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The attractive force of the galactic system as determined from the distribution of RR Lyrae variables, by *J. H. Oort* and *A. J. J. van Woerkom*.

Summary.

Data by SHAPLEY on the distribution of RR Lyrae variables in high latitudes combined with recent information on the velocity distribution from JOY's measures have been used to estimate the acceleration due to the galactic system in a direction perpendicular to the galactic plane at distances between 2000 and 10000 parsecs from this plane. The acceleration appears to vary little over this range, and is on the average $8{\cdot}4 \times 10^{-9}$ cm/sec^2.

This knowledge permits an independent estimate of the mass of the galaxy. If the distance of the sun to the centre is assumed to be 8000 parsecs the total mass of the galactic system is found to be approximately $1{\cdot}0 \times 10^{11}$ solar masses, in good agreement with the value derived from the rotation of the galaxy. Inversely, the velocity of rotation corresponding with the acceleration found in the present article would come out as 240 km/sec.

In a recent publication by SHAPLEY [1]), extensive data are given concerning the decrease of density of RR Lyrae variables in a direction perpendicular to the galactic plane. We have compared this distribution with the distribution of radial velocities as determined by JOY [2]), in order to investigate whether these new data are compatible with the mass and dimensions of the galactic system as derived from the rotation of the galaxy. In the case of a steady state and a Gaussian distribution of velocities in the direction perpendicular to the galactic plane the following relation should exist between the acceleration K_z and the space density $\Delta(z)$ of the stars considered [3])

$$K_z = 7{\cdot}48 \times 10^{-9}\ \overline{Z^2}\ \partial \log \Delta(z)/\partial z,$$

where K_z is expressed in cm/sec^2, z in parsecs and $\overline{Z^2}$ is the average square velocity in (km/sec)^2. The value of $\partial \log \Delta(z)/\partial z$ may be found from SHAPLEY's article. We adopt $-{\cdot}00022$, as determined from the 10 fields between $-40°$ and $-90°$ galactic latitude; this value would appear to hold roughly for the region between 1000 and 10000 parsecs which is covered by these observations.

The distribution of the peculiar velocities Z is shown in Table 1.

TABLE 1.

Interval of velocity (km/sec)	O	C	O − C
0— 30	7	9·5	− 2·5
30— 60	10	7·8	2·2
60— 90	5	5·6	− ·6
90—120	6	3·3	2·7
120—150	0	1·7	− 1·7
150—180	1	·7	·3
$>$180	0	·3	− ·3

It has been obtained from the radial velocities of those 29 stars in JOY's list which have latitudes in excess of 50°; the velocities have first been corrected for the effect of the sun's motion with respect to the system of RR Lyrae variables, which was assumed to be 130 km/sec towards $\alpha = 297°$, $\delta = 52°$ [1]). The third column of Table 1 gives the numbers corresponding to a normal or Gaussian distribution with an average velocity of 57 km/sec $\pm$ 5 km/sec (p.e.). It will be seen that the observed numbers are well represented by the Gaussian distribution. Accordingly, we have $\overline{Z^2} = 5112$ (km/sec)^2, and K_z is found to be $8{\cdot}4 \times 10^{-9}$ cm/sec^2, with a probable error of about 20%. We now wish to compare this value with the value inferred from the rotation of

[1]) *Proc. Nat. Ac. Washington*, **25**, 423, 1939.
[2]) *P. A. S. P.* **50**, 302, 1938.
[3]) Cf. *B.A.N.* No. 238.

[1]) *B.A.N.* No. 318, p. 338, 1939.

the galaxy and the known density distribution in the neighbourhood of the sun.

In order to have a definite starting point, computations were begun with the aid of the following schematic model of the galactic system. The distance of the sun to the centre of the system is adopted at 8000 parsecs [1]). The mass of the galaxy is divided into a "central mass" and a number of homogeneous spheroids, the surfaces of which are equidensity surfaces. The densities are chosen in such a way that for points situated on a line perpendicular to the galactic plane and passing through the sun they reproduce, or extrapolate, the density distribution in the *z*-direction found for stars with absolute magnitude fainter than + 4·5, as shown in Figure 5, *B.A.N.* No. 290. The semi-axes assumed for these equidensity surfaces are shown in Table 2.

TABLE 2.

a (parsecs)	c (parsecs)	c/a	relative star-density in shell	mass (in 10^8⊙)
10000	500	·050	·76	96
10200	660	·064	·36	27
10400	1000	·100	·20	26
10600	1600	·150	·10	23
11200	1900	·170	·046	17
12250	2450	·200	·017	5
14050	3150	·225	·0114	8
16800	4200	·250	·0061	9
20500	5650	·275	·0031	8
26400	7950	·300	·00150	10
32500	10300	·320	·00075	19

The axes of the 4th and 5th spheroids are such that they reproduce approximately the shapes of the outer equidensity surfaces as derived in *B.A.N.* No. 308 (cf. the inclined straight lines marked ·03 and ·05 in Figure 8, p. 261). The major axis of the first spheroid was rather arbitrarily assumed to be 10000 parsecs, and a regular transition was made for the spheroids in between. For the still larger spheroids the axial ratio was assumed to increase gradually to a value of nearly $^1/_3$ for the outermost surface considered. The latter was taken as the boundary, the density being supposed to be zero outside.

The mean relative densities in the shells, as inferred from Figure 5, *B.A.N.* No. 290, are shown in the fourth column of the table. In accordance with the results found in *B.A.N.* No. 238, p. 284, the absolute value of the density near the sun was assumed to be ·087 solar masses per cubic parsec. As the interstellar dust is strongly concentrated near the galactic plane the unit of density for the shells outside the first spheroid was taken $^2/_3$ of this value. The density of one of the homogeneous spheroids in this schematic galaxy is, of course, the absolute density in the shell corresponding to this spheroid diminished by the density in the next larger shell. The masses of these homogeneous ellipsoids are in the last column of Table 2. It is evident that there is a large margin of uncertainty as to the exact shapes of the spheroids used, but this is not of much consequence, as the whole attraction by the larger spheroids is only a minor fraction of the total attraction, which comes mainly from the "central mass". For this "central mass" the two following rather extreme assumptions were used alternatively:

a) a spherical mass,

b) a central mass consisting of a homogeneous spheroid with semi-axes of 7000 and 700 parsecs. In each of the cases *a*) and *b*) the calculations were carried out for two different values of the central mass, corresponding to a circular velocity near the sun of 240 km/sec (I) and 270 km/sec (II), respectively. The central acceleration K_ϖ in the neighbourhood of the sun is partly caused by the "outer" spheroids of Table 2, and partly by the central mass. The "central masses" needed to supply the additional force needed beyond that exerted by the "outer ellipsoids" are as follows

I*a*) $M = \cdot 863 \times 10^{11}$ solar masses,
II*a*) $M = 1{\cdot}145 \times 10^{11}$,, ,,
I*b*) $M = \cdot 605 \times 10^{11}$,, ,,
II*b*) $M = \cdot 802 \times 10^{11}$,, ,,

The accelerations in a direction perpendicular to the galactic plane due to these central masses as well as to the outer ellipsoids have been computed for various points on a line through the sun perpendicular to the galactic plane.

The results are shown in Table 3, K_{z1} referring to the attraction by the central mass, K_{z2} to that by the outer spheroids. The total accelerations K_z are given in columns 5, 8, 11 and 14. All accelerations are expressed in units of 10^{-9} cm/sec². In the accompanying figure the accelerations for the case $\Theta_c = 240$ km/sec are represented graphically. The residuals in the columns O—C are the differences between the "observed" value K_z in the second column (i.e. the value derived from the distribution of the RR Lyrae variables) and the computed quantities K_z.

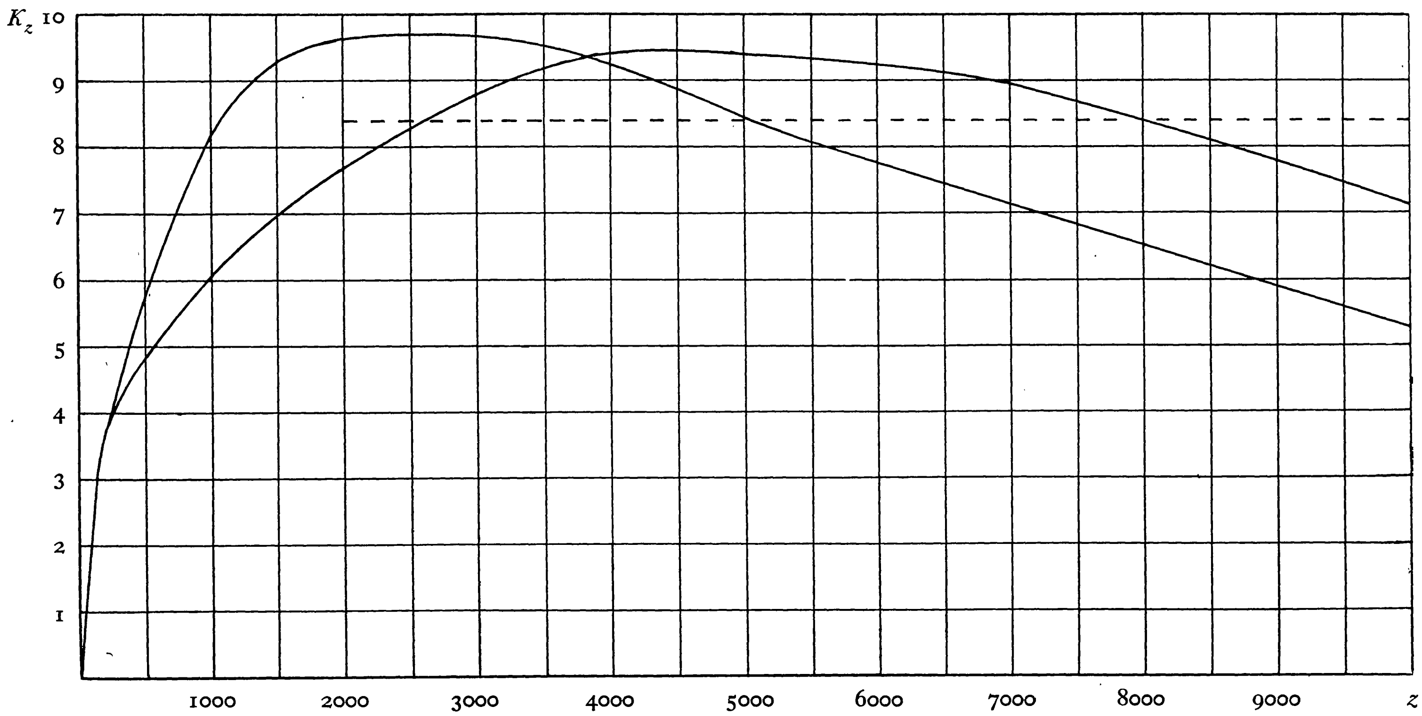
If we take the average of the solutions with spherical and spheroidal central masses a reasonable agreement with the "observed" value of K_z appears to be obtained for $\Theta_c' = 240$ km/sec; the probable error of this result is about ± 25 km/sec.

[1]) J. H. OORT, *M.N.* **99**, 374, 1939.

TABLE 3.

Acceleration perpendicular to the galactic plane (unit 10^{-9} cm/sec^2)

z in kps	$K_{z_{obs}}$	K_{z_2}	$\Theta_c = 240$ km/sec						$\Theta_c = 270$ km/sec					
			Spherical centr. mass			Spheroidal centr. mass			Spherical centr. mass			Spheroidal centr. mass		
			K_{z_1}	K_z	O—C	K_{z_1}	K_z	O—C	K_{z_1}	K_z	O—C	K_{z_1}	K_z	O—C
1	8·4	3·75	2·31	6·06	+ 2·3	4·42	8·17	+ ·2	3·07	6·82	+ 1·6	5·86	9·61	− 1·2
2	8·4	3·37	4·32	7·69	+ ·7	6·27	9·64	− 1·2	5·74	9·11	− ·7	8·31	11·68	− 3·3
3	8·4	2·96	5·83	8·79	− ·4	6·71	9·67	− 1·3	7·74	10·70	− 2·3	8·89	11·85	− 3·4
4	8·4	2·62	6·78	9·40	− 1·0	6·62	9·24	− ·8	9·00	11·62	− 3·2	8·77	11·39	− 3·0
5	8·4	2·18	7·22	9·40	− 1·0	6·23	8·41	·0	9·59	11·77	− 3·4	8·26	10·44	− 2·0
6	8·4	1·98	7·28	9·26	− ·9	5·76	7·74	+ ·7	9·66	11·64	− 3·2	7·63	9·61	− 1·2
7	8·4	1·89	7·07	8·96	− ·6	5·29	7·18	+ 1·2	9·38	11·27	− 2·9	7·01	8·90	− ·5
8	8·4	1·71	6·70	8·41	·0	4·77	6·48	+ 1·9	8·89	10·60	− 2·2	6·33	8·04	+ ·4
9	8·4	1·54	6·25	7·79	+ ·6	4·35	5·89	+ 2·5	8·30	9·84	− 1·4	5·77	7·31	+ 1·1
10	8·4	1·36	5·77	7·13	+ 1·3	3·93	5·29	+ 3·1	7·67	9·03	− ·6	5·21	6·57	+ 1·8



Acceleration perpendicular to the galactic plane.

Abscissae are distances from the galactic plane in parsecs, ordinates accelerations K_z along a line passing through the sun perpendicular to the galactic plane. The unit of K_z is 10^{-9} cm/sec^2. The distance from the sun to the centre has been taken as 8000 parsecs. The curve rising most slowly corresponds to case I*a*), in which the "central mass" was assumed to be either small or spherical, while the curve rising steeply for small values of z corresponds to case I*b*) with a strongly flattened and extended "central mass" (semi-axes 7000 and 700 parsecs). The dashed line gives the observed values.

The agreement between the rotational velocity of 240 km/sec derived from this entirely different set of data with the values of between 200 and 300 km/sec found from more direct data concerning the rotation of the galaxy is extremely satisfactory. It should be kept in mind, however, that the value determined in this article may be slightly too small for the following reason: The distribution of peculiar velocities has been assumed to be of the normal Gaussian type; to some extent this is supported by the observations, but the number of velocities is too small to prove that there is no excess of large velocities, such as occurs so often; even a relatively slight excess might cause the real average velocity at $z = 6$ kps to be sensibly larger than the value found from the near-by variables of which the radial velocities have been measured; in such case the values of $K_{z\text{obs}}$ would evidently have been found too small.

It should also be noted that the present result is, of course, dependent on the value assumed for the distance of the centre. It only gives a new numerical relation between the rotational velocity and this distance. We might better put the result this way: The observed velocity- and density-distribution of RR Lyrae variables fit in very well with the accepted values for the rotation and distance to the centre. Had we assumed ϖ = 10000 ps instead of 8000, the resulting value for Θ_c would have been 276 km/sec; with ϖ = 7000 ps on the other hand, Θ_c would have become 225 km/sec.

It may be observed that, on the whole, the run of the computed values of K_z conforms very well with the observed constancy of the gradient from $z = 2$ to $z = 10$ kps. There is, however, a slight tendency in the residuals to increase towards the larger values of z, which indicates that the relation between $\log \Delta$ and z is probably not an entirely straight line, but slightly curved, with the convex side down; the data on the distribution of the variables are not yet sufficient to discover such a slight curvature from observation.

A somewhat uncertain element in the above determination of K_z is the influence of an eventual incompleteness in the numbers of cluster-type variables, or rather, of the change of this incompleteness with apparent magnitude; however, we do not believe that the influence is likely to have been serious. That an important percentage of variables may have remained undiscovered is illustrated in a careful discussion of incompleteness of this type of variables by VAN GENT in *B.A.N.* No. 243. We may refer to Table 8 of his article, which shows that after comparison of 30 pairs of plates in the blink microscope, about 40% of the variables between $13^{m}\cdot85$ and $15^{m}\cdot65$ have remained undiscovered. The table indicates also that within this interval (which would correspond with the interval from z = 5400 to z = 12000 ps in the present study) the incompleteness is not much dependent upon the magnitude. This shows that, if the incompleteness in the Harvard survey is analogous to that of VAN GENT's investigation, its influence upon the present determination of K_z will not have been important.

As SHAPLEY gives no details, it is not possible to get any precise information concerning the incompleteness of the discoveries. Since the Harvard material was obtained with an instrument which is similar to that used by VAN GENT and reaches down to about the same limit [1]), a rough comparison is possible. The average number of pairs of plates compared in the Harvard survey appears to be about 40 [2]). As a rough estimate we might assume that on the average perhaps 70% of the variables brighter than $15^{m}\cdot0$ have been found.

It may be of interest to compute the number of RR Lyrae variables we should expect within 1000, or, respectively, 500 ps from the sun. The density near $z = 0$ is related to the density at an arbitrary height by

$$\frac{\Delta(0)}{\Delta(z)} = e^{\frac{1}{Z^2}\int_0^z K_z\,dz}, \text{ if } Z \text{ is expressed in cm/sec.}$$

Using the densities at 6000 and 8000 parsecs as given by SHAPLEY's formula (2), and using the mean of the accelerations found in the cases I*a*) and I*b*) we find, respectively, 17·5 and 16·6 stars per cubic kiloparsec near $z = 0$, in good agreement with SHAPLEY's density of 19. If we assume, tentatively, that the correction for incompleteness as found by VAN GENT would hold also for the Harvard material the true density near $z = 0$ would become 28. The total number of RR Lyrae variables within 1000 ps would then be 110, that within 500 ps, 15. The *Katalog und Eph. Veränd. Sterne* for 1940 gives 101 variables of the RR Lyrae type with median visual magnitude brighter than or equal to 10·0 (corresponding to the distance limit of 1000 ps); of this number 62 are in the northern hemisphere, and 39 in the southern. Eight of these variables may be supposed to be nearer than 500 ps. The agreement of these numbers with those predicted from the faint variables is satisfactory.

In conclusion we give the values for the velocity of escape and the total mass of the galactic system as derived from the models considered above. They are shown in Table 4. The velocities are expressed in km/sec, the masses in 10^{11} times the solar mass as unit.

TABLE 4.

Case	velocity of escape	total mass of the model
Θ_c = 240 km/sec *a*)	350	1·111
" *b*)	320	·853
Θ_c = 270 km/sec *a*)	391	1·393
" *b*)	356	1·050

1) SHAPLEY, l.c. p. 424.
2) SHAPLEY and Miss HUGHES, *Harvard Annals*, **90**, 164, 1934.