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An attempt to determine the true distribution of the double stars,

by Dr. *E. A. Kreiken.*

In the preceding paper we have considered the distribution curve $\psi(L)$ of the double stars of one and the same distance and of the same apparent magnitude. The most frequent distance of the stars of magnitude m was [1])

$$\rho_1 = \frac{b-q_1}{2r} + \frac{r}{l+r} m \qquad (1)$$

It appeared that the observed numbers could be represented by the function

$$\left.\begin{array}{l} \psi(L) = 10^{u + vL + wL^2} \\ u = +1{\cdot}876 \\ v = +0{\cdot}286 \\ w = -0{\cdot}223 \end{array}\right\} \qquad (2)$$

In these formulae $\rho_1 = 5 \log r$ indicates the most frequent distance of the F type dwarfs with $m = 7{\cdot}5 \pm \frac{1}{2}$. At this distance $L = 0$ for $d = 1''$.

For any other distance we have

$$L = \log D + \log r = L_1 + 0{\cdot}2\,(\rho - \rho_1) \qquad (3)$$

The total number of double stars up to a certain limit of L will then be equal to

$$\log \Sigma D(m) = \int_{-\infty}^{L} 10^{u+vL+wL^2} dL = 10^{u'+v'L+w'L^2} \qquad (4)$$

$$(u' = +1{\cdot}776;\ v' = +0{\cdot}546;\ w' = -0{\cdot}146)$$

Let us now consider the total numbers of double stars of different spectral types and of different magnitudes up to the limit $d = 15''$. For the F type stars with $m = 7{\cdot}0$—$8{\cdot}0$ we have evidently

$$L = \log d = 1{\cdot}18.$$

For the F stars of other magnitudes we have

$$L = \log d + 0{\cdot}2\,(\rho - \rho_1) \qquad (5)$$

in which according to (1)

$$\rho_1 - \rho = \frac{r}{l+r}(7{\cdot}5 - m).$$

Now $\frac{r}{l+r} = 0{\cdot}97$ (see the preceding paper) and, therefore,

$$L_1 = \log d - 0{\cdot}2 \times 0{\cdot}97 (7{\cdot}5 - m) \qquad (6)$$

If stars of different spectral types are considered we have $L_1 = \log d + 0{\cdot}2\,(\rho - \rho_1)$ and, according to (1),

$$\rho_1 - \rho = \frac{q - q_1}{2r} + \frac{r}{l+r}(7{\cdot}5 - m). \qquad (7)$$

For q we have found (the preceding paper, p. 110)

$$q = 2r\,(M_0 - K)$$

and, therefore, we have

$$\rho_1 - \rho = +(K_1 - K) + \frac{r}{l+r}(7{\cdot}5 - m),$$

where $K_1 - K$ denotes the difference between the absolute magnitude of the top of the luminosity curve of the F stars and that of another spectral type.

Accordingly

$$L_1 = \log d + 0{\cdot}2\,(K - K_1) - 0{\cdot}2 \times 0{\cdot}97\,(7{\cdot}5 - m) \qquad (8)$$

TABLE 1. Absolute magnitudes of the maxima in the luminosity curves of the dwarfs of different spectral types.

Spectral type	M_0 M. W.	M_0 Br.	Adopted Mean	$K-K_1$	$0.2 \times (K-K_1)$
B	0.0	0·0	0·0	+3·5	+0·70
A	+1·6	+1·8	+1·7	+1·8	+0·36
F	+3·4	+3·6	+3·5	0·0	0·00
G	+5·3	+5·3	+5·3	−1·8	−0·36
K	+7·2	+7·8	+7·5	−4·0	−0·80
(1)	(2)	(3)	(4)	(5)	(6)

TABLE 2. Values of $L = \log d + 0{\cdot}2\,(K - K_1) - 0{\cdot}2 \times 0{\cdot}97\,(7{\cdot}5 - m)$.

Magn. \ Spectr.	B	A	F	G	K
3·0	0·91	0·57	0·21	9·85	9·41
4·0	1·10	0·76	0·40	0·04	9·60
5·0	1·30	0·96	0·60	0·24	9·80
6·0	1·49	1·15	0·79	0·43	9·99
7·0	1·69	1·35	0·99	0·63	0·19
8·0	1·88	1·54	1·18	0·82	0·18

TABLE 3. Numerical values of formula (9) for different magnitudes and different spectral types.

Magn. \ Spectr.	B	A	F	G	K
3·0	1·16	1·50	2·14	3·36	6·52
4·0	1·04	1·29	1·76	2·63	4·81
5·0	0·95	1·13	1·46	2·09	3·60
6·0	0·89	1·01	1·26	1·71	2·79
7·0	0·86	0·93	1·10	1·43	2·20
8·0	0·85	0·88	1·00	1·23	1·80

[1]) A. PANNEKOEK, *Publ. Astron. Inst. Amst.* Vol. I.

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From (7) and (8) we see that the absolute limit of L corresponding to $d = 15''$ varies with spectral type and magnitude. Evidently L increases for the earlier types and decreases for the smaller magnitudes. This is understood at once if we bear in mind that the early type stars are highly luminous and, therefore, very distant. For these stars the absolute distance D corresponding to the same apparent distance between the two components of a double star must accordingly be larger than for the later types. For the brighter stars the distances of the systems are smaller and thus the absolute value of D must decrease.

It is evident that reliable results as to the distribution of the double stars over different apparent magnitudes, spectral types and galactic latitudes will be obtained only if, for the different groups of stars, equal intervals of L are considered, or if the observed numbers are reduced to equal intervals.

This reduction to equal intervals is rather easy.

If all F stars up to the limit $d = 15''$ and with apparent magnitudes 7·0 — 8·0 are observed, the limits of L are $-\infty$ and 1·18.

The total number of double stars is then equal to

$$\Sigma D(m) = \int_{-\infty}^{+1\cdot 18} 10^{u + vL + wL^2}\, dL$$
$$= 10^{u' + 1\cdot 18\, v' + 1\cdot 18^2 w'}.$$

If stars of another spectral type and an arbitrary magnitude m are observed, the limits of L are $L = -\infty$ and

$$L = \log d + 0\cdot 2\,(K - K_1) - 0\cdot 2 \times 0\cdot 97\,(7\cdot 5 - m).$$

The total numbers up to $d = 15''$ are then represented by

$$\Sigma D(m) = \int_{-\infty}^{L} 10^{u + vL + wL^2}\, dL = 10^{u' + v' L + w' L^2}.$$

In order to reduce the latter numbers to the same interval of L the observed numbers must be multiplied by (9)

$$\frac{\int_{-\infty}^{+1\cdot 18} 10^{u + vL + wL^2}\, dL}{\int_{-\infty}^{L} 10^{u + vL + wL^2}\, dL} = \frac{10^{u' + 1\cdot 18\, v' + 1\cdot 18^2\, w'}}{10^{u' + v' L + w' L^2}} \qquad (9)$$

For the different spectral types and different magnitudes the numerical values of (9) are easily found if the fractions 0·2 $(K - K_1)$ are known. In Table 1 the numerical data concerning the maxima of the different absolute luminosity curves are given. The second column indicates the absolute magnitude in the Mount Wilson diagram by E. P. HUBBLE [1]), the third the values according to A. BRILL [2]), the next the adopted mean value. Column 5 indicates the values of $(K - K_1)$ and finally the last column contains the values of 0·2 $(K - K_1)$.

If these fractions are inserted in formula (8) we obtain the limits of L corresponding to different spectral types and different magnitudes. The results appear in Table 2. Next the total numbers of double stars up to the limit $L = \log d = \log 15\cdot 0$ are obtained from (9). The values of the different fractions (9) are given in Table 3. The fractions in Table 3 must be applied to the observed numbers in Table 4. In this way the true distribution of the double stars in Table 5 is obtained.

TABLE 4. Observed distribution of double stars with $d < 15''$ and $\Delta m < 2\cdot 1$.

(The numbers apply to 1000 square degrees).

Galactic latitude < 20°.

Spectr. / m	B	A	F	G	K
3·0	0·18	0·00	0·00	0·00	0·00
4·0	0·54	0·36	0·18	0·00	0·00
5·0	1·08	0·00	0·36	0·18	0·00
6·0	2·52	1·44	0·90	0·36	0·90
7·0	5·58	6·66	1·98	1·26	0·54
8·0	9·54	15·30	9·90	4·50	2·88

Galactic latitude 20°—40°.

Spectr. / m	B	A	F	G	K
3·0	0·00	0·00	0·00	0·00	0·00
4·0	0·00	0·00	0·00	0·00	0·24
5·0	0·49	0·00	0·00	0·00	0·24
6·0	0·49	0·97	0·73	0·49	0·00
7·0	1·94	3·16	1·70	1·70	0·73
8·0	2·43	8·26	8·02	3·40	2·43

Galactic latitude 40°—90°.

Spectr. / m	B	A	F	G	K
3·0	0·00	0·00	0·00	0·00	0·00
4·0	0·24	0·00	0·00	0·24	0·00
5·0	0·24	0·48	0·00	0·00	0·24
6·0	0·24	0·96	0·49	0·24	0·24
7·0	0·24	4·13	2·19	1·21	0·49
8·0	0·00	4·13	8·26	4·62	0·73

[1]) *Carn. Inst. Year Book*, No. 20, 270, 1921.
[2]) *Veröff. Berlin Babelsberg*, Band VII, Heft 1.

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TABLE 6. Galactic concentration.

Lat. \ Spectr.	Ordinary stars					Double stars, observed number					Double stars, reduced number				
	B	A	F	G	K	B	A	F	G	K	B	A	F	G	K
0—20	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00
20—40	0·52	0·40	1·00	0·99	0·63	0·28	0·52	0·79	0·89	0·85	0·27	0·52	0·77	0·90	0·90
60—90	0·07	0·27	0·84	0·89	0·63	0·05	0·41	0·82	1·00	0·40	0·05	0·41	0·80	1·04	0·44

Lat. \ Spectr.	Ratio $\frac{\text{red. concentr. double stars}}{\text{concentr. ordinary stars}}$					All spectral types together		
	B	A	F	G	K	Ord. stars	Double stars	Ratio
0—20	—	—	—	—	—	1·00	1·00	—
20—40	0·54	1·30	0·77	0·90	1·43	0·56	0·60	1·08
40—90	0·71	1·52	0·95	1·17	0·74	0·44	0·48	1·10

TABLE 5.

Distribution after reducing to equal intervals of *L*.

Galactic latitude $< 20°$

Magn. \ Spectr.	B	A	F	G	K
3·0	0·21	0·00	0·00	0·00	0·00
4·0	0·56	0·46	0·32	0·00	0·00
5·0	1·03	0·00	0·53	0·38	0·00
6·0	2·24	1·45	1·13	0·62	2·51
7·0	4·80	6·19	2·18	1·80	1·19
8·0	8·11	13·46	9·90	5·53	5·18
Σ	16·95	21·56	14·06	8·33	8·88

Galactic latitude 20°—40°

Magn. \ Spectr.	B	A	F	G	K
3·0	0·00	0·00	0·00	0·00	0·00
4·0	0·00	0·00	0·00	0·00	1·15
5·0	0·47	0·00	0·00	0·00	0·86
6·0	0·44	0·98	0·92	0·84	0·00
7·0	1·67	2·94	1·87	2·43	1·61
8·0	2·07	7·27	8·02	4·18	4·37
Σ	4·65	11·19	10·81	7·45	7·99

Galactic latitude 40°—90°

Magn. \ Spectr.	B	A	F	G	K
3·0	0·00	0·00	0·00	0·00	0·00
4·0	0·25	0·00	0·00	0·63	0·00
5·0	0·23	0·54	0·00	0·00	0·86
6·0	0·21	0·97	0·62	0·41	0·67
7·0	0·21	3·84	2·41	1·73	1·08
8·0	0·00	3·63	8·26	5·68	1·31
Σ	0·90	8·98	11·29	8·65	3·92

TABLE 7. Frequency of spectral types.

Spectr. \ Lat.	Ordinary stars			Observed numbers of double stars			Reduced numbers of double stars		
	0—20	20—40	40—90	0—20	20—40	40—90	0—20	20—40	40—90
B	·21	·09	·03	·29	·14	·03	·24	·11	·02
A	·32	·23	·18	·35	·33	·33	·31	·27	·27
F	·13	·23	·24	·20	·28	·37	·20	·26	·33
G	·11	·18	·21	·09	·15	·21	·12	·18	·26
K	·23	·27	·34	·07	·10	·06	·13	·18	·12

TABLE 8.

The numbers in Table 7 reduced to F = 10 and ratios.

Spectr. \ Lat.	Ordinary stars			Reduced double stars			Ratios		
	0—20	20—40	40—90	0—20	20—40	40—60	0—20	20—40	40—90
B	16	4	1.2	12	4.3	0.7	0·75	1·07	0·58
A	25	10	7.5	15.5	10	8	0·62	1·00	1·06
F	10	10	10	10	10	10	—	—	—
G	8.5	8	9	6	7	8	0·71	0·85	0·89
K	18	12	14	6.5	7	4	0·36	0·58	0·28

TABLE 9. Percentages of giants and dwarfs.

Spectr. \ Lat.	Giants			Dwarfs		
	0—20	20—40	40—90	0—20	20—40	40—90
B	25	0	42	75	100	58
A	38	0	0	62	100	100
F	0	0	0	100	100	100
G	29	15	11	71	85	89
K	64	42	72	36	58	28
All spectra	45	14	26	55	86	74

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The galactic concentration from the true numbers of double stars.

For the different spectral types the surface densities in the various galactic latitudes were expressed in the density of the region 0°—20° as unit. From the reduced numbers we see how the surface density varies with increasing latitude. A survey of the results is given in Table 6. Only the total numbers of stars up to $m = 8{\cdot}0$ are considered. In the table are shown: 1° the galactic condensation of the stars at large with $m \leq 8{\cdot}0$; 2° the condensation from the observed numbers of double stars; 3° the same from the reduced numbers and 4° the ratios of the relative surface density of double stars to the relative surface density of the stars at large.

The observed numbers of double stars are rather small and owing to this there are large fluctuations of the individual values. In the last part of the table I have combined the stars of all spectral types into one set and from these numbers we see that the galactic concentration of the double stars as a whole is slightly smaller than that of the stars at large. If the individual values are considered it appears that a very high concentration is obtained for the B type double stars. On the whole the differences between the concentration obtained from the observed numbers and the reduced ones are small.

Frequency of different spectral types.

In Table 7 the frequencies of the different spectral types among the ordinary stars are compared to the observed and reduced numbers of double stars. It now appears that the distribution of the double stars over different spectral types is greatly influenced by the dispersion curve $\psi(L)$.

For the early type double stars the true frequency is smaller, than the one obtained from the observed numbers. For the later types it is larger. In order to obtain a better insight into the distribution of the double stars over different spectral types the numbers in Table 7 were reduced in such a way that the number of F type stars is always equal to 10. For instance, the numbers in the second column were multiplied by $\frac{10}{13}$, and so on. From these reduced numbers we obtain the ratios

$$\frac{\text{frequency of spectral types among the double stars}}{\text{frequency of spectral types among the stars at large}}.$$

The reduced numbers and the ratios appear in Table 7. It has been pointed out several times that the F type stars among the stars in general are largely dwarfs. On the other hand it appeared that the double stars of other spectral types are also largely dwarfs. [1]) From the ratios in Table 7 we can, therefore, obtain the percentages of giants and dwarfs among the stars at large (Table 9).

From Table 9 we see that, as a rule, the percentage of dwarfs increases with increasing latitude. The individual values given in this table will no doubt be greatly improved if additional material can be taken into account.

The percentage of the „dwarf stars" among the A and B types is rather high. It is probable that among the A and B types the double stars are not entirely confined to the main sequence. According to JEANS [2]) and TEN BRUGGENCATE [3]) a double star may develop from a variable in the giant series. If, therefore, the star develops in the usual way, two white giants will issue. From the same consideration it is evident that among the giants of later types no double stars should occur. For these reasons I think that the separation of the observed numbers into giants and dwarfs is real for the later types only.

Among the G stars giants are very infrequent, while, on the other hand, the observed K stars are largely giants.

The last line of Table 9 shows the percentages of giants and dwarfs among the stars of all spectral types together for which $m < 8{\cdot}0$.

From a statistical point of view the most important result is that statistically we are able to separate the giant and dwarf K stars when a sufficient number of double stars is available. For practical calculations it will be advisable to combine all galactic latitudes into one set.

Finally it should be remarked that the results obtained for the B stars are rather uncertain. For the various subdivisions of the B type stars the top of the luminosity curve shifts rapidly toward the brighter magnitudes. A large fraction of the B type stars must, therefore, be brighter than the limit assumed in the present paper. However, the scantiness of the material did not allow to take the subclasses into account.

The apparent distribution of double stars.

Usually the observers of double stars take account only of those pairs for which d is smaller than a given limit. In the following I have tried to determine the fractions of the total numbers of double stars of a given magnitude and spectral type for which d lies between the limits 0″—5″; 0″—15″; 0″—30″ and 0″—60″. This was done for each separate spectral type and

1) *B. A. N.* No. 173.
2) *M. N. R. A. S.*, **85**, 797, 1925.
3) *Naturw.*, **14**, 905, 1926.

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for each apparent magnitude from 3·0 up to 8·0. Evidently these fractions are equal to (10)

$$\frac{\int_{-\infty}^{L} 10^{u+vL+wL^2}\,dL}{\int_{-\infty}^{+\infty} 10^{u+vL+wL^2}\,dL} \qquad (10)$$

The upper limit of the first integral may be found from (8)

$$L_1 = \log d + 0{\cdot}2\,(K - K_1) - 0{\cdot}2 \times 0{\cdot}97\,(7{\cdot}5 - m).$$

But the simplest way to determine the limits L_1 is by deriving them from the values of L given in Table **2**. To the values of L given there we simply add the constants $-0{\cdot}48$; $+0{\cdot}30$ and $+0{\cdot}60$. We then obtain from (10) the fractions in Table 10.

The actual numbers of double stars per thousand square degrees between the limits mentioned before are obtained by using the observed numbers up to $d = 15''$. The fractions in Table 10 are multiplied with the ratios

$$\frac{\text{observed numbers} < 15''}{\text{fractions} < 15''}.$$

I have not given the separate results. Future observations will probably largely influence the numbers derived in this way.

But it appears from the fractions in Table 10 that, if the observations are limited to a definite value of d, many double stars will escape observation. For instance, in Table 11 I have given the percentages of double stars with an angular distance $d > 15''$. Especially for the later types these percentages are very high. Still it is very important that they should be observed. A more accurate knowledge of the distribution of the wider pairs will enable us to derive far more accurate values for the coefficients of the distribution curve $\psi(L)$. There is no doubt that if the wider pairs are observed several optical pairs will also occur. With the small interval $\Delta m < 2{\cdot}1$ and the brighter magnitudes used here their number cannot be very large. (Cf. *B. A. N.* 154). In any case corrections could be applied to take account of their occurrence.

TABLE 10. Numerical values of (10).

Spectr.	B				A				F			
m \ d	0—5″	0—15″	0—30″	0—60″	0—5″	0—15″	0—30″	0—60″	0—5″	0—15″	0—30″	0—60″
3·0	·445	·575	·650	·720	·355	·485	·560	·640	·270	·390	·465	·545
4·0	·490	·620	·695	·760	·405	·535	·610	·685	·315	·440	·515	·595
5·0	·545	·670	·740	·800	·455	·585	·660	·730	·365	·490	·565	·645
6·0	·595	·715	·775	·830	·505	·635	·705	·770	·410	·540	·615	·690
7·0	·650	·760	·815	·865	·560	·685	·750	·810	·465	·595	·670	·740
8·0	·695	·795	·850	·890	·610	·725	·790	·840	·515	·640	·715	·775

TABLE 10 (*continued*).

Spectr.	G				K			
m \ d	0—5″	0—15″	0—30″	0—60″	0—5″	0—15″	0—30″	0—60″
3·0	·200	·300	·370	·450	·125	·205	·265	·335
4·0	·235	·345	·420	·500	·155	·245	·310	·385
5·0	·275	·395	·470	·550	·190	·285	·360	·440
6·0	·320	·445	·525	·600	·225	·330	·405	·490
7·0	·370	·500	·580	·655	·265	·385	·460	·540
8·0	·420	·545	·630	·700	·300	·435	·510	·590

TABLE 11. Percentages of the double stars of different magnitudes and spectral types, with apparent angular distances $d > 15''$.

Spectr. \ m	B	A	F	G	K
3·0	42	51	61	70	80
4·0	38	46	56	66	75
5·0	33	42	51	60	71
6·0	29	37	46	55	67
7·0	24	31	40	50	61
8·0	20	27	36	45	56

Extension of the results to fainter magnitudes.

Approximate values for the coefficients of the distribution curve being known we may extend our results to the fainter magnitudes.

In *B. A. N.* 154 (Table 5) extensive tables have been given indicating the apparent distribution of the double stars with $\Delta m < 2{\cdot}4$ and $d < 15''$ with respect to apparent magnitude and galactic latitude. The tables are complete down to $m = 12$.

The distribution of these numbers does not indicate the true distribution of the double stars, as the absolute value of D corresponding to a given limit of angular distance d varies with magnitude and latitude. If we want to find the true distribution equal intervals of D must be considered for all groups of stars.

In the following all observed numbers have been reduced to the value $L = \log d = \log 15$ corresponding to the F type stars with $m = 7{\cdot}0$—$8{\cdot}0$.

For each magnitude and each latitude we therefore determine the fractions of the total numbers of

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double stars with apparent angular distance $d < 15''$. For each group of stars we then want the absolute value of L corresponding to $d = 15''$.

Now we have

$$\rho_m = \frac{b-q}{2r} + \frac{r}{l+r}\, m \qquad (11)$$

For b, q, r and l we must insert the values as obtained from all spectral types together. We use the following data, derived by KAPTEYN and VAN RHIJN [1]):

$$\left.\begin{array}{lllll} \text{latitude } 0-20 & l = 0{\cdot}0237 & r = 0{\cdot}0345 & b = 0{\cdot}725 & q = 0{\cdot}186 \\ \text{latitude } 20-40 & l = 0{\cdot}0262 & r = 0{\cdot}0345 & b = 0{\cdot}690 & q = 0{\cdot}186 \\ \text{latitude } 40-60 & l = 0{\cdot}0534 & r = 0{\cdot}0345 & b = 0{\cdot}733 & q = 0{\cdot}186 \end{array}\right\} \quad (12)$$

From (11) and (12) we find the most frequent distance ρ_m of the stars with apparent magnitude m.

TABLE 12. Values of ρ_m, 0.2 (ρ_m — 9.11) and (L_1 — 0.65) H.

Magn. \ Lat.	ρ_m 0—20	ρ_m 20—40	ρ_m 40—60	0·2 (ρ_m — 9·11) 0—20	0·2 (ρ_m — 9·11) 20—40	0·2 (ρ_m — 9·11) 40—60	(L_1 — 0·65) H 0—20	(L_1 — 0·65) H 20—40	(L_1 — 0·65) H 40—60
7·5	12·24	11·58	10·85	0·63	0·49	0·35	0·52	0·45	0·39
8·5	12·83	12·15	11·24	0·74	0·61	0·43	0·56	0·51	0·43
9·5	13·42	12·72	11·63	0·86	0·72	0·50	0·62	0·56	0·46
10·5	14·01	13·29	12·02	0·98	0·84	0·58	0·67	0·61	0·49
11·5	14·60	13·86	12·41	1·10	0·95	0·66	0·72	0·66	0·53

TABLE 13. Fractions of the total numbers of double stars with $d < 15''$ and values of ratios $\frac{\text{fraction}}{0.640}$.

Magn. \ Lat.	Fractions 0—20	Fractions 20—40	Fractions 40—60	Ratios 0—20	Ratios 20—40	Ratios 40—60
7·5	0·770	0·737	0·709	0·831	0·868	0·903
8·5	0·786	0·764	0·728	0·814	0·838	0·879
9·5	0·809	0·786	0·742	0·791	0·814	0·863
10·5	0·828	0·806	0·756	0·773	0·794	0·847
11·5	0·845	0·824	0·773	0·757	0·777	0·828

TABLE 14. Observed numbers of double stars (*B. A. N.* 154, Table 5) and reduced numbers.

Magn. \ Lat.	log Σ $D(m)$ observed 0—20	observed 20—40	observed 40—60	log Σ $D(m)$ reduced 0—20	reduced 20—40	reduced 40—60
7·0	9·83	9·65	—	9·75	9·59	—
8·0	0·39	0·30	9·56	0.31	0·24	9·51
9·0	0·96	0·75	0.06	0.88	0·68	0·00
10·0	1·44	1·15	0·86	1·34	1·07	0·80
11·0	1·84	1·59	1·47	1·73	1·50	1·40
12·0	2·34	1·90	1·91	2·22	1·80	1·84

TABLE 15. Galactic concentration of double stars and stars in general. Differences gal. conc. double stars minus gal. conc. stars in general.

Magn. \ Lat.	Gal. conc. double stars 20—40	Gal. conc. double stars 40—60	Gal. conc. stars in general 20—40	Gal. conc. stars in general 40—60	Differences 20—40	Differences 40—60
7·0	·16	—	·23	·37	— ·07	—
8·0	·07	·80	·23	·38	— ·16	+ ·42
9·0	·20	·88	·23	·38	— ·03	+ ·50
10·0	·27	·54	·24	·40	+ ·03	+ ·14
11·0	·23	·33	·25	·42	— ·02	— ·09
12·0	·42	·38	·27	·46	+ ·15	— ·08

The spread of the individual values of ρ about ρ_m is indicated by the Gaussian error curve

$$10^{-(l+r)(\rho-\rho_m)^2}.$$

For L we have $L = \log D = \text{const.} + \log d = 0{\cdot}2\,\rho$.

Thus, if we consider a number of stars with the same value of L_0 their apparent distribution will be (13)

$$\left.\begin{array}{l} \text{Freq.} = \text{const. } 10^{-h_2^2 (L - L_0)^2} \\ h_2 = 5\sqrt{l+r} \end{array}\right\} \qquad (13)$$

For the true dispersion curve $\psi(L)$ we write

$$\text{Freq.} = \text{const. } 10^{-h_1^2 (L - L_0)^2} \qquad (14)$$

The apparent dispersion curve will then be of the form:

$$\left.\begin{array}{l} \text{Freq.} = \text{const. } 10^{-H^2 (L - L_0)^2} \\ H = \dfrac{h_1 h_2}{\sqrt{h_1^2 + h_2^2}} \end{array}\right\} \qquad (15)$$

Inserting the values $h_1 = 0{\cdot}472$ and $h_2 = 5\sqrt{l+r}$ in (15) we obtain in the three galactic belts 0° — 20°; 20° — 40° and 40° — 60°:

$$H = 0{\cdot}93 \times 0{\cdot}472,$$
$$H = 0{\cdot}94 \times 0{\cdot}472,$$
$$H = 0{\cdot}95 \times 0{\cdot}472.$$

We adopt $H = 0{\cdot}94 \times 0{\cdot}472 = 0{\cdot}444$.

The absolute value of L corresponding to $d = 15''$ is

$$L = \log d + 0{\cdot}2\,(\rho - \rho_1) \qquad (16)$$

where ρ_1 is the most frequent distance of the F stars with $m = 7{\cdot}0 - 8{\cdot}0$.

In a previous paper we obtained $\rho_1 = 9{\cdot}11$.

The fractions of the total numbers of double stars with $d = 15''$ can now be found from (17)

[1]) *Astroph. Journ.* **52**, 23, 1920.

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TABLE 16. Increase of the numbers of double stars and stars in general with magnitude. Differences log number of double stars minus log number of stars in general.

Lat. / Magn.	log $N(m+1)$ — log Nm for double stars			log $N(m+1)$ — log Nm for stars in general			Differences		
	0—20	20—40	40—60	0—20	20—40	40—60	0—20	20—40	40—60
7·0									
	·56	·65	—	·44	·44	·44	+ ·12	+ ·21	—
8·0									
	·57	·44	·49	·44	·44	·43	+ ·13	+ ·00	+ ·06
9·0									
	·46	·39	·80	·43	·42	·41	+ ·03	— ·03	+ ·39
10·0									
	·39	·43	·60	·42	·41	·40	— ·02	+ ·03	+ ·20
11·0									
	·49	·30	·44	·42	·40	·38	+ ·07	— ·10	+ ·06
12·0									
Σ							+ ·33	+ ·11	+ ·71

$$\frac{\int_{-\infty}^{H(L-0{\cdot}65)} e^{-H^2(L-0{\cdot}65)^2}\, d\,\{H(L-0{\cdot}65)\}}{\int_{-\infty}^{+\infty} e^{-H^2(L-0{\cdot}65)^2}\, d\,\{H(L-0{\cdot}65)\}} \qquad (17)$$

In Table 12 the upper limits of the first integral have been derived for the different apparent magnitudes and the different latitudes. This table contains the values ρ_m according to (11); 0·2 $(\rho_m - 9{\cdot}11)$; and from formula (16) $H(L_m - 0{\cdot}65)$.

In Table 13 the numerical values of (17) are inserted. From Table 10 it appears that for the F stars with m between 7·0 and 8·0 the observed fraction is 0·640. So in order to make our results directly comparable with the previous ones the observed numbers have been multiplied by

$$\frac{0{\cdot}640}{\text{computed fraction}} \qquad (18)$$

In Table 14 the observed numbers of double stars with $d < 15''$ are given. These values were reprinted from Table 5 in *B. A. N.* 154. In the second part of this same table the reduced numbers are given.

In Table 15 the galactic concentration from the reduced numbers of double stars is compared with that of the stars at large.

The numbers inserted in our table represent the differences between the logarithm of the numbers of stars up to a given magnitude in a certain latitude and the corresponding value in the latitudes 0°—20°. The numbers for the stars in general were obtained from the tables by SEARES, VAN RHIJN, JOYNER and RICHMOND [1]).

On the whole the galactic concentration of the double stars seems to be somewhat smaller than that of the stars in general. The evidence however is not conclusive.

The increase of the numbers of double stars and of the stars in general with magnitude appears in Table 16. In the last part of the table we have compared the values of this increase for the two groups of stars.

It appears that the numbers of double stars increase much more rapidly with magnitude than the numbers of the stars in general.

This was anticipated, as it has been proven previously that the double stars are largely dwarfs.

Summary.

1. The true distribution of the double stars with respect to apparent magnitude, angular distance d, spectral type and galactic latitude has been derived.

2. The distribution of the double stars over the different spectral types mainly indicates the distribution of the dwarf stars over the different spectra. This is probably best fulfilled for the later types. The very high percentages of B and A "dwarfs" indicate that the general distribution of the B and A type double stars over absolute magnitude is the same as for the stars in general.

3. The galactic condensation of the double stars is, on the whole, slightly smaller than that of the stars in general.

4. The increase of the number of double stars with magnitude is larger than the corresponding increase for the stars in general.

5. For the stars brighter than 8·0 tables have been given which indicate the fractions of the total numbers of double stars of each spectral type which have an angular separation $d < 5''$; $d < 15''$; $d < 30''$ and $d < 60''$.

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[1]) *Astroph. Journ.* **62**.

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