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# AN ATTEMPT TO ANALYSE THE INFLUENCE OF THE MOON ON THE RESULTS OF THE INTERNATIONAL LATITUDE SERVICE,

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From the latitudes as published under the auspices of the International Latitude Service from 1900 to 1931 the variation of the vertical due to the attraction of the moon has been determined, in order to obtain information about the rigidity of the earth. A differential method of reduction was used, to avoid all errors in declination and, for the greater part, those in assumed latitude variation. By this method the influence of the sun is greatly reduced. The residuals might have been compared with cos $t$ and cos $2t$, but as the general expression $d\varphi = - P \sin 2z \cos a$ shows a rather large modulation, the attempt was made to determine the complete coefficient $P$. The mean value found is $''\cdot 0133$. At first, long periods of observation, mostly covering one volume of the Latitude Results, were treated together, until it was found that the coefficient showed unexpected variations in time; then, shorter periods were taken.

No explanation for the variation in $P$ is offered. An attempt to trace the origin of this behaviour to small nutation terms, hitherto neglected, failed. To account for it by an error in the adopted value of the constant of nutation seems to require too great a change in this constant.

**1.** The observations performed by the International Latitude Service have repeatedly been the subject of investigations into the influence of the moon on the vertical[1]). Only a restricted number of the observations then available were treated and the results were rather discordant.

In 1931 BROWN and BROUWER published their analysis of the influence of the moon upon the pendulum clock as opposed to quartz clocks, which showed the theoretical effect very nicely, without an appreciable earth-tide lag[2]).

Sir HAROLD SPENCER JONES' result[3]) concerning the Greenwich Cookson telescope also showed a small earth-tide lag, if any.

In his book 'Some problems of geodynamics', page 56, 1926, LOVE states that the earth tide is in phase with the tidal forces.

In 1939 I considered the possibility of analysing all observations published by the International Latitude Service at that date, and because of the results just mentioned I decided to refrain from determining a possible earth-tide lag.

Only after the war, when this work was near to its actual state, I learned about the work done at Washington[4]).

[1]) E. PRZYBYLLOK, *A. N.* **213**, 201; **218**, 85. H. T. STETSON, *Nat.* **131**, 437, 1933. KAWASAKI, *Proc. Imp. Ac. Tokyo*, **11**, 1935.
[2]) E. W. BROWN and D. BROUWER, *M. N.* **91**, 575, 1931.
[3]) *M. N.* **99**, 196, 1939.
[4]) W. MARKOWITZ and S. M. BESTUL, *A. J.* **49**, 81, 1941.

**2.** Development of the expression

$$d\varphi = - P \sin 2z \cos a^{1})$$

according to the arguments $t$ and $\delta$ yields the form:

$$- \tfrac{1}{2} P \sin 2\varphi \,(3 \cos^2\delta - 2) - P \cos 2\varphi \sin 2\delta \cos t - \tfrac{1}{2} P \sin 2\varphi \cos^2\delta \cos 2t,$$

where $P = 206265''.\ \tfrac{3}{2}\,\mu \sin^3 p$, $a$ = moon's azimuth, $\delta$ = moon's declination, $t$ = moon's hour-angle, $\mu$ = moon's mass/earth's mass, $\varphi$ = latitude of observing station, $p$ = moon's parallax.

Evidently there exists a quite large variable term not containing $t$. I decided to try to evaluate $P$ without omitting a term or splitting up the material in groups according to $\delta$ as was usually done.

In order to minimize the influence of several sources of uncertainty I compared for each pair the individual observations during two consecutive months with their mean. In doing this residuals are derived which are entirely free from errors in declination. The reduction of the individual observations to a mean latitude implies the knowledge of the variation in latitude, but for so short a period as two months, the error entering into this reduction cannot be very troublesome.

The observations performed at the International Latitude Stations are all made close around midnight and it is easily understood that in taking the residuals

[1]) HELMERT, 'Höhere Geodäsie' II, 384.

as explained, the influence of the sun is of no importance at all.

A value of $\sin 2z \cos a$ was read from a diagram for every observation and these values were averaged for the same periods as the latitudes. The residuals of $\sin 2z \cos a$ from their means were compared with the corresponding residuals in latitude, yielding an equation of the form:

$$P\,(\overline{\sin 2z \cos a} - \sin 2z \cos a) = \bar{\varphi} - \varphi$$

for every observation.

In solving these equations their number was greatly reduced by taking means. The final least-squares solution depended mostly on 25 equations, each weighted according to the number of contributing equations.

**3.** Originally, in the solution for $P$ all observations contained in one volume of the Latitude Results were used, for each station separately. In the course of the work it became obvious that the values for $P$ yielded by the different stations for one period showed a decidedly better agreement than did the values for $P$ resulting from the observations at one station for different periods. To obtain a clearer insight into this unexpected behaviour the original intervals were reduced and new solutions for $P$ derived. Table 1 shows the results for $P$ in units of $1'' \times 10^{-4}$. The mean errors derived from the solutions of the linear equations are added; they are independent quantities.

TABLE 1

| (1) | Mizusawa (2) | Tschardjui (3) | Carloforte (4) | Gaithersbg (5) | Cincinnati (6) | Ukiah (7) | (8) | (9) | (10) |
|---|---|---|---|---|---|---|---|---|---|
| 99—06 | + 77 ± 63 | + 274 ± 56 | + 215 ± 28 | + 213 ± 51 | + 252 ± 50 | + 76 ± 52 | + 170 ± 46 | + 197 ± 29 | ± 18 |
| 06—09 | + 76 ± 54 | + 34 ± 76 | + 66 ± 28 | + 36 ± 64 | + 175 ± 75 | + 108 ± 62 | + 74 ± 10 | + 75 ± 15 | ± 20 |
| 09—12 | + 161 ± 79 | + 92 ± 86 | + 45 ± 55 | − 26 ± 65 | + 139 ± 68 | − 81 ± 100 | + 55 ± 55 | + 58 ± 34 | ± 29 |
| 12—15 | − 73 ± 73 | + 83 ± 58 | + 88 ± 45 | + 15 ± 63 | + 22 ± 80 | + 143 ± 70 | + 66 ± 54 | + 51 ± 33 | ± 25 |
| 15—18 | − 36 ± 76 | | + 180 ± 45 | | | + 325 ± 108 | + 147 ± 78 | + 147 ± 78 | ± 36 |
| 18—22 | + 102 ± 87 | | + 170 ± 39 | | | + 93 ± 58 | + 140 ± 26 | + 140 ± 26 | ± 30 |
| 22—25 | + 271 ± 67 | | + 106 ± 48 | | | + 244 ± 71 | + 181 ± 54 | + 181 ± 54 | ± 34 |
| 25—28 | + 314 ± 117 | | + 86 ± 91 | | | + 169 ± 54 | + 170 ± 47 | + 170 ± 47 | ± 43 |
| 28—31 | + 161 ± 91 | | + 234 ± 58 | | | + 249 ± 61 | + 227 ± 22 | + 227 ± 22 | ± 38 |
| *w.* | + 101 ± 40<br>± 24 | + 141 ± 57<br>± 33 | + 138 ± 24<br>± 14 | + 79 ± 57<br>± 30 | + 174 ± 47<br>± 32 | + 144 ± 31<br>± 22 | | | |
| *u.* | + 117 ± 42 | + 121 ± 53 | + 132 ± 23 | + 60 ± 53 | + 147 ± 48 | + 147 ± 40 | | | |

Column 1 gives the limits of the intervals in years, omitting the number of centuries. Columns 2—7 give the values for $P$; its theoretical value, all secondary effects being excluded, is + 174.

Column 8 shows the means of the values of $P$ for Mizusawa, Carloforte and Ukiah only, with the mean errors as derived from the weighted differences between these three.

Column 9 contains the means of $P$ for all stations concerned, with the mean errors again derived from the weighted individual values. From the total weight of a horizontal row of $P$'s the mean errors of the means of $P$ would be as given in column 10. On the average these mean errors agree satisfactorily with those in column 9.

Computation of the weighted mean of a vertical column yields the results in the line designated by $w$, again with the mean errors derived from the differences between the various periods. Directly under these mean errors are given the mean errors expected from the total weight of a column. In this case the latter mean errors come out smaller than the former.

The bottom line $u$ contains the unweighted arithmetical mean with its mean error.

Three stations did not continue observations during the whole interval of time considered. Table 2 gives an answer to the question whether the values in column 9 should have been corrected first for a possible systematic difference between the two groups of stations.

Even if the differences in Table 2 were real, and half of their mean value, + 8, were added to the values in Table 1, where only Mizusawa, Carloforte and Ukiah

TABLE 2

| | Ti, Gg, Ci | Ma, Ce, Uh | diff. |
|---|---|---|---|
| 99–06 | + 245 | + 170 | + 75 |
| 06–09 | + 77 | + 74 | + 3 |
| 09–12 | + 62 | + 55 | + 7 |
| 12–15 | + 45 | + 66 | – 21 |

had contributed, the general impression that the values of column 9 show distinctly a variation in *P* would have been fortified instead of weakened.

The mean errors as given show without doubt that for every station additional observations do not yield an increased accuracy for the mean of *P*.

On the other hand, taking the mean of the values for different stations over one period yields on the average a more accurate value, nearly as accurate as could be expected from the total weight. It should be kept in mind that it is reasonable to expect a difference between the values from Tschardjui, Carloförte and Cincinnati on the one hand, compared with the other values, as it is most likely that the former stations do not need a correction for the direct influence of the water tides. I have divided the material according to these two groups, just to see whether, even with a greatly reduced weight, the general indications given by the two groups would differ or agree (see Table 3).

TABLE 3

| Ti, Ce, Ci | Ma, Gg, Uh |
|---|---|
| + 245 | + 129 |
| + 75 | + 74 |
| + 84 | + 24 |
| + 76 | + 30 |
| **Ce** | **Ma, Uh** |
| + 180 | + 84 |
| + 170 | + 96 |
| + 106 | + 258 |
| + 86 | + 195 |
| + 234 | + 222 |

I believe that, notwithstanding the decreased weights, the general character has remained the same.

**4.** I cannot offer a solution for the discrepancy found in section 3 between the different values of *P* I will only discuss shortly the possible causes which have occurred to me.

The term $- P \sin 2z \cos a$ is only the first term of a development. The following one contains *P* as well, but, as it is only of the order of about 3% of the first one, its neglect cannot explain column 9 of Table 1. The same holds for the influence of the variable factor remaining in *P*, apart from the smoothing process used.

There is a slight indication that, if the variation of *P* were periodic, its period would be of the order of some 15–20 years.

The declinations of the latitude stars are reduced to apparent place, in which process many small nutation terms are omitted. If there happened to exist among these a term with the argument $2g$ or, perhaps, $g$ (VON OPPOLZER's notation), with an additional argument like $\Omega$, so that it would be a modulated term with long period, it would have been possible to trace the period in *P* as a consequence of the neglect of this term. The investigation into this detail was kindly carried out by Miss H. A. KLUYVER, as, on account of the preparation of the Kenya expedition, I had no time to spend on the work any more. Miss KLUYVER investigated the total influence of some of VON OPPOLZER's neglected terms on the actual distribution of the latitude variations, and her conclusion was that no such variation as demanded by column 9, Table 1, could be found.

An error in the constant of nutation would affect other small terms in the same proportion as the main term. It is out of the question that the error in some term with argument $(2g \pm \Omega)$ would account for the effect which the terms investigated by Miss KLUYVER could not explain.

A work of this extent could only be finished by the aid of many. During the oppressing years 1940–1945 nearly everybody on the computing staff of the observatory worked on it; this number was reduced near the end of 1945. The work, as it stands, was finished at the end of 1946. I wish to express my sincere thanks to all who collaborated.

The publication of this work was delayed, among other reasons, because Professor OORT wished to discuss the results with Professor BROUWER during a visit to the U.S.A. As no solution was finally offered, the results are given as they stand.

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