Flux-cancellation effect on narrow-channel magnetoresistance fluctuations

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The magnetic-field correlation function of the universal conductance fluctuations (UCF) in a laterally confined two-dimensional electron gas is calculated in the high-mobility regime. Boundary scattering induces cancellations of the magnetic flux enclosed by electron trajectories, which leads to an enhanced correlation field. The theory is discussed in relation to a recent observation of UCF in a narrow GaAs-$Al_xGa_{1-x}As$ heterostructure.

An interesting development in the field of quantum transport in disordered metals is the Al'tshuler-Lee-Stone theory[1,2] of universal conductance fluctuations (UCF). This theory predicts aperiodic fluctuations in the conductance as a function of magnetic field, with an rms amplitude at $T=0$ of order $e^2/\hbar$, independent of sample size or degree of disorder. Such UCF have been observed in metal wires and rings and in quasi-one-dimensional (1D) semiconductor devices.[3] Their origin is the quantum interference of electrons on different trajectories, which surprisingly does not average out to zero within a few (elastic) collisions. (For a simple physical picture, see Ref. 4.) To destroy phase coherence, inelastic processes are necessary, but these become increasingly rare at low temperatures. An increment in magnetic-field shifts the phases of the electrons so that a different interference pattern results, and this is seen in the magnetoresistance fluctuations.

Recently, several groups[5-10] have observed fluctuations in the perpendicular field magnetoresistance of laterally confined 2D electron gases in GaAs-$Al_xGa_{1-x}As$ heterostructures. Because of the high electron mobility in these quasi-1D channels, the elastic mean free path l_e (associated with impurity scattering) can be much larger than the channel width W. This leads to an interesting modification of the current theory, in which the dirty metal limit $l_e \ll W$ is assumed.[11] The effect we have in mind is the *flux cancellation* known from superconductivity,[12] and studied recently[13-15] in relation to the weak localization peak in the magnetoresistance. As illustrated in Fig. 1, the intersecting trajectories of two electrons moving ballistically from one boundary to the other can enclose zero flux, due to a geometric cancellation. As a consequence, the electrons acquire no relative phase shift in a magnetic field. One would, therefore, expect the correlation field for magnetoresistance fluctuations to be *enhanced* in high-mobility channels with $l_e > W$. We note that finite-size enhancement of the correlation field was seen, and understood as a geometrical effect, in previous numerical work by Lee, Stone, and Fukuyama.[11] The calculation of the enhancement is the main issue of the present paper.

For simplicity, we limit ourselves here to the quasi-1D geometry of the experiments mentioned above (a thin metal film in parallel magnetic field is analogous). We consider the case $W \ll l_\phi \ll L$, where L is the length of the channel, and l_ϕ the phase-coherence length. There is an additional restriction, which is essential, that the motion along the channel is diffusive on the phase-coherence time scale, that is to say $\tau_\phi \gg \tau_e$. [We use the definitions $l_\phi \equiv (D\tau_\phi)^{1/2}$, $l_e \equiv v_F \tau_e$, with D the diffusion coefficient and v_F the Fermi velocity.] We will return to this restriction below. The quantity to be calculated is the correlation function[11,16]

$$F(\Delta B) \equiv \langle \delta G(B) \delta G(B+\Delta B) \rangle \ . \tag{1}$$

Here $\delta G(B) \equiv G(B) - \langle G(B) \rangle$, with $G(B)$ the conductance in a magnetic field B (the angle brackets denote an average over impurity configurations). Note that F is B independent, under the assumption that B is outside the region of the weak localization peak in the magnetoresis-

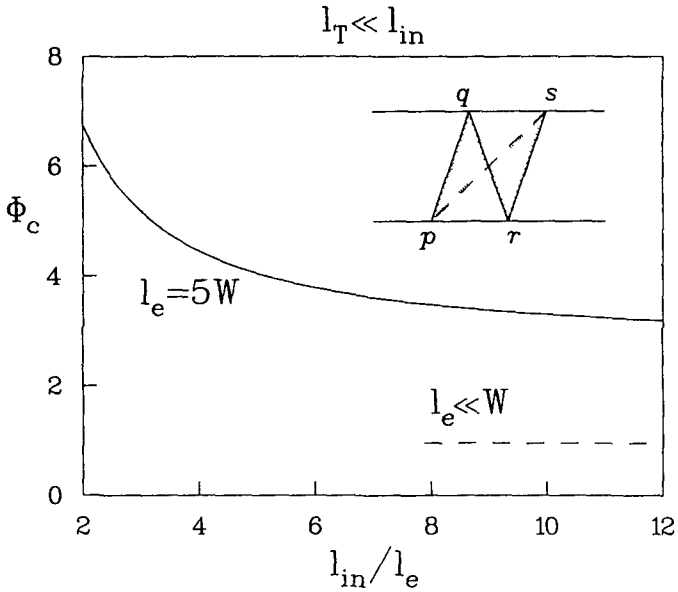

FIG. 1. Plot of the correlation flux Φ_c (in units of $2\pi\hbar/e$) vs the zero-field phase-coherence length $l_{in} \equiv l_\phi(0)$ (normalized by the elastic mean free path l_e). The solid curve is for a high-mobility channel [Eq. (10)], the dashed line is the dirty metal value. The inset shows two trajectories of an electron in a narrow channel (dashed and solid lines) to illustrate the characteristic flux cancellation: The *relative* phase shift acquired in a perpendicular magnetic field is proportional to the flux enclosed by the contour *pqrsp*, which is zero.

tance around zero field. (Then only the diffusion propagator contributes to F, which is sensitive to field differences ΔB, not to B itself.[11,16]) The value $F(0)$ equals the variance of the conductance fluctuations, and the field increment ΔB_c such that $F(\Delta B_c)=\frac{1}{2}F(0)$ is by definition the correlation field.

The variance is affected by boundary scattering only implicitly via the diffusion coefficient, so that no new analysis is required. At $\Delta B=0$ one has the two formulas[11]

$$F=\alpha\left(\frac{e^2}{2\pi\hbar}\right)^2\left(\frac{l_\phi}{L}\right)^3 \text{ if } l_\phi\ll l_T\ , \tag{2a}$$

$$F=\beta\left(\frac{e^2}{2\pi\hbar}\right)^2\frac{l_T^2 l_\phi}{L^3} \text{ if } l_\phi\gg l_T\ . \tag{2b}$$

The thermal length is defined by $l_T\equiv(\hbar D/k_BT)^{1/2}$. The above asymptotic expressions are given in Ref. 11 up to unspecified numerical coefficients α and β. This has caused considerable uncertainties in the experimental literature. To eliminate these, we have evaluated the series and integrals of Lee *et al.*[11] (which can be done analytically in the asymptotic regimes) and find $\alpha=6$, $\beta=\frac{4}{3}\pi$. For the analysis of experiments in which l_ϕ and l_T are comparable, we also give a formula which interpolates between Eqs. (2a) and (2b):

$$F\approx 6\left(\frac{e^2}{2\pi\hbar}\right)^2\left(\frac{l_\phi}{L}\right)^3\left[1+\frac{9}{2\pi}\left(\frac{l_\phi}{l_T}\right)^2\right]^{-1}\ . \tag{3}$$

This formula is approximate, but the differences with the full expressions of Lee *et al.*[11] are not significant (less than 10%).

The correlation field ΔB_c depends on the geometry of the trajectories (via the enclosed flux), and this is where boundary scattering comes in explicitly. The diagrammatic analysis of Lee *et al.*[11] gives F in terms of the diffusion propagator $P(\mathbf{r},\mathbf{r}';t)$. This quantity is the product of three terms:[17] (1) the classical probability to diffuse from $\mathbf{r}$ to $\mathbf{r}'$ in a time t (independent of B in the field range of interest); (2) the relaxation factor $\exp[-t/\tau_\phi(0)]$, with $\tau_\phi(0)$ the zero-field phase-coherence time (also referred to in the literature as the inelastic scattering time τ_{in}); (3) the average phase factor $\langle e^{i\Delta\phi}\rangle$, with $\Delta\phi$ the phase shift induced by the field increment ΔB. More explicitly,

$$\langle e^{i\Delta\phi}\rangle\equiv\left\langle\exp\left[\frac{ie}{\hbar}\int_{\mathbf{r}}^{\mathbf{r}'}\Delta\mathbf{A}\cdot d\mathbf{l}\right]\right\rangle$$

$$=\exp(-t/\tau_{\Delta B})\ , \tag{4}$$

where the line integral of the vector potential increment $\Delta\mathbf{A}$ is along a classical trajectory which goes from $\mathbf{r}$ to $\mathbf{r}'$ in a time t, and the average is taken over all such trajectories.[18] As a consequence of Eq. (4), proven in Ref. 14, the effect of a nonzero ΔB on the diffusion propagator is simply to increase its relaxation rate by an amount $1/\tau_{\Delta B}$,

$$1/\tau_\phi(\Delta B)=1/\tau_\phi(0)+1/\tau_{\Delta B}\ . \tag{5}$$

It follows that the correlation function $F(\Delta B)$ is given by Eqs. (2) and (3), with a ΔB-dependent length $l_\phi(\Delta B)\equiv[D\tau_\phi(\Delta B)]^{1/2}$. By solving $F(\Delta B_c)=\frac{1}{2}F(0)$ we then find that the correlation field is determined by the relation

$$\tau_{\Delta B_c}=\gamma\tau_\phi(0)\ , \tag{6}$$

where $\gamma=\frac{1}{3}$ if $l_\phi\gg l_T$, and $\gamma=1/(4^{1/3}-1)\approx 1.7$ if $l_\phi\ll l_T$. [For comparable l_T and l_ϕ the value of $\tau_{\Delta B_c}$ can be obtained using the interpolation formula (3).]

It remains to determine $\tau_{\Delta B}$. In Ref. 14, the relaxation time τ_B relevant to weak localization in a magnetic field B was calculated for the present geometry. The relaxation time $\tau_{\Delta B}$ relevant to UCF is obtained from those results simply by replacing B by $\frac{1}{2}\Delta B$ [compare the phase definitions in Eq. (2.2) of Ref. 14 and Eq. (4) above]. One thus finds, in the dirty metal limit ($l_e\ll W$),

$$\tau_{\Delta B}=12(\hbar/e\Delta B)^2/DW^2\ , \tag{7}$$

and, for a high-mobility channel ($l_e\gg W$),

$$\tau_{\Delta B}=4C_1(\hbar/e\Delta B)^2/v_FW^3+2C_2(\hbar/e\Delta B)l_e/v_FW^2\ . \tag{8}$$

Equation (8) is a numerically obtained interpolation formula, the limits of small and large ΔB being exact.[14] The coefficients C_1, C_2, and the diffusion coefficient D depend on the type of boundary scattering. In the heterostructures considered the scattering is predominantly specular;[15] then $C_1\approx 9.5$, $C_2=\frac{24}{5}$, and $D=\frac{1}{2}v_Fl_e$. We stress that the results (7) and (8) are only valid for sufficiently small-field increments ΔB, such that $\tau_{\Delta B}\gg\tau_e$, and $D\tau_{\Delta B}\gg W^2$. These inequalities are implied by the conditions $\tau_\phi\gg\tau_e$, $l_\phi\gg W$, mentioned earlier. The resulting restriction[14] is that the flux increment $W^2\Delta B$ should be less than $\hbar/e$.

Upon substitution of Eq. (7) into Eq. (6), the criterion of Lee *et al.*[11] for the dirty metal correlation field is recovered,[19]

$$\Phi_c\equiv\Delta B_cWl_\phi(0)=\text{const}\times 2\pi\hbar/e,\ \text{if } l_e\ll W\ . \tag{9}$$

The numerical constant (left undetermined in Ref. 11) is given by $(3/\pi^2\gamma)^{1/2}$, which equals 0.95 for $l_\phi\gg l_T$ and 0.42 for $l_\phi\ll l_T$. In a high-mobility channel, the "correlation flux" Φ_c resulting from Eqs. (6) and (8) is many times larger than the dirty metal value (9). For specular scattering we find

$$\Phi_c=0.38\gamma^{-1}(2\pi\hbar/e)l_e^2[l_\phi(0)W]^{-1}\{1+[1+3.3\gamma l_\phi(0)^2Wl_e^{-3}]^{1/2}\}\ \text{if } l_e\gg W\ . \tag{10}$$

In the regime $l_\phi(0)\gg l_e\sqrt{l_e/W}$, we see that Φ_c is larger than the dirty metal result (9) by a factor of order $\sqrt{l_e/W}$, while for $l_e\ll l_\phi(0)\lesssim l_e\sqrt{l_e/W}$ the enhancement factor can increase up to l_e/W. As a typical example, we have plotted in Fig. 1 Φ_c as a function of $l_\phi(0)/l_e$ for a channel with $l_e=5W$, in the case $l_\phi\gg l_T$. For comparison, the constant dirty metal result is plotted as well. This figure illustrates the importance of the flux cancellation effect on magnetoresistance fluctuations in high-mobility channels.

We have analyzed the UCF seen by van Houten *et al.*,[6] using the above results. The system studied is a narrow conducting channel, etched in the 2D electron gas of a GaAs-$Al_xGa_{1-x}As$ heterostructure. Estimates for the relevant lengths of the narrowest channel studied, at $T=2.4$ K, are $L=10$ μm, $W=138$ nm, $l_e=314$ nm, $l_T=344$ nm. The conducting width W of the channel is much smaller than the lithographic width of 0.5 μm, due to side wall depletion. The value of W given above is obtained from an analysis[6] of the magnetic depopulation of 1D subbands at high fields. An analysis of the low-field weak localization effect gives a value which is 20% lower,[15] and comparable uncertainties exist in the values of l_e and l_T.[20]

In Fig. 2 we have plotted the correlation function $F(\Delta B)$ obtained from the magnetoresistance fluctuations shown in Fig. 3 of Ref. 6. As usual, the impurity average in Eq. (1) is replaced by an average over B. A linear fit through the experimental data points was subtracted before calculating F, to correct for a systematic trend in $G(B)$. We find $F(0)=1.9\times10^{-4}(e^2/2\pi\hbar)^2$, $\Delta B_c=0.05$ T, with error estimates of 30%. Substitution of this value for $F(0)$ into Eq. (3) gives a zero-field phase-coherence length $l_\phi(0)$ of 500 nm. Equations (3), (5), and (8) then predict $\Delta B_c=0.12$ T, more than twice the experimental value. This discrepancy seems rather large to attribute entirely to uncertainties in W. More likely, the reason that the correlation field turns out smaller than predicted is that, as we increase the field increment, more and more electrons lose phase coherence *before entering the regime of diffusive motion.* This breakdown of coherent diffusion is beyond the UCF theory, which assumes $\tau_\phi(\Delta B)\gg\tau_e$,

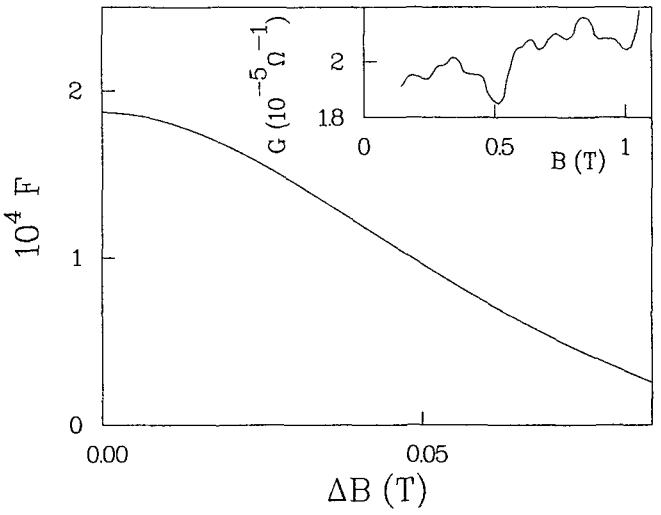

FIG. 2. Correlation function $F(\Delta B)$, in units of $(e^2/2\pi\hbar)^2$, obtained from the magnetoresistance measurements of Ref. 6. The inset shows the experimental data.

but certainly plays a role in systems where the phase-coherence time is comparable to the elastic scattering time.[21] In the case above, the ratio τ_ϕ/τ_e is about 5 at zero field, which is not especially large (and becomes even smaller as we increase ΔB). The situation in the similar experiment of Thornton *et al.*[5] is even less favorable, with τ_ϕ/τ_e about 2 at the lowest temperature studied. The presented theory calls for experiments with a larger ratio of inelastic to elastic scattering times.

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[18]The simple relation (4) holds (Ref. 14) only in the special gauge $\Delta A=\Delta B(z,0,0)$, with the boundaries of the channel at $y=0$, $z=\pm W/2$. For other choices of the vector potential the average phase factor depends on the initial and final coordinates of the trajectories, which would complicate a calculation. The ultimate result for the correlation function F does not, of course, depend on the gauge.
[19]For completeness we also give the $T=0$ results: Lee *et al.* (Ref. 17) find $\Delta B_c WL=1.2(2\pi\hbar/e)$, in the limit $l_e\ll W$. In the opposite limit $l_e\gg W$, we find that the correlation field is determined by the equation $D\tau_{\Delta B_c}=0.22L^2$, with $\tau_{\Delta B_c}$ given by Eq. (8). For specular boundary scattering and in the regime $L\gg l_e\sqrt{l_e/W}$ this simplifies to $\Delta B_c WL=1.5(2\pi\hbar/e)\sqrt{l_e/W}$.
[20]The diffusion coefficient D, appearing in the definition of l_T, is determined from an estimate of the (classical) conductance at higher temperatures,

$$G_{cl}=(me^2/\pi\hbar^2)(W/L)D\approx 23\times10^{-6}\ \Omega^{-1},$$

which gives $D=0.037$ m^{2}s^{-1}. The elastic length l_e follows from the formula $D=\frac{1}{2}v_F l_e$, for specular boundary scattering. (The Fermi velocity $v_F=2.4\times10^5$ ms^{-1} results from the electron density obtained from the Shubnikov–de Haas oscillations).
[21]An *ad hoc* formula, which accounts to some extent for a finite τ_ϕ/τ_e ratio, has been used for the analysis of the weak localization effect in Ref. 15.