The total weight of the velocity-curve, derived by combining the series of MOORE and JACOBSEN is $72{\cdot}8$ km^{-2} sec^{2}.

The velocity of the centre of gravity is on MOORE's system: $-16{\cdot}8$ km/sec. A smooth curve was drawn through the points in Figure 1 of the next paper. The 25 ordinates read from this curve at the equidistant phases: $^{P}{\cdot}00$, $^{P}{\cdot}04$, $^{P}{\cdot}08$, etc. have been subjected to a Fourier analysis of which the result is given in Table 4.

The mean errors of the coefficients have been calculated from the total weight of the curve. The Fourier development has been helpful for the derivation of the displacement of the star's surface as a function of the phase in the next article.

The observations of brightness, colour and radial velocity of δ Cephei and the pulsation hypothesis, by *A. J. Wesselink.*

A survey of previous investigations and an introduction to the present one is given in section 1. A description is given of BAADE's suggestion to determine the mean radius of a Cepheid variable from measures of brightness, colour and radial velocity. BOTTLINGER noticed that the assumption of black body radiation leads to contradictory results for ζ Geminorum. W. BECKER assumed that among Cepheids and throughout their variation there exists a single-valued relation between colour and surface-brightness. BECKER determined this relation and found it possible to derive reasonable mean radii for a number of Cepheids by means of it. In the present article it is assumed that there is a single-valued relation throughout the variations of one and the same Cepheid. This assumption, shortly called "basic assumption" assumes less than BECKER's hypothesis. The arguments in favour and against the „basic assumption" are discussed, with the conclusion that it cannot be regarded as well established. Nevertheless the consequences are investigated for δ Cephei. In section 2 the displacement of the surface of the variable as determined from radial velocities, is discussed. According to BRÜCK and GREEN the results found by some observers that different lines yield different radial velocity-curves are spurious and are due to incomplete resolution of the lines on low dispersion spectrograms. The radial velocity-curve of the preceding article is free from this criticism and refers to the moving surface of δ Cephei. Integration of that part of the Fourier development of the radial velocity that depends on the phase gives the function D. If p is a factor depending on the degree of darkening towards the limb pD is the displacement; $p = 3/2$ for uniform discs and $4/3$ for a disc darkened towards the limb according to the "cos-law". In section 3 we show the consistency of basic assumption and pulsation hypothesis for δ Cephei using the photovisual lightcurve by the writer, the photoelectric lightcurve by GUTHNICK and SMART and the radial velocity-curve by MOORE and JACOBSEN. We derive the lightcurve due to the change in area of the variable m_r and the photovisual surface-brightness as a function of the phase. The mean radius follows directly from the slope of the straight line in Figure 2. The "basic relation" is found practically linear. In section 4 we determine by least squares the value $\bar{R}/p$ and the slope of the linear "basic relation". We find $\bar{R}/p = (18{\cdot}8 \pm 1{\cdot}6) \times 10^6$ km and: photovisual surface-brightness $= (2{\cdot}32 \pm {\cdot}06) \times$ colour-index. With a coefficient of darkening $\beta = 2/3$ in the conventional form of the law of darkening: $p = 1{\cdot}4$, $\bar{R} = (26{\cdot}3 \pm 2{\cdot}2) \times 10^6$ km or 38 R(sun). The uncertainty in $\bar{R}$ due to the uncertainty in the degree of darkening is only a few per cent. The colour-index curve practically determines the weight of the solution; therefore in future work along these lines accurate determinations of colour are especially needed. We have compared the factor $2{\cdot}32$ found for the ratio of changes in surface-brightness and colour-index with the value valid for black body radiation; this is $4{\cdot}1$. From the dwarf stars Castor C and the sun we find for the same ratio 5. The mean surface-brightness of δ Cephei is $^{m}{\cdot}8$ fainter than that of the sun. This result is rather uncertain as it involves the zero-point of the period-luminosity relation. In section 5 we give the mean radius (apart from a known numerical factor) as the quotient of the areas of two closed curves. This result is less accurate than that derived above by means of least squares. In section 6 we have abandoned the "basic assumption" and with $\bar{R}/p$ as parameter a set of closed curves is given representing the relation between colour-index and surface-brightness in each case, as required by the pulsation hypothesis.

1. *Previous investigations and introduction.*

In 1926 BAADE[1]) remarked that the pulsation theory of Cepheid variation could be tested with measures of brightness, colour and radial velocity. The test, if successful, leads to a determination of the mean radius of the variable in absolute measure. The argument is essentially as follows:

If black body radiation is assumed for the radiation of the variable the surface-brightness may be computed from the observed colour, the area then follows in terms of an arbitrary unit, by dividing the observed light by the surface-brightness. Hence the radius becomes known as a function of the phase, in an arbitrary unit. On the other hand, the displacement of the star's surface may be found in kilometres by integrating the radial velocity-curve. It is clear that the radius and the displacement so determined should be in phase and that if they prove to be so the mean radius can be calculated since a known fraction is known in absolute measure.

Because of lack of suitable data at the time, BAADE did not follow up his suggestion himself, but BOTTLINGER[1]) made the first actual attempt in 1928 with observations of ζ Geminorum. The attempt was not successful, the radius and the displacement were found out of phase. Of course, no determination of the radius in kilometres could be made. BOTTLINGER rightly did not blame the pulsation hypothesis, but concluded the assumption of black body radiation to be fallacious.

It is well known from investigations on constant stars[2]), that different parts of the spectrum lead to different colour temperatures, a fact which shows

[1]) *A.N.* **228**, 359 (1926).

[1]) *A.N.* **232**, 3 (1928).

[2]) H. JENSEN, *A.N.* **248**, 15 (1933); A. J. WESSELINK, *B.A.N.* **7**, 239 (1935).

that the stars do not radiate as black bodies. Later investigations [1]) on variable stars proved the same state of things to hold for them, thus confirming BOTTLINGER's conclusion that a calculation of the surface-brightness from colour-temperatures and black body formulae is inadmissible. BAADE's suggestion cannot be carried out unless one is able to put the calculation of the surface-brightness on a sound basis which evades the assumption of black body radiation.

The problem was taken up 12 years later by WILHELM BECKER [2]), who replaced the assumption of black body radiation by the hypothesis that among Cepheids and throughout their variation, there exists a single-valued relation between colour and surface-brightness. Though this hypothesis of BECKER is much better founded and assumes less than the one of black body radiation, which is a particular case of BECKER's hypothesis, we should not consider it as proved. A more detailed discussion of BECKER's hypothesis is given below.

For the derivation of the relation between colour and surface-brightness BECKER made use of the colours at both the minimum and the maximum light and of the total amplitude of a number of Cepheids. From the few radial velocity curves that have been determined of these variables it is known that the displacements and thus the areas at the phases of minimum and maximum light are very nearly equal. BECKER concludes that the amplitudes refer to the magnitudes of the surface-brightness. Hence in the relation between colour and surface-brightness, the abscissae of the points corresponding to minimum and maximum light of the same variable are the known colours of the star at the phases of extreme light. Of the ordinates (magnitude of surface-brightness = σ) only the difference is known and equals the amplitude of the lightvariation. BECKER solves the problem of finding a curve when a number of difference-quotients are known by trial and error.

The relation thus found was subsequently used by BECKER to follow up BAADE's suggestion for a number of Cepheids of which radial velocity curves have been determined. In nearly all cases the radius (in an arbitrary unit) proved to be in phase with the displacement and reasonable values for the mean radii in kilometres were found. The uncertainty in the results must be still rather large as the relation between surface-brightness and colour was derived from data taken from many different sources and there is no doubt that the considerable inhomogeneity impairs the determination. A satisfactory determination of the relation would require the evaluation of colours and amplitudes of a number of Cepheids on a homogeneous system.

It is known from both astrophysical theory and practice that the surface-brightness is the main controlling factor for the spectral type of the constant stars[1]). On the other hand there is a close observational relation between the spectral type and the colour, which relation depends slightly on the density or the absolute magnitude [2]). It follows, that if, as far as the relation between surface-brightness and colour is concerned, the Cepheid variables may be considered as constant stars, there is a definite justification for BECKER's hypothesis.

In the present article we shall, first on the hypothesis that there is a single-valued relation between colour-index and surface-brightness throughout the variations of one and the same Cepheid variable, follow up BAADE's suggestion for δ Cephei, using the accurate data collected in the two preceding papers. The hypothesis just mentioned shall in the following be denoted shortly as "basic assumption". It assumes less than BECKER's hypothesis in so far that the relation valid for δ Cephei is not necessarily supposed to hold for other variables as well. Though still subject to criticism, of which the most severe one seems to be that the atmosphere of a variable is not in equilibrium at any moment, we shall examine its consequences [3]).

Finally, we shall consider our problem without making any assumption concerning any relation between surface-brightness and colour-index. Of course no test of the pulsation theory is possible then, nor can a determination of the mean radius in absolute measure be made. But, on the basis of the pulsation hypothesis, with the mean radius as parameter, we are able to derive a series of curves representing the surface-brightness as a function of the phase, making use of both the displacement-curve and the light-curve. Figure 4 shows the closed curves, representing the relation between colour and surface-brightness corresponding to a number of adopted values for the mean radius. The curve corresponding to the radius found on the basic assumption is (no. 3) and represents the corresponding single-valued relation between colour and surface-brightness.

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1) FRED. L. WHIPPLE, *L.O.B.* **16**, 1 (1932); WILH. BECKER, *Zs. für Aph.* **13**, 69 (1937); BECKER and STROHMEIER, *Zs. für Aph.* **13**, 317 (1937); **14**, 218 (1937); **15**, 85 (1938); **16**, 1 (1938); **17**, 137 (1939); **17**, 182 (1939).
2) *Zs. für Aph.* **19**, 249 (1940).

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1) A.S. EDDINGTON, *Internal constitution of the stars*, page 2, (1926).
2) WILH. BECKER, *Veröff. Berlin-Babelsberg* **10**, Heft 6 (1935).
3) In *Ciel et Terre* 1943, page 369, it is announced that Prof. VAN HOOF, University of Louvain, has determined the mean radii of three Cepheid variables, making use of radial velocities, colours and brightnesses. The method seems to be the same as that outlined here, but particulars are not known yet.

2. *The displacement-curve.*

Before we shall derive the displacement-curve from the radial velocity-curve derived in the preceding article, we shall discuss the evidence that the measured radial velocity refers to the moving surface of the variable. Several observers [1]) have found that the radial velocity-curves determined for Cepheid variables were dependent on the spectral line of which the Doppler shift had been measured. It was further believed that the velocity varied with the depth in the atmosphere at which the line originated [2]). From these results it is clearly not possible to derive the velocity of the surface and hence the displacement.

H. A. BRÜCK and H. E. GREEN [3]) measured a number of high dispersion spectrograms of δ Cephei taken at Cambridge, England. They were careful to select for measurement only lines that were well isolated. They did not find any dependence of the velocity on the line used and conclude that the results mentioned above are due to the measurement of poorly isolated lines on spectrograms with relatively low dispersion. The measured centre of gravity of two blended lines, as a consequence of the varying relative intensity of the components, may show shifts, which if interpreted as Doppler shifts, lead to spurious velocity-curves dependent on the line.

Another argument against the view that different lines actually would correspond to different velocities, is that the differences in displacement calculated from them are much too large to be consistent with what is known about the atmospheres of giant stars. The radial velocity-curve of the preceding article by MOORE and JACOBSEN was made with high dispersion. Its shape is in excellent agreement with the curve by BRÜCK and GREEN. We therefore conclude that it refers to the surface of the star.

If $-\bar{v}$ denotes the difference between measured radial velocity and the velocity of the star's centre ($-16{\cdot}84$ km/sec) we have: $\bar{v}$ = (variable part of Fourier development on page 90).

The velocity of the star's surface with respect to its centre, v, differs from $\bar{v}$ by an averaging factor. $\bar{v}$ is the average value over the disc of the component of v in the direction of the observer. The ratio $v/\bar{v} = p$ depends on the law of darkening towards the limb. With a law of darkening of the form $\sigma(\gamma) = \sigma(0) \times (1-\beta+\beta\cos\gamma)$, where $\sigma(\gamma)$ is the surface-brightness in a direction, making an angle γ with the normal on the surface and β is the coefficient of darkening, we have $v/\bar{v} = p = 2\,\dfrac{3-\beta}{4-\beta}$. For $\beta = 0$ (uniform disc), $p = 3/2$; for $\beta = 1$, $\sigma(\gamma) = \sigma(0)\cos\gamma$ and the star is completely dark at the limb: $p = 4/3$. The ratio $p(\beta = 0)/p(\beta = 1) = 9/8$ and thus so near to 1 that an uncertainty in the assumed law of darkening evidently does not affect the calculation of v from $\bar{v}$ to more than a few per cent. The displacement is $\int v\,dt$. If D represents $\int \bar{v}\,dt$ we have: Displacement $= pD$.

[1]) *Michigan Publications* **4** (1932).

[2]) The levels have been taken as equivalent to the heights of the same lines in the solar chromosphere.

[3]) *Mon. Not. R.A.S.* **101**, 376 (1942).

$D(\varphi)$ has been determined by integrating term for term the Fourier development for $\bar{v}$. The result of the integration is given in Table 1. The mean errors

TABLE 1

Coefficients of Fourier development of D (unit 10^6 km).

n	coeff. of $\cos 2\pi n\varphi$	coeff. of $\sin 2\pi n\varphi$	(m.e.)
1	+1·121	+·259	±·010
2	− ·223	−·115	±·005
3	+ ·080	+·055	±·003
4	− ·027	−·024	±·003
5	+ ·012	+·012	±·002
6	− ·009	−·004	±·002

of the Fourier coefficients of the integrated curve are seen to be inversely proportional to the order of the term [1]), quite otherwise as in the case of a Fourier development of a directly observed quantity, where the mean errors of all coefficients are equal.

The numerical results for $D(\varphi)$ are given in Table 3. The additive constant has been determined such that the minimum value of D equals zero. The mean error of a computed value of $D(\varphi)$ is independent of φ and is equal to $\pm 14 \times 10^3$ km, which is $6^0/_{00}$ of the total amplitude.

3. In this section we shall carry out BAADE's suggestion or rather we shall investigate in how far the "basic assumption" and the pulsation hypothesis are consistent.

The observational data for δ Cephei that have been used are:

1) the photovisual lightcurve by the writer.

Photovisual magnitude $= m$ (Table 2 and Figure 4 on page 86);

2) the photoelectric lightcurve by GUTHNICK and SMART, discussed in the preceding article.

Photoelectric magnitude $= m'$ (Table 2 on page 89 and Figure 1 on page 94);

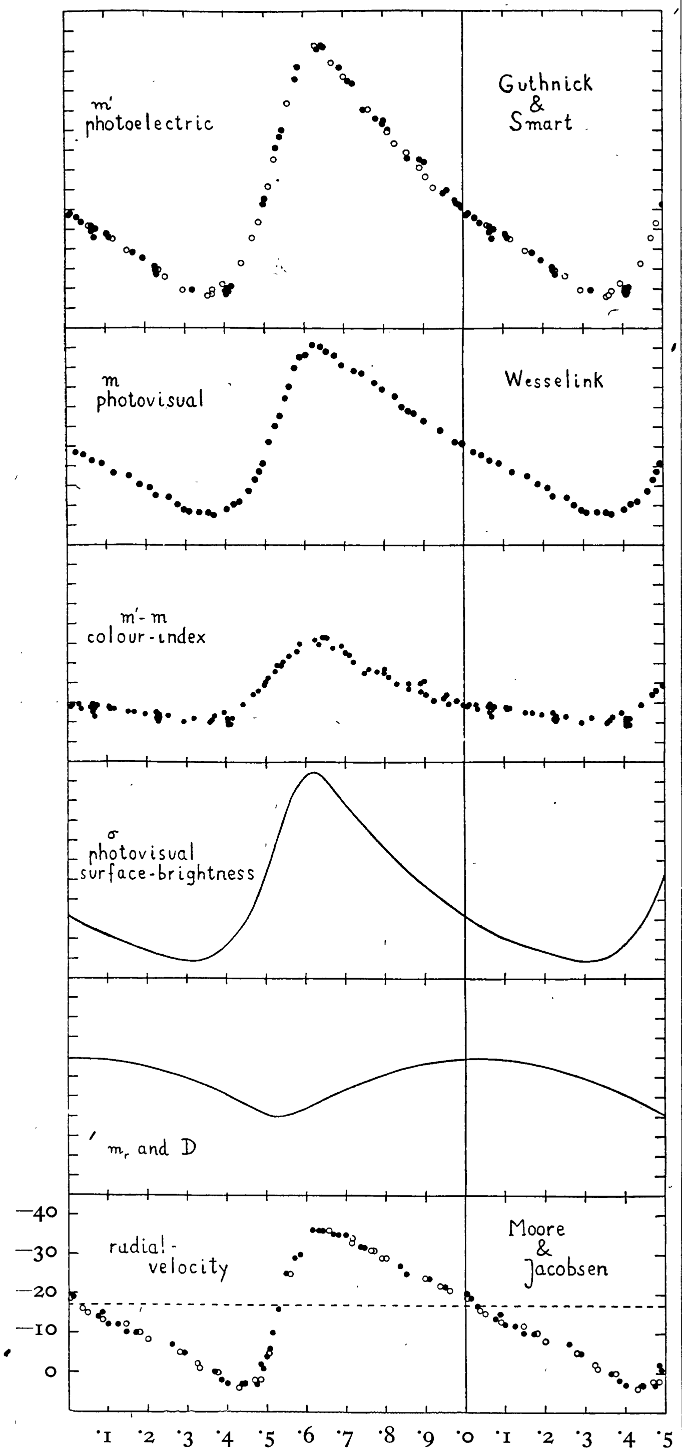
3) the function $D(\varphi) = 1/p$ times the displacement.

In Figure 1 the colour-index $m'-m$ is shown as a function of the phase. Each dot represents an observed m' minus a value m interpolated from a smoothed photovisual lightcurve at the same phase as that of m'.

We consider the two phases corresponding to some definite value of the colour-index. According to the

[1]) We do not include the constant term among the coefficients.

FIGURE 1



Abscissa is phase.
From top to bottom: photoelectric lightcurve at $\lambda_{\text{eff.}} = \mu{\cdot}44$ by GUTHNICK and SMART. Photovisual lightcurve by the writer at $\lambda_{\text{eff.}} = \mu{\cdot}55$. Colour-index curve $m'-m$. Photovisual surface-brightness σ; variation m_r as a consequence of the variation of the size of the star; the same curve represents D, the vertical dimension of the diagram being 10^6 km in that case. In all these curves one division in the ordinates corresponds to ${}^{m}{\cdot}1$. The last curve shows the radial velocity-curve by MOORE and JACOBSEN; a division in the ordinates is 10 km/sec.

basic assumption the surface-brightness is the same at these phases. The magnitudes in the lightcurves are unequal and the difference in magnitude $\Delta m = \Delta m' = \Delta\frac{m+m'}{2}$ is a consequence of the difference in size of the star at these phases, according to the pulsation hypothesis.

In fact we have:

$$\Delta m = \Delta m' = \Delta\frac{m+m'}{2} = 2\tfrac{1}{2}\ \Delta \log R^2 = 5\ \Delta \log R \doteq 5 \log e\ \Delta R/\bar{R} = 5\ \log e\ \frac{p}{\bar{R}}\ \Delta D;$$

since D is known at the same phases, we could compute $\bar{R}$ if a plausible value for p is inserted. The above mentioned procedure has been repeated for 11 different colours, and the corresponding values of $\Delta\frac{m+m'}{2}$ and ΔD have been plotted in Figure 2. The numerical data are contained in Table 2. The dots in Figure 2 appear to scatter reasonably well round a straight line through the origin showing pulsation hypothesis and basic assumption to be consistent.

The straight line through the origin corresponds to $\bar{R}/p = 18{\cdot}8 \times 10^6$ km. It is clear that the uncertainty of the colour-index curve has a great effect on the accuracy of the results. In fact we shall show that this accuracy is determined almost solely by it.

The lightcurve m_r due to the variation of the area of the disc of the variable can now be calculated. We have with sufficient approximation $m_r = -5\frac{\log e}{\bar{R}/p} D +$ const. It is shown in Figure 1 by the same diagram as D.

The photovisual surface-brightness $\sigma = m - m_r$ is shown graphically in Figure 1 [1]). We note that both σ and m_r are independent of p. We could also find the photoelectric surface-brightness as a function of the phase but it is of less interest than the photovisual one because of the variation of the effective wavelength with the phase [2]). As both the photovisual surface-brightness and the colour-index are known as functions of the phase the relation between them can be found. It is shown in the third diagram in Figure 4. The relation proves to be practically linear. Though this result could not have been anticipated and must

1) Another manner of separating m into σ and m_r is the following, analogous to the method employed by BECKER: Select pairs of phases for which the displacement (thus D) is the same. The difference in brightness is then equal to the difference in σ, according to the "basic assumption". Of the relation between σ and the colour-index, each pair of such phases furnishes two colour-indices and the corresponding $\Delta\sigma$. The complete relation between σ and the colour-index has to be found by trial and error. m_r is found from $m_r = m - \sigma$, and $\bar{R}/p$ follows from a comparison with D. The advantage of the method followed in the text is that it is straightforward and does not involve a process of trial and error.

2) Compare page 83.

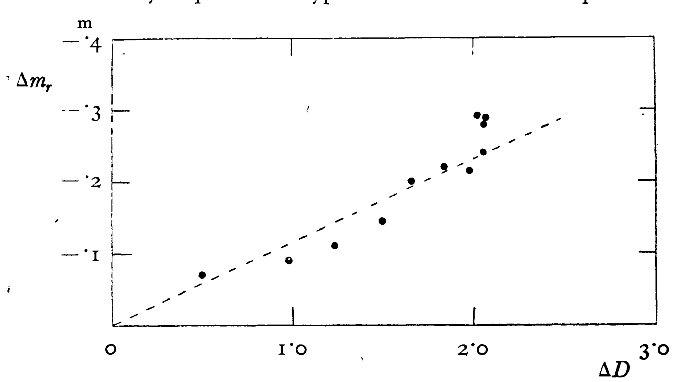
TABLE 2

Investigation of the consistency of basic assumption and pulsation hypothesis.

colour-index $m'-m-1·00$	φ_1	φ_2	m_1	m_2	m_1'	m_2'	Δm	$\Delta m'$	$\Delta(m+m')$	D_1	D_2	ΔD
m	P	P	m	m			m					
·40	·455	·960	−·16	−·46	+1·23	+·94	−·30	−·29	−·59	·44	2·46	2·02
·37	·470	·920	−·22	−·51	1·14	·87	−·29	−·27	−·56	·30	2·36	2·06
·34	·483	·891	−·27	−·55	1·08	·78	−·28	−·30	−·58	·19	2·26	2·07
·31	·497	·861	−·33	−·59	·95	·73	−·26	−·22	−·48	·08	2·14	2·06
·28	·510	·830	−·41	−·63	·87	·66	−·22	−·21	−·43	·04	2·02	1·98
·25	·520	·800	−·46	−·68	·79	·57	−·22	−·22	−·44	·01	1·85	1·84
·22	·532	·771	−·52	−·72	·72	·52	−·20	−·20	−·40	·00	1·66	1·66
·19	·544	·746	−·60	−·75	·60	·46	−·15	−·14	−·29	·01	1·51	1·50
·16	·558	·716	−·68	−·80	·47	·37	−·12	−·10	−·22	·05	1·28	1·23
·13	·570	·688	−·76	−·83	·40	·29	−·07	−·11	−·18	·10	1·08	·98
·10	·590	·651	−·84	−·89	·27	·18	−·05	−·09	−·14	·26	·76	·50

FIGURE 2

Consistency of pulsation hypothesis and basic assumption.



be considered to be quite accidental, it is of practical value as it makes the adaptation of our problem to least squares easy and hence the mean error of the mean radius can be found.

TABLE 3

Details of least squares solution for $\bar{R}/p$ and a.

Phase	m	m'	$m+m'$	D	O $m'-m$	C $m'-m$	O−C
P	m	m			m		m
·00	−·41	+1·00	+ ·59	2·51	+·41	+·40	+·01
·04	−·36	1·07	·71	2·52	·43	·42	+ 1
·08	−·31	1·12	·81	2·50	·43	·44	− 1
·12	−·28	1·16	·88	2·42	·44	·45	− 1
·16	−·24	1·22	·98	2·32	·46	·46	0
·20	−·19	1·26	1·07	2·18	·45	·48	− 3
·24	−·15	1·32	1·17	2·01	·47	·48	− 1
·28	−·10	1·38	1·28	1·81	·48	·49	− 1
·32	−·07	1·43	1·36	1·56	·50	·50	0
·36	−·06	1·43	1·37	1·29	·49	·49	0
·40	−·08	1·41	1·33	·96	·49	·47	+ 2
·44	−·13	1·30	1·17	·58	·43	·42	+ 1
·48	−·25	1·10	·85	·22	·35	·35	0
·52	−·46	·79	·33	·01	·25	·25	0
·56	−·70	·47	− ·23	·05	·17	·16	+ 1
·60	−·87	·22	− ·65	·31	·09	·09	0
·64	−·91	·18	− ·73	·66	·09	·09	0
·68	−·85	·27	− ·58	1·00	·12	·13	− 1
·72	−·79	·36	− ·43	1·32	·15	·17	− 2
·76	−·73	·50	− ·23	1·60	·23	·22	+ 1
·80	−·68	·57	− ·11	1·85	·25	·25	0
·84	−·62	·68	+ ·06	2·06	·30	·27	+ 3
·88	−·56	·75	+ ·19	2·22	·31	·32	− 1
·92	−·51	·87	+ ·36	2·36	·38	·36	+ 2
·96	−·46	·94	+ ·48	2·46	·40	·38	+ 2

4. We have $m = \sigma + m_r = \sigma - 5\,\frac{\log e}{\bar{R}/p}\,D + \text{const.}$ Because of the linearity of σ and $m'-m$ just mentioned we write $\sigma = a\,(m'-m) + \text{const.}$

Figure 4, diagram 3, shows that $a = 2·35$, but we shall introduce the quantity as an unknown in our solution. Elimination of σ between the two preceding equations yields:

$$\frac{1}{2a+1}\,(m+m') + \frac{2}{2a+1}\,\frac{5\log e}{\bar{R}/p}\,D + \text{const.} = m'-m.$$

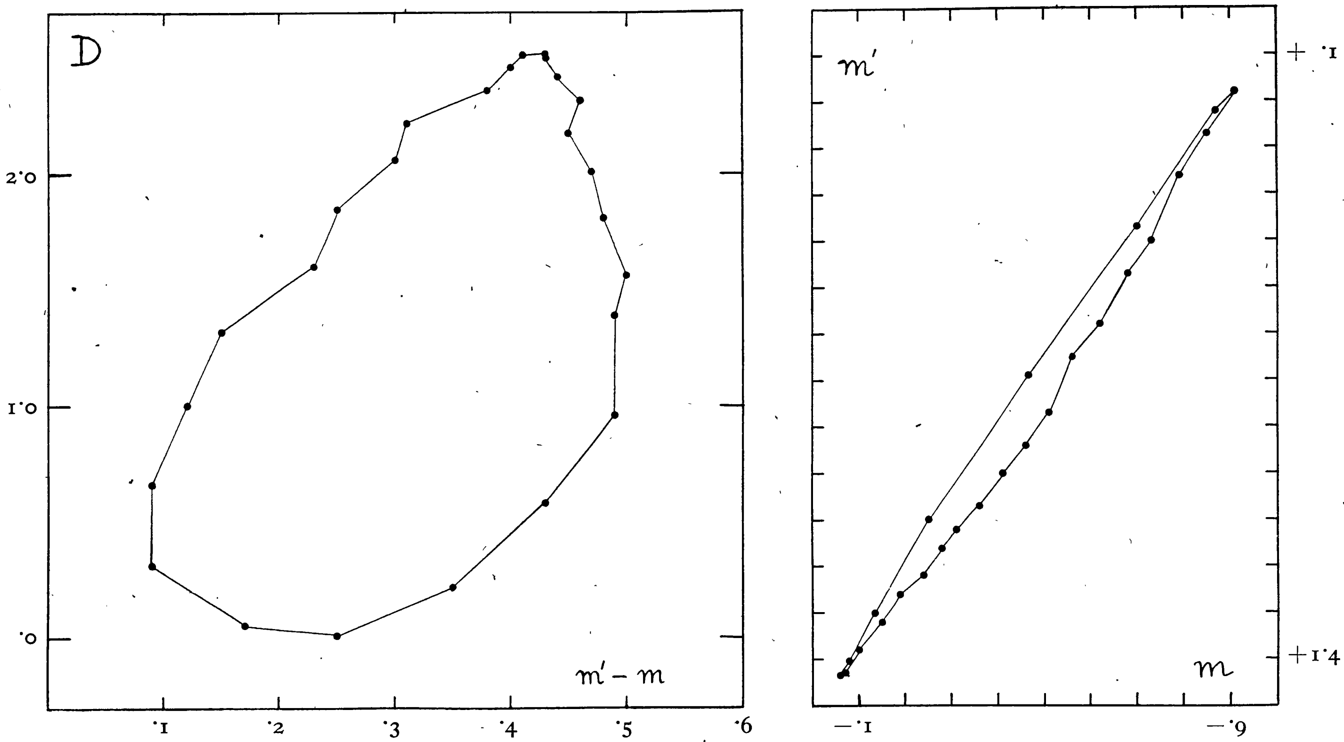
In this equation of condition, which is linear in the unknowns $\frac{1}{2a+1}$ and $\frac{2}{2a+1}\,\frac{5\log e}{\bar{R}/p}$ and the constant, the coefficients $m + m'$ and D are like the right-hand member $m'-m$ subject to accidental error. It is however perfectly legitimate to carry out the least squares solution in the normal way assuming the coefficients $m'+m$ and D to be exact[1]). This is in agreement with the argument already given before that the uncertainty in the unknowns is in practice determined by that of the colour-index alone. The values m and m' in the 25 equations of condition were interpolated at the phases ·00, ·04, ·08, ·12 etc.; D was computed from Table 1. Details of the solution are given in Table 3. As the 25 right-hand members $m'-m$ were read from a smoothed colour-index curve it follows that the corresponding equations of condition just mentioned are not independent. Correct

1) It is well known that in the theory of least squares only the right-hand members are subject to accidental errors, whereas the coefficients of the unknowns in the left-hand member are exact. Though the mean error of $m + m'$ is equal to that of $m'-m$, the values of the unknowns are such that the uncertainty in the left-hand members as a consequence of the uncertainties of both $m + m'$ and D is negligible in comparison to that of the right-hand member $m' - m$. If the equation is written otherwise, for example with either $m + m'$ or with D in the right-hand member, the uncertainty in the left-hand member is larger than that of the right-hand member. This is why the treatment indicated in the text should be followed.

An analogous problem occurs when two photometric series m and m' are to be compared. In that case one does not determine regression of m on m' or of m' on m, but one solves by least squares a set of equations of condition of the form:

$$m' - m = \mathrm{A}(m + m') + \mathrm{B}.$$

FIGURE 3



Left diagram: abscissa: colour-index $m'-m$, ordinate D. Each division in the abscissae is $^{m}{\cdot}1$, in the ordinates it is 10^6 km. Right diagram: abscissa: $m = m_{pv}$; ordinate: $m' = m_{pe}$. Each division in each co-ordinate equals $^{m}{\cdot}1$. Both curves are described clockwise during a complete period.

mean errors for the unknowns therefore cannot be obtained by the usual treatment followed when a set of independent equations of condition is solved according to the method of least squares; the effect of the dependence of the equations on the results themselves is negligible.

However correct mean errors for the unknowns may be obtained from the above solution, for which purpose it is necessary to take into account the known total weight of the colour-index curve ($190000\ m^{-2}$).

The results and their mean errors are:

$$a = 2{\cdot}32 \pm {\cdot}06 \text{ (m.e.)}$$

$$\bar{R}/p = (18{\cdot}8 \pm 1{\cdot}6) \times 10^6 \text{ km}$$

The mean error of the mean radius is thus 9% of the mean radius itself. The total variation of the radius during a period is 13% of the mean radius. With $p = 1{\cdot}4$, corresponding to a coefficient of darkening $\beta = 2/3$, the mean radius is found to be $\bar{R} = (26{\cdot}3 \pm 2{\cdot}2) \times 10^6$ km or $38\ r_\odot \pm 3\ r_\odot$ (m.e.).

A rather independent check on this computation has been made as follows. A set of six equations of condition has been written down in which the unknowns to be solved were the same as in the set just treated (apart from the constant, which is now absent). The coefficients were the coefficients in the Fourier development of some order respectively of $m + m'$ and D, whereas the right-hand member was the Fourier coefficient of the same order of $m'-m$.

The solution by least squares of the set of six independent equations of condition (corresponding to orders 1 to 6) yielded practically the same results and mean errors as found above.

The relative uncertainty in $\bar{R}$ as a consequence of the uncertainty in p is not more than a few per cent (see page 91). We have compared the mean surface-brightness of δ Cephei with the surface-brightness of the sun. With a mean absolute magnitude of δ Cephei equal to $-2^{M}{\cdot}19$ [1]) and an absolute magnitude of the sun $= +4^{M}{\cdot}73$ [2]) and the above value of the radius we find σ (δ Cephei) $^{m}{\cdot}8$ fainter than σ (sun). This result is of course rather uncertain.

According to KUIPER [3]) the absolute visual magnitude of the average component of Castor C is $4^{m}{\cdot}3$ fainter than that of the sun; $\log r/r_\odot = -{\cdot}20$ [4]). Hence σ (Castor C) is $3^{m}{\cdot}3$ fainter than σ (sun). The colour-index, in the scale used throughout this article, of the sun is $^{m}{\cdot}65$ [5]) smaller than that of Castor C.

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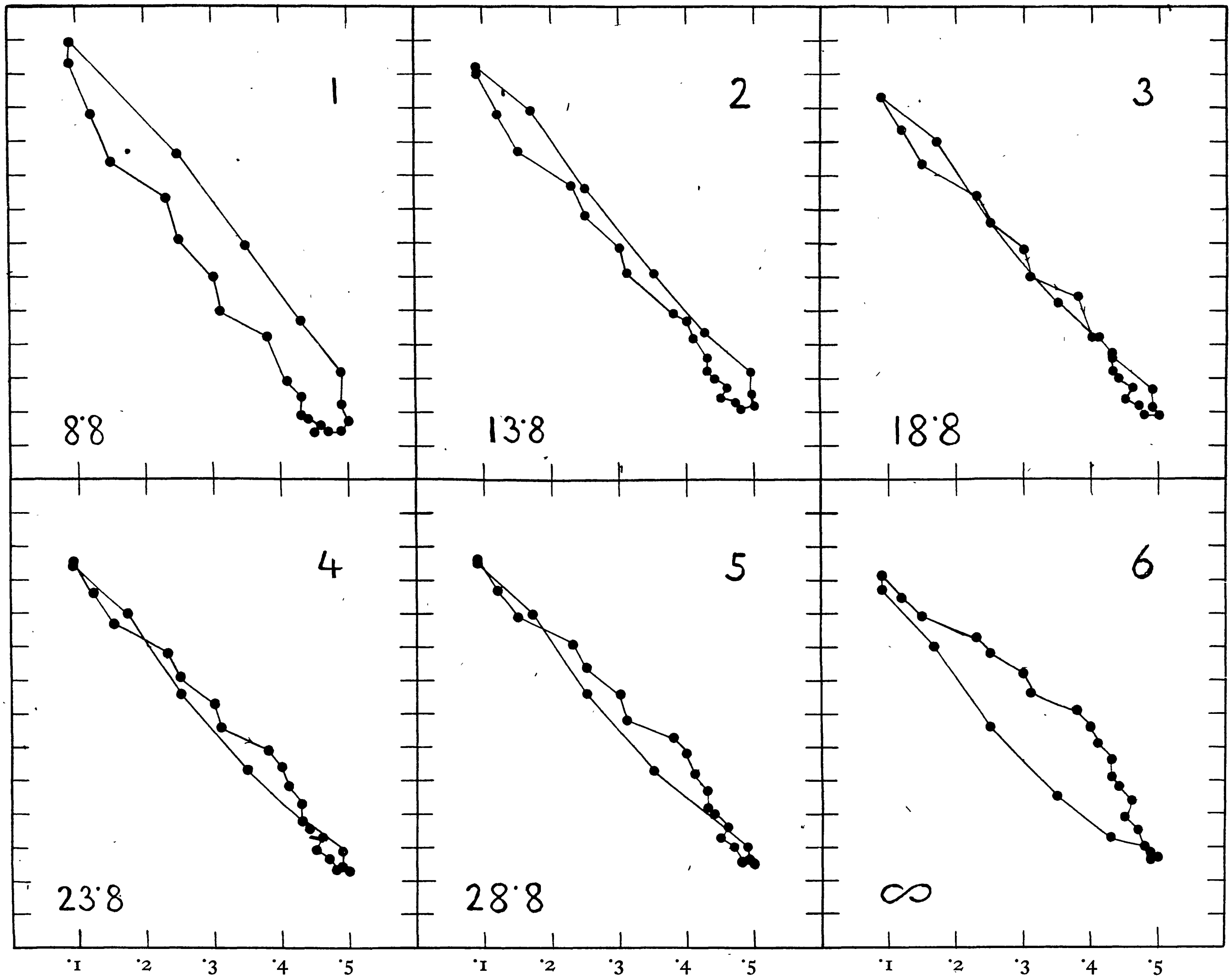
1) SHAPLEY, *Star clusters*, page 129.
2) G. P. KUIPER, *Ap. J.* **88**, 438 (1938).
3) *Ap. J.* **88**, 458 (1938).
4) *B.A.N.* **6**, 108 (1931), *Ap. J.* **88**, 458 (1938).
5) The spectra of the sun and of Castor C according to KUIPER are dG2 and K6+ respectively (see *Ap. J.* **88**, 432, 458); the corresponding colour-indices in BECKER's system are $+^{m}{\cdot}72$ and $1^{m}{\cdot}48$. The scale used in the present paper is $\cdot 85 \times$ BECKER's scale so that the difference between the colour-indices of the sun and Castor C is found to be $\cdot 85 \times (1^{m}{\cdot}48 - {}^{m}{\cdot}72) = {}^{m}{\cdot}65$.

FIGURE 4



In each of the six diagrams the abscissa is colour-index; the ordinate is the photovisual surface-brightness σ. In the lower left corner the value of $\bar{R}/p$ in 10^6 km used in the construction of the diagram is given. Diagram 3 corresponds to the basic assumption.

Hence the ratio a found from two dwarf stars is found to be 5. We conclude that the result for a for dwarf stars does not resemble our result for δ Cephei. We can also calculate a value a valid for black body radiation. When WIEN's approximation to PLANCK's law is used, differences in surface-brightness are proportional to differences in colour-index. We have a (black body) $= \frac{\lambda_{pe}}{\lambda_{pv}-\lambda_{pe}} = 4{\cdot}1$, so that for a given change in surface-brightness the colour-index of δ Cephei changes nearly twice as much as that of a black body.

5. A very simple determination of the mean radius could be made as follows, though the result has less weight than the result derived above.

In Figure 3 the photoelectric magnitude m' has been plotted against the photovisual magnitude m. We note that apart from a rotation over an angle of 45° and a factor $\sqrt{1/2}$, the diagram is identical with one in which $m + m'$ is plotted against the colour-index $m'-m$.

Dots following each other in phase have been connected by parts of straight lines. The rising and descending branches of the lightcurve correspond respectively with the upper and lower branch of the loop in Figure 3. During a cycle the closed curve is described clockwise. Lines of equal colour-index are straight lines running upwards under 45° from left to right. Each line cuts the curve in two points, the distance of which is proportional to the difference in magnitude on the lightcurve, corresponding to phases with the same colour-index; it is due to a difference in area of the stellar disc.

We have $\Delta m_r = -\frac{5 \log e}{\bar{R}/p} D$, here Δ means a difference corresponding to the same colour-index.

In Figure 3, diagram 1, D has been plotted against the colour-index.

We have

$$\bar{R}/p = \frac{\text{area of closed curve diagram 1, Figure 3}}{\text{area of closed curve diagram 2, Figure 3}} \times 5 \log e$$

This simple representation of the situation following from the basic assumption and the pulsation hypothesis does not give however the mean radius with the greatest possible precision.

6. In the preceding sections we have studied the consequences of the basic assumption and the pulsation hypothesis. We have found that the data for δ Cephei are consistent with both hypotheses. The resulting mean radius does not deviate much from independent estimates which are based on the absolute magnitude and an assumed value of the surface-brightness. However we still do not consider the basic assumption as being well founded; we cannot argue at the present state of our knowledge that the relation between colour-index and surface-brightness for a non-static star does not include the phase.

We have therefore studied our problem without making any assumption concerning the dependence of the surface-brightness on the colour-index. Of course no test of the pulsation hypothesis is then possible, nor can we derive a value for the mean radius. But on the pulsation hypothesis and with the mean radius as parameter we are able to derive a series of closed curves showing the course of simultaneous values of the colour-index and the surface-brightness during a cycle.

As the total range of D is about 2·50 (in 10^6 km) and the minimum value of D is chosen equal to zero, we have calculated $m_r(\varphi)$ for the values of the parameter $\bar{R}/p = 8{\cdot}8$, $13{\cdot}8$, $18{\cdot}8$, $23{\cdot}8$, $28{\cdot}8$ and ∞ with the formula: $m_r(\varphi) = -5 \log \{\bar{R}/p + D(\varphi) - 1{\cdot}25\}$. From the formula $\sigma = m - m_r$ we find σ.

In each of the six diagrams of Figure 4 the thus calculated values have been plotted against the simultaneous value of the colour-index; each diagram corresponds to one of the assumed values of $\bar{R}/p$. The diagram for $\bar{R}/p = 18{\cdot}8$ corresponds to the basic assumption and shows the relation between colour-index and surface-brightness in that case. The diagram for $\bar{R}/p = \infty$ is obviously simply m against the colour-index.

In Table 4 the photovisual surface-brightness as a function of the phase is given for five different assumed values of $\bar{R}/p$.

TABLE 4.

Photovisual surface-brightness as a function of the phase for five different assumed values of $\bar{R}/p$.

The zeropoint in each column is arbitrary.

phase	$\bar{R}/p = 8{\cdot}8 \times 10^6$ km	$\bar{R}/p = 13{\cdot}8 \times 10^6$ km	$\bar{R}/p = 18{\cdot}8 \times 10^6$ km	$\bar{R}/p = 23{\cdot}8 \times 10^6$ km	$\bar{R}/p = 28{\cdot}8 \times 10^6$ km
P	m	m	m	m	m
·04	+ ·21	− ·02	− ·12	− ·18	− ·22
·04	+ ·26	+ ·04	− ·07	− ·13	− ·17
·08	+ ·31	+ ·08	− ·02	− ·08	− ·12
·12	+ ·32	+ ·10	·00	− ·06	− ·10
·16	+ ·34	+ ·13	+ ·03	− ·03	− ·06
·20	+ ·36	+ ·16	+ ·06	+ ·01	− ·03
·24	+ ·36	+ ·17	+ ·08	+ ·03	·00
·28	+ ·36	+ ·19	+ ·11	+ ·07	+ ·04
·32	+ ·33	+ ·18	+ ·11	+ ·07	+ ·05
·36	+ ·28	+ ·15	+ ·09	+ ·06	+ ·04
·40	+ ·18	+ ·08	+ ·03	+ ·01	− ·01
·44	+ ·03	− ·03	− ·06	− ·08	− ·09
·48	− ·19	− ·21	− ·22	− ·23	− ·23
·52	− ·46	− ·46	− ·46	− ·46	− ·46
·56	− ·69	− ·69	− ·70	− ·70	− ·70
·60	− ·79	− ·82	− ·83	− ·84	− ·85
·64	− ·73	− ·80	− ·83	− ·85	− ·86
·68	− ·58	− ·68	− ·73	− ·76	− ·77
·72	− ·44	− ·57	− ·63	− ·67	− ·69
·76	− ·31	− ·47	− ·54	− ·58	− ·61
·80	− ·21	− ·38	− ·46	− ·51	− ·54
·84	− ·10	− ·29	− ·38	− ·43	− ·46
·88	·00	− ·21	− ·30	− ·36	− ·39
·92	+ ·08	− ·14	− ·24	− ·29	− ·33
·96	+ ·15	− ·07	− ·18	− ·24	− ·28

Note added in proof.

When this article was in press Mr VAN HOOF's paper on the same subject appeared (*Koninklijke Vlaamsche Academie voor Wetenschappen* etc. **5**, No. 12, 1943; see the footnote on page 92).

VAN HOOF's article is essentially on the same principle as has been exposed by W. BECKER, and by the writer in the present paper, and contains a discussion of the variables ζ Gem, δ Cep and S Sge.

It is of interest to compare VAN HOOF's results for δ Cephei with those obtained in this paper. The data used by VAN HOOF are: STEBBINS's visual lightcurve, WIRTZ's photographic lightcurve (*A.N.* **154**, 334, 1901) and MOORE's radial velocity-curve.

The photometric data used by VAN HOOF and by the writer are therefore entirely independent. This is not quite so for the radial velocity-curve, as the curve used by the writer is a combination of MOORE's and JACOBSEN's curves. As has been explained above the accuracy of the mean radius is determined chiefly by that of the colour-index curve and the effect of the radial velocity-curve on the result is only slight. VAN HOOF's result and that of the writer can therefore be considered to be practically independent, which