km.
Figure 2c Observed $\log g_{eff}$ as function of χ and for fixed values of ψ. Horizontal lines, indicating equilibrium value $\log g_0 = 3.75$, are displaced vertically by 0.25 in log.
Figure 2d $\log g_{eff}$ calculated from the time-derivative of radial velocity.

doubtful. What we actually observe is not gravity but rather density or the differential motions of adjacent layers of mass, which is a more complicated problem. One such complication is the systematic delay of the observed peaks. This can only be seen as a phase lag of the motion of higher layers relative to the lower ones. It is, therefore, remarkable that in other respects the agreement between observed and computed curves is so good. Both sets of curves show how the phase χ of the peaks is constant for $\psi = 0$ through $\psi = 0.4$ then, at $\psi = 0.5$ the peaks disappear; at $\psi = 0.6$ they appear again but have jumped in phase χ by 0.5 and remain there for higher values of ψ. This behaviour of phase and amplitude can be explained only by the addition of two sinusoidal variations of equal amplitude. But the high amplitude curves are far from sinusoidal, so here we have an extreme case of distortion, both in time and in ordinate. And yet this distortion is very similar in the observed and computed curves.

We conclude that to a rather good degree of approximation the variations of all observed quantities of AI Velorum can be derived from a single set of sinusoidal variations characterized by two numbers, i.e. the two amplitudes. The distortions depend on an single number e.g. the coefficient of the S-distortion. For the description of the variation in temperature two more numbers are needed i.e. the scale factor and a coefficient which characterizes the M-distortion.
The fact that such a simple and elegant description of the behaviour of AI Velorum is possible suggests that the star pulsates radially and simultaneously in two modes according to the linear adiabatic wave equation. When the amplitude of the sum of the pulsations becomes large non-linear effects produce the distortions.

We comment on the equilibrium radius for which we found $r_0 = 2 \times 10^6$ km. This is very close to the value derived by Balona and Stobie (1980) who used an entirely different photometric system (Cape-Kron BVRI) combined with simultaneous radial velocity observations. Together with log $g_0 = 3.75$ we find a mass $M = 1.7$ $M_\odot$. This kind of mass was derived earlier by Breger (1977) and is in contrast with the much lower mass proposed by Bessel (1969).

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