NOTE ON THE DETERMINATION OF K_z AND ON THE MASS DENSITY NEAR THE SUN [1])

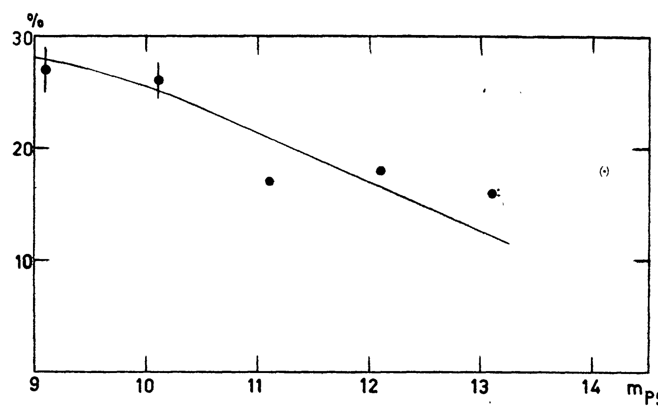
BY J. H. OORT

A force K_z has been determined which is consistent with the observations discussed in the preceding article and which at the same time fulfills the requirement that K_z must be due to the attraction by stars and interstellar matter. Our knowledge of the distribution of this attracting mass, though incomplete, is sufficient to put rather stringent conditions on K_z, so that it can be determined much more reliably than if the requirements of Poisson's law are left out of consideration. On the supposition that the z-distribution of the total stellar density is the same as that of the K giants, the variation of K_z with z was found to be like that indicated by crosses in Figure 3. The total mass density at $z = 0$ was found to be 10.0×10^{-24} g/cm^3, or 0.15 solar masses per pc^3, with an estimated uncertainty of about 10%. This density agrees well with that derived by HILL without using Poisson's equation. Its precision is considered to be rather greater. A comparison is made with the results of other recent investigations.
The ratio of mass- to light-density, M/L, in solar units, is 2.4. For the total contents of a cylinder perpendicular to the galactic plane $M/L = 4.2$. The curve in Figure 4, marked K giants, is believed to give a fair representation of the variation of the total star density with z.

As has been pointed out by HILL (1959), the result for K_z obtained in the preceding article is unsatisfactory in so far as, for $z > 500$ pc, it conflicts with the requirements set by Poisson's law. It begins to deviate from these requirements between 300 and 400 pc from the galactic plane; beyond 500 pc the deviations become quite serious. Mr HILL's aim was to derive K_z simply from the observed velocity and density distributions of K giants, without considering other conditions. In the present note an attempt is made to combine HILL's data with the condition that the gravitational force must be due to stars and interstellar matter, the approximate distribution of which may, at least partly, be supposed to be known. In other words, a solution for K_z is sought which satisfies Poisson's law, and at the same time gives a reasonable agreement with the observed data for the K giants.

I begin by reconsidering the data concerning the numbers of fainter K stars. As described on p. 12 of the preceding article, these were determined from general star counts combined with percentages of K stars derived from the *Bergedorfer Spektral Durchmusterung*. These percentages are given in Table 3 of HILL's article. As pointed out by VAN RHIJN and SCHWASSMANN (1935, p. 168) the classifications in the BSD become increasingly uncertain beyond $12^m.5$. Less weight should therefore be given to the result for $m_{pg} = 13.1$, and hardly any weight to that at $14^m.1$. The percentages given in HILL's Table 3 have been plotted in Figure 1. The vertical lines at the two brightest magnitudes are equal to twice the statistical mean errors. For the fainter stars these statistical errors are small, but systematic errors come in. If, for the reasons explained, we discard the point at $14^m.1$, the smooth curve in Figure 1 would appear to give the most plausible representation of the data. A simple estimate shows that even at $13^m.1$ the dwarfs still constitute only a small fraction (about 1/4 or 1/5) of the total number of K stars, so that the run of the curve must be largely determined by the decrease in density of K giants relative to that of G and F dwarfs. As the former are at rather greater distances from the galactic plane, their numbers must diminish more rapidly than those of the G and F dwarfs. Accordingly, the percentage of K stars must continue to drop fairly rapidly until we get to magnitudes where K dwarfs begin to preponderate—which does not take place within the limits of Figure 1. Combining this smooth curve with VAN RHIJN's total star counts as given in HILL's Table 3 and with the percentages of

FIGURE 1



Percentages of stars classified as G8-K9 in the *Bergedorfer Spektral Durchmusterung*.

giants as derived from the proper-motion distributions (HILL's Table 19), we obtain log $A(m_{pg})$ for K giants as shown in the right-hand curve in Figure 2. The curve for the brighter magnitudes is for visual magnitudes, and has been directly copied from HILL's data.

As has been pointed out by HILL, the curves have a considerable uncertainty, in particular at the faintest magnitude. In view of this uncertainty, to which should be added that regarding the frequency and

[1]) The publication of this article, written in 1957, was delayed by various circumstances. The comparison with other recent results, on p.p. 52-53, was added in 1959.

the distribution of high Z-velocities, as well as the uncertainty of the luminosity distribution of the K giants, it is clearly impossible to determine from these data the variation of K_z with z for the whole range of z with which we are concerned (from $z = 0$ to about $z = 2000$ pc). However, the condition that K_z must satisfy Poisson's equation prescribes more or less the way in which it has to vary with z. With the aid of this equation one can, in fact, deduce a fair estimate of K_z as a function of z, except, mainly, for an unknown multiplication factor. It is in the first place this multiplication factor which has to be determined from dynamical studies such as contained in the preceding article.

Poisson's equation may be used in the following form

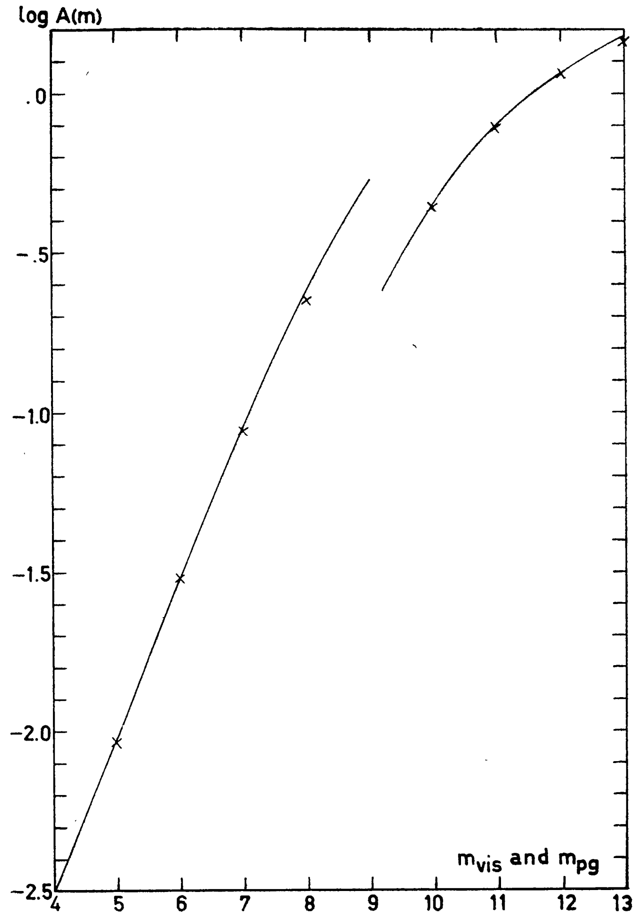
$$\frac{\partial K_\varpi}{\partial \varpi} + \frac{K_\varpi}{\varpi} + \frac{\partial K_z}{\partial z} = -\,4\pi G \rho(z)\,, \tag{1}$$

where ϖ is the distance from the galactic axis, and $\rho(z)$ the mass density. In the galactic plane the first two terms are given by

$$\frac{\partial K_\varpi}{\partial \varpi} + \frac{K_\varpi}{\varpi} = 2\,(A-B)\,(A+B)\,, \tag{2}$$

in which A and B are the constants of galactic rotation. With $A = 19.5$ km/sec.kpc $= 6.32 \times 10^{-16}$ sec^{-1}, and $B = -6.9$ km/sec.kpc $= -2.24 \times 10^{-16}$ sec^{-1} the expression becomes 0.70×10^{-30} sec^{-2}. In order to find these terms outside the plane, we need a model of the mass distribution in the Galactic System. With SCHMIDT's (1956) model we obtain the values given in the second column of Table 1. Near $z = 0$ these are slightly lower than the result given above.

FIGURE 2



Observed and computed numbers of K giants per square degree. The curve for the bright stars is for visual magnitudes, that for the faint stars for photographic magnitudes. The crosses show the values of log $A(m)$ computed with K_z and velocity distribution as found in the present article.

TABLE 1

Estimates of K_z from Poisson's equation.

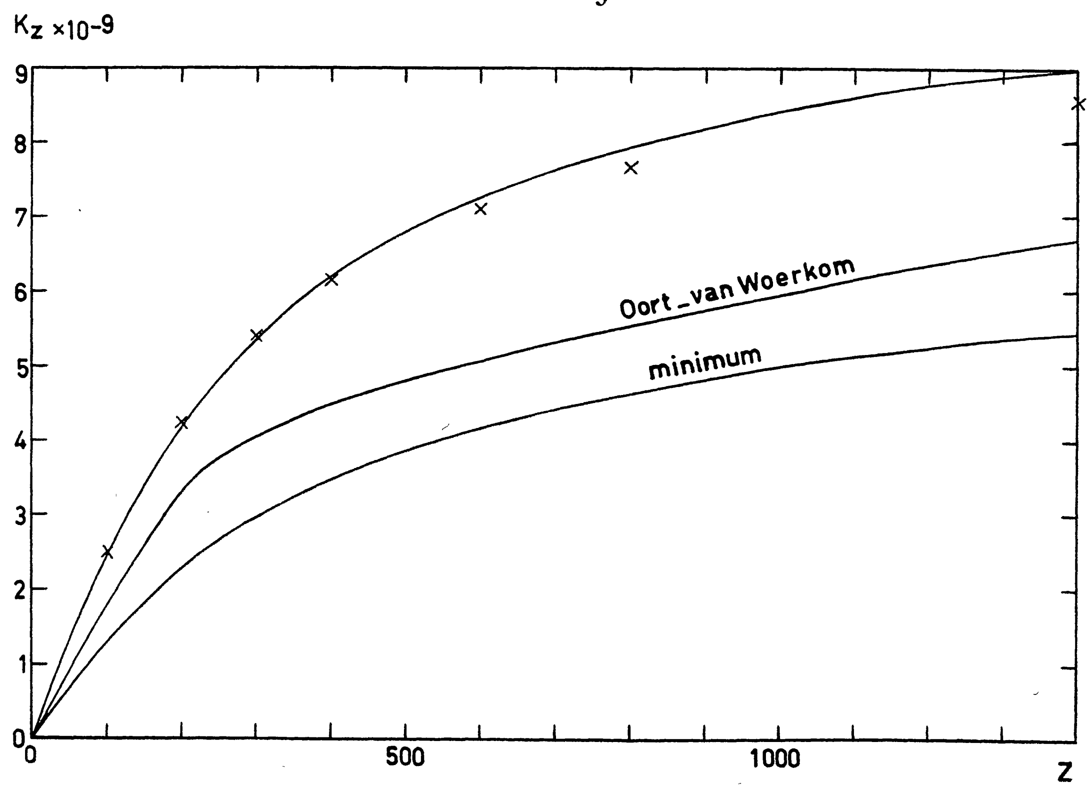
(1) z (pc)	(2) $\frac{\partial K_\varpi}{\partial \varpi} + \frac{K_\varpi}{\varpi}$ (10^{-30} sec^{-2})	(3) ρ_{gas} (10^{-24} g/cm^3)	(4) ρ_{stars}	(5) $4\pi G \int_0^z \rho_{min}\, dz$ gas (10^{-9} cm/sec^2)	(6) stars	(7) total	(8) $\int_0^z \left(\frac{\partial K_\varpi}{\partial \varpi} + \frac{K_\varpi}{\varpi}\right) dz$ (10^{-9} cm/sec^2)	(9) $K_z^{min} =$ (7) + (8) (10^{-9} cm/sec^2)	(10) K_z (a) (10^{-9} cm/sec^2)	(11) (b)	(12) (c)
0	.60	2.00	3.10								
50	.60	1.32	2.96	.22	.39	.62	.09	.71	1.08	1.33	1.58
100	.60	.87	2.83	.36	.77	1.13	.18	1.32	1.99	2.44	2.89
200	.60	.38	2.05	.52	1.39	1.91	.37	2.28	3.43	4.19	4.95
300	.58	.16	1.39	.58	1.83	2.41	.55	2.97	4.41	5.37	6.33
400	.53	.07	.94	.61	2.13	2.74	.72	3.47	5.10	6.20	7.30
600	.48	.014	.47	.63	2.48	3.11	1.05	4.16	6.03	7.27	8.51
800	.44		.28	.63	2.67	3.30	1.34	4.64	6.62	7.94	9.26
1000	.39		.17	.64	2.78	3.42	1.58	5.00	7.05	8.42	9.79
1500	.14		.074	.64	2.93	3.57	2.00	5.56	7.71	9.14	10.57
2000	.09		.043	.64	3.00	3.64	2.16	5.80	7.98	9.44	10.90
3000	−.03		.019	.64	3.07	3.71	2.19	5.90	8.13	9.61	11.09

This is of no consequence for the present purpose, since $\partial K_z/\partial z$ near the galactic plane is 8.6×10^{-30}, so that a change of 0.1×10^{-30} has a negligible effect. Near the galactic plane the uncertainty in $\frac{\partial K_\varpi}{\partial \varpi} + \frac{K_\varpi}{\varpi}$ is considerably less than that in $\frac{\partial K_z}{\partial z}$. It is only above $z = 500$ pc that the calculations become appreciably affected by uncertainties in the model.

If $\rho(z)$ were known, $\partial K_z/\partial z$, and therefore also K_z, could be derived from Poisson's equation. In reality we do not know $\rho(z)$, principally because we do not know the numbers and masses of dwarfs fainter than about +15 visual absolute magnitude. In fact, it is just this density of unknown stars that we primarily hope to determine from the data in the present article. But we do know something about the *relative* densities of common stars at different z. Such knowledge can, for instance, be derived from general star counts near the galactic poles. As a first approximation, the z-density distribution for stars with $M_{pg} > +4.5$ as derived previously (Oort 1936) has been adopted as representative for the stars in general.

We can now calculate K_z for the hypothetical case that there would be no stars fainter than about +15 absolute visual magnitude. The total density of the "ordinary" stars brighter than this limit was taken to be 0.046 solar masses per pc³, or 3.1×10^{-24} g/cm³. This value was taken from Gliese (1956). He gives an average of 0.052 solar masses per pc³ for the known stars; in the present analysis it was assumed that one third of the stars fainter than $M_v = +10$, or 0.006 ⊙/pc³, belonged to the class of Me dwarfs which according to Baade and Delhaye (1953) should be counted among the extreme population I; these were therefore counted with the interstellar gas (see below). With the relative densities quoted above, $\rho(z)$ can be computed for the stars down to $M_v = +15$. The result is shown in the fourth column of Table 1, marked ρ_{stars}. In addition there are the interstellar material and the extreme-population I objects which one may combine with the interstellar matter. For the interstellar matter a density of 1.6×10^{-24} g/cm³ was taken (cf. section 2 of the preceding article). Including the Me dwarfs the total density of material other than ordinary stars was estimated to be 2.0×10^{-24}. The density distribution of this material was assumed to be proportional to $e^{-q|z|}$, with $1/q = 120$ pc, as derived by van Rhijn (1946 and 1949). The densities so found are shown under ρ_{gas} in the third column of Table 1. Columns (5), (6) and (7) give the integrals from 0 to z of these minimum densities multiplied by $4\pi G$. The

Figure 3



The variation of K_z with z.
The upper curve shows K_z as found in the present article, case (b). An alternative solution, referred to in the text, is shown by crosses. The middle curve, derived by Oort and van Woerkom, has been added for comparison. The lower curve shows the minimum K_z as described in the text.

next column contains the integral of the numbers given in column (2). The sum of columns (7) and (8), in column (9), shows the minimum absolute values of K_z which are to be expected on the basis of the densities used. These quantities, plotted as the curve marked "minimum" in Figure 3, show the general run of K_z with z. The actual $|K_z|$ must everywhere be larger than the value given by this curve and moreover, for every z, the derivative of the actual $|K_z|$ must exceed the derivative of the minimum $|K_z|$.

The distribution of the stars which are absolutely fainter than about +15 is unknown. We shall assume provisionally that they have the same distribution as that given in Table 1 for the total density of known stars and interstellar matter (other distributions will be tested later on). On this hypothesis the true K_z may be obtained by multiplying column (7) of Table 1 by a constant factor, and then adding column (8) to it. The adequacy of the hypothesis is to be tested by investigating whether a K_z so obtained can represent the counts of K giants.

The computations were made for the following three cases:

$$(a)\quad \rho(z) = 1.60\, \rho_{min}$$
$$(b)\quad \rho(z) = 2.00\, \rho_{min}$$
$$(c)\quad \rho(z) = 2.40\, \rho_{min}$$

The corresponding values of K_z are shown in columns (10) to (12) of Table 1. With these K_z, $\Delta(z)/\Delta(0)$ for the K giants was now calculated from relation (4b) in the preceding article, using the Z-velocity distribution found in section 11. The numbers of stars per square degree for various apparent magnitudes are then found from

$$A(m) = 7.01 \times 10^{-4} \int_{\log r_0}^{\infty} \Phi(M) \frac{\Delta(r)}{\Delta(0)} r^3 \, d(\log r), \quad (3)$$

in which $r = z/\sin 59°$, and r_0 is the distance corresponding to the limiting absolute magnitude of the giants (+3.4 visual, +4.6 photographic). For $\Phi(M)$ the luminosity curves adopted by VAN RHIJN and SCHWASSMANN (1935), and reproduced in Figure 5, have been used.

The resulting values of $\log A(m)$ are given in Table 2, columns (a), (b), and (c). Comparison with the observed values in the second column of this table indicates that a density slightly lower than (b) may fairly represent the counts of K giants up to $m_{pg} = 11.0$, but that this fails to represent the counts for fainter magnitudes. In particular it appears impossible to reproduce the ratio of the number observed at $m_{pg} = 13.0$ to that at 12.0.

TABLE 2

Log $A(m)$ for K giants at an average latitude of 59° computed with various assumptions for the mass density near the galactic plane. (a): $\rho = 1.6\, \rho_{min}$; (b): $\rho = 2.0\, \rho_{min}$; (c): $\rho = 2.4\, \rho_{min}$. In each case the distribution of the unknown stars was assumed to be the same as that of the gas plus known stars with $M_{pg} > +4.5$. In the columns (a′), (b′), (c′) 1% halo population II has been added. The luminosity curve used was that given by VAN RHIJN and SCHWASSMANN.

m	obs.	(a)	(a′)	(b)	(b′)	(c)	(c′)
5.0 vis	−2.02	−2.02		−2.03		−2.04	
8.0 vis	− .62	− .60		− .65		−.71	−.70
11.0 pg	− .10	−.01	.00	−.12	.11	−.22	−.20
12.0 pg	.06	.15	.19	.00	.06	−.13	−.06
13.0 pg	.18:	.20	.31	.02	.16	−.18	−.01

Although the observed numbers are uncertain, especially for $m_{pg} = 13$, it appears unlikely that there should be so large an error in the ratio of the counts at 13.0 and 12.0. The excess of the observed increment over the computed values indicates the presence of a class of K giants with still higher average velocity than the high-velocity group in HILL's Table 14. There is rather convincing evidence from other data of the existence of such a component among main-sequence stars, as may be seen from the general counts of faint stars. In Table 3 these counts are given for 17.0 and 18.0 photographic magnitude, at 80° galactic latitude. The observed values in the second column were taken from *B.A.N.* No. 238 (OORT 1932), after provisional corrections had been applied for systematic errors in the Mt Wilson magnitude scale. These corrections were kindly supplied in

TABLE 3

Comparison of observed and computed numbers of faint stars at 80° latitude

m_{pg}	log $A(m)$ obs.	log $A(m)$ computed S_1	S_3	S_4
17.0	2.17	1.65	1.79	2.20
18.0	2.37	1.72	1.85	2.35

advance of publication by Dr BAUM. The numbers in the further columns were computed from formula (3), in which the integral was extended from $-\infty$ to $+\infty$ and $\Phi(M)$ was taken from an investigation by VAN RHIJN (1936). The densities were derived with the aid of K_z as found in the present article (the curve defined by the crosses in Figure 3) and three different Z-velocity distributions of the form $\sum_i \theta_i \frac{l_i}{\sqrt{\pi}} e^{-l_i^2 Z^2}$, the parameters of which are as follows:

	S_1	S_3	S_4
$l_1 = .046$	$\theta_1 = .90$	$\theta_1 = .85$	$\theta_1 = .90$
$l_2 = .020$	$\theta_2 = .10$	$\theta_2 = .15$	$\theta_2 = .08$
$l_3 = .010$			$\theta_3 = .02$

They are practically identical with the velocity distributions used in *B.A.N.* No. 238 (p. 270 and Table 25).

It is clear that the numbers computed with S_1 fall far short of what is observed. They will presumably be too small for any K_z compatible with Poisson's law and the observed densities. We see from column S_3 that no reasonable increase in the proportion of the component with moderately high velocities (i.e., the component with $l = 0.020$, corresponding to $\overline{|Z|} = 28$ km/sec) can give an important improvement. The only way in which the computed numbers can be sufficiently increased, is by adding a component of much higher velocity, as has been done in S_4.

The above calculations appear to give sufficient proof of the existence of a component with very high velocity dispersion among the main-sequence stars. As the K giants have probably evolved from main-sequence stars, it is reasonable to assume that they likewise contain such a component of "halo population II" stars. Only a very small percentage of this group is required to remove the discrepancy encountered in Table 2; so small that it would hardly make itself felt in the velocity distribution near the galactic plane. In fact, with the amount of velocity data at present available, the only way of observing such an admixture of halo population II is by studying counts of faint stars. Assuming, somewhat arbitrarily, a Gaussian distribution with an average Z-velocity of 48 km/sec—twice that of the high-velocity component of Table 14 in the preceding article—it is found that the difference between the observed and calculated increment between m_{pg} 12.0 and 13.0 disappears if 1% of this group is added to the velocity distribution at $z = 0$; this 1% has been taken from the ordinary high-velocity group, the fraction of which is thereby reduced to 0.14. The effects of the change may be seen in columns (a′), (b′), (c′) of Table 2, giving log $A(m)$ as computed with the amended velocity distribution. There is now an almost perfect agreement of (b′) with the observed values. The fact that all computed values are about 0.02 too small may be attributed to a slight error in the luminosity curve, an error which is well within the uncertainty of this curve. For case (b′) log $A(m)$ has likewise been computed for other magnitudes. The results, shown by crosses in Figure 2, agree satisfactorily with the observed curves. The force K_z corresponding to cases (b) and (b′) is shown graphically in Figure 3 (upper curve).

We have thus found that, with the slight extension of the velocity distribution discussed, a density of 2.0 ρ_{min} represents adequately the numbers of K giants up to $m_{pg} = 13.0$, while the density distribution used satisfies the conditions set by Poisson's law.

Table 4 gives an impression of how the stars with which we are principally concerned are distributed in distance.

Since $\rho_{min} = 5.1 \times 10^{-24}$, the actual density at $z = 0$ becomes 10.2×10^{-24} g/cm^3, or 0.151 solar masses per pc^3. In a previous investigation (1932) I had derived 6.3×10^{-24} g/cm^3. As may be seen from Figure 4 of that publication, where K_z was taken to increase linearly with z up to $z = 200$ pc, this value must be considered as representing the average density in a layer extending to 200 pc from the galactic plane. The density which was found above refers strictly to the density at $z = 0$. With the aid of the data in columns (3) and (4) of Table 1 the average density for the layer between $z = 0$ and $z = 200$ is found to be 0.84 times that at $z = 0$, or 8.6×10^{-24} g/cm^3. This is 1.36 times the value I found in *B.A.N.* No. 238.

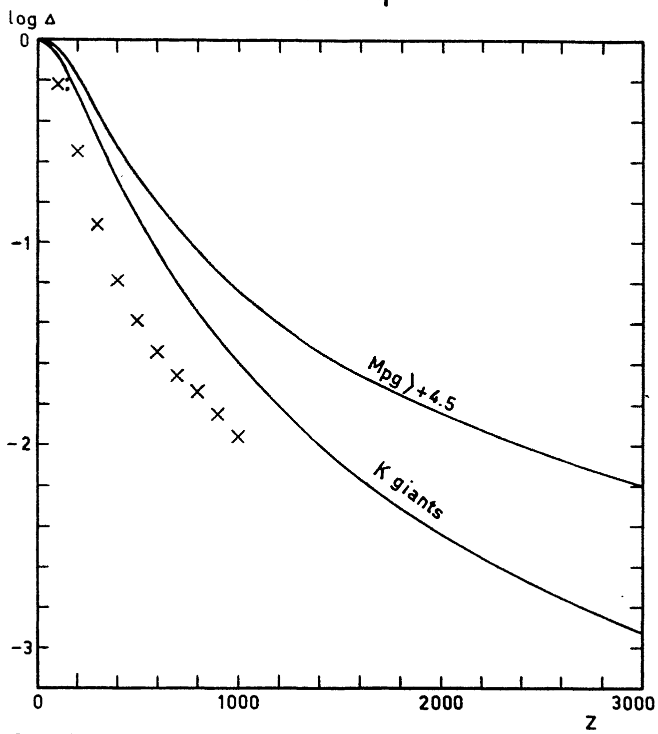
TABLE 4

Distance distribution of K giants of m_{pg} 11.0 and 13.0 at 59° galactic latitude.
Numbers of stars per square degree per interval of 0.2 in log r, solution (b′).

log r	z (pc)	$m_{pg} = 11.0$ M	$A(m)$	$m_{pg} = 13.0$ M	$A(m)$
2.4	215	3.8	.03		
2.6	341	2.7	.21		
2.8	541	1.7	.42	3.7	.11
3.0	857	0.7	.10	2.7	.48
3.2	1360	−0.3	.01	1.7	.63
3.4	2150	−1.3	.001	0.7	.12
3.6	3410			−0.3	.01

It is of interest to form an estimate of the uncertainty in the mass density at $z = 0$ which may result from the uncertainty of the density distribution of the known and unknown stars. In the above computation it was assumed that the unknown stars have the same distribution as the minimum mass density of gas plus stars. It is perhaps more reasonable

FIGURE 4



The density distribution of K giants as determined in the present article and the mass distribution of stars used in the first approximation.
Crosses indicate the densities, in a somewhat arbitrary unit, derived by ELVIUS from spectrophotometric observations.

to assume that their distribution resembles that of the known *stars* alone. In that case the factor by which the minimum *star* density must be multiplied in order to obtain a good representation of $A(m)$ is 2.30, and the *total* mass density near $z = 0$ becomes 9.1×10^{-24} g/cm^3, instead of 10.2×10^{-24} as found above.

The density distribution of K giants corresponding to the K_z of case (b) and to the amended velocity distribution is shown in Figure 4, together with the density distribution for stars with $M_{pg} > +4.5$ as computed in *B.A.N.* No. 290, which was used in the above calculations of K_z. There is evidently a considerable difference between the two curves; at $z = 1000$ pc the relative density indicated by the upper curve is 2.2 times higher than that now found for the K giants. We should not conclude that there is a real difference in distribution of giants and dwarfs. For the densities of the stars with $M_{pg} > +4.5$ were based on rather poor data for the Z-velocity distribution, combined with a K_z considerably smaller in absolute value than the K_z found in the present paper. The density distribution of K giants in Figure 4 should be much more reliable.

For comparison, I have also plotted in Figure 4, as crosses, the densities found for giant G and K stars by ELVIUS (1951, Table 29). The point at $z = 100$ has a low weight; a still more uncertain point near $z = 25$ has been omitted. Apart from a difference in zero point — which was arbitrary in ELVIUS' table — the agreement between the crosses and the curve is quite satisfactory.

Since it does not appear unreasonable to assume that the distribution of the stars in general resembles that of K giants, an alternative solution was made in which the stars (known and unknown) were supposed to be distributed like the K giants, while the distribution of the gas (including stars of extreme population I) was left as before, with a total density at $z = 0$ of 2.0×10^{-24} g/cm^3. In this case the factor by which the minimum star density must be multiplied in order to obtain a good fit with the observed K-giant counts comes out as 2.58. The corresponding *total* mass density in the galactic plane is 10.0×10^{-24} g/cm^3. Comparing this with the value of 9.1×10^{-24} derived for the corresponding case computed with the *upper* curve in Figure 4, one notes that the very considerable change in the relative density distribution used has brought about only 10% change in the absolute density. The final K_z is still less sensitive to these changes, as may be seen from Figure 3, where the crosses indicate the values found in this last solution, while the curve just above the crosses shows the results of the first solution.

The values of $\log A(m)$ for the K giants as computed from the two alternative solutions just described are shown in columns (3) and (4) of Table 5. The representation of the observed counts is equally good in the two cases. In both cases the amended velocity distribution was used (i.e., the velocity distribution in which 1% halo population II stars was added). The slight excess of the observed numbers of bright stars over the computed values that was already noted in Table 2, appears to persist.

As the most likely total density derivable from the present material one may adopt $\rho(0) = 10.0 \times 10^{-24}$ g/cm^3. This is somewhat higher than what might have been expected from an extrapolation of the known stars. Using data largely taken from GLIESE's (1956) recent discussion of near-by stars, I arrive at a total of 5.9×10^{-24} g/cm^3. The recent data concerning the large optical thickness of some interstellar clouds (MULLER 1957 and 1959) may bring about an increase in the estimates of the average density of interstellar material by a factor of perhaps 1.5, which would bring the total to 6.7×10^{-24}. About one third of the total density would thus remain unexplained.

For the neighbourhood of the sun the above data indicate a value of 2.4 for M/L, where M is the total mass in units of the mass of the sun and L the total light expressed in the photographic luminosity of the sun. L was computed from VAN RHIJN's luminosity curve (1936). An estimation of the same ratio in a cylinder perpendicular to the galactic plane gives

TABLE 5

Effect of different assumptions concerning the mass distribution and $\Phi(M)$ on the computed numbers of K giants.
In columns (3) and (4) the total stellar mass density was assumed to be distributed like the upper and lower curve in Figure 4, respectively. The values in column (5) were calculated with a changed luminosity curve.

(1) m	(2) log $A(m)$ observed	(3)	(4)	(5)
		log $A(m)$ computed		
5.0 vis	−2.02	−2.03	−2.03	−2.03
8.0 vis	− .62	− .64	− .66	− .65
11.0 pg	− .10	− .11	− .11	− .08
12.0 pg	.06	.05	.06	.08
13.0 pg	.18:	.15	.18	.18

$M/L = 4.2$. Here it was assumed that the distribution of stellar mass was like that of the K giants in Figure 4, an estimate of the total amount of interstellar matter being added. The integrated light was computed from the contribution of the star light to the surface brightness at high latitudes as estimated by VAN RHIJN (1929; cf. also *B.A.N.* No. 238, p. 285 [1])). It is of interest to compare the value of M/L for the cylinder with that observed in the Andromeda nebula. With the aid of the rotational velocities resulting from recent measures of the 21-cm line SCHMIDT (1957) finds $M/L = 27$ for distances between 3 and 16 kpc from the centre. For comparison with the above results for the Galactic System the value of L should be corrected for absorption within the nebula. There are no numerical data on this absorption, but at an inclination of 77°.7 it must be considerable. DE VAUCOULEURS (1958) has estimated that L would have been reduced by $0^m.6$; the true value of L/M, according to him, would be 13.5. This is an average between spiral arms and inter-arm regions. In an arm the ratio would be lower, but only slightly so. There may thus be a real difference between the ratio M/L in the Andromeda nebula and that in the Galactic System.

We must still investigate whether the small admixture of halo-population II stars required by the above solutions does not conflict with the data on the Z-velocity distribution as discussed by HILL (cf. his section 11). The only comparison which is seriously influenced by the halo population II is that with EDMONDSON's data for faint stars at $z = 1000$ pc. With the K_z and velocity distribution used in solution (b′) one obtains for $z = 1000$ pc, $\overline{|Z|} = 28.8$ km/sec. From EDMONDSON's data HILL found an average Z-velocity of 24.1 km/sec ±1.3 (p.e.). The difference is larger than one would have wished. However, upon closer inspection it appears likely that the discrepancy is due to an overestimate of the distance. This may be seen from the following calculation.

The average magnitude of the K giants observed by EDMONDSON and used in the above result for $\overline{|Z|}$ is 10.6 pv, corresponding with $m_{pg} = 11.8$. Using the distance distribution computed from the K_z derived in the present paper (solution (b′) in Table 2) and the velocity distribution given in the preceding article (Table 14) amended by adding 1% stars with $l = 0.010$, one finds for $m_{pg} = 12.0$ the following fractions in the various velocity groups:

strong-line	.015
weak-line	.250
high-velocity	.577
halo pop. II	.158

The average velocity corresponding to this mixture is $\overline{|Z|} = 25.3$ km/sec. A similar computation for $m_{pg} = 11.0$ gives $\overline{|Z|} = 21.3$ km/sec. For $m_{pg} = 11.8$ one thus obtains 24.5 km/sec, which is very close to the observed value of 24.1 km/sec. This shows that the new velocity distribution is in good agreement with EDMONDSON's velocities for distant stars.

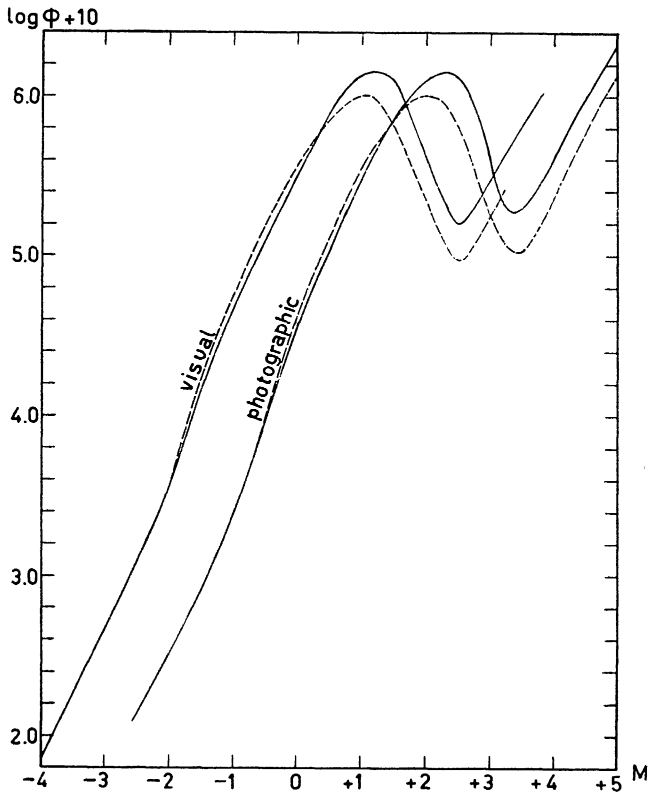
It is not surprising that the true average distance from the galactic plane is considerably lower than that obtained by HILL. The latter was computed on the assumption that the absolute magnitudes were equal to those found by MORGAN, KEENAN and KELLMAN from a study of relatively bright stars. Even within the restricted group of K giants the average intrinsic brightness decreases for fainter apparent magnitudes, so that the luminosity calibration derived from the bright stars cannot be directly applied to the fainter ones. For instance, from the data in the present section we find that the average visual absolute magnitude of the K giants for $m_{vis} = 5.0$ is 0.2, while for $m_{pg} = 12.0$ we obtain $M_{pg} = 2.1$, corresponding to $M_{vis} = 0.9$.

It is of interest to inquire what influence the uncertainty of $\Phi(M)$ has on the determination of the density and of K_z. To this end the computations of $A(m)$ have been repeated with an altered $\Phi(M)$, as indicated by dashed curves in Figure 5. The alterations were made in such a way as to leave the counts of the brightest stars practically unchanged. In order to get well-determinable differences the changes introduced are somewhat larger than the uncertainty which may be expected to exist.

If the density distribution of the unknown stars is supposed to be like that assumed in our first solution for the stars, with the gas added, as for the computations (a′), (b′), (c′) given in Table 2, and the velocity distribution is again extended by adding 1% halo population II, one finds that the observed giant counts can be represented by taking $\rho = 1.6\,\rho_{min}$.

[1]) In that article the unit of light was taken to be a star of +6.0 photographic magnitude. We have changed this to units of a star of +5.26 absolute magnitude, corresponding to KUIPER's photographic absolute magnitude of the sun.

FIGURE 5



Luminosity curves for K stars.
The full curves show the luminosity curves adopted by VAN RHIJN and SCHWASSMANN. The dashed curves are alternative luminosity curves used to investigate the effect of the uncertainty in $\Phi(M)$ on the computation of $A(m)$.

This gives $\rho(0) = 8.2 \times 10^{-24}$ g/cm³, i.e., 20% smaller than the value found by using the original $\Phi(M)$. The values of log $A(m)$ found in this case are shown in the last column of Table 5. A similar change in the opposite direction can, of course, be brought about by an alteration of $\Phi(M)$ in the inverse sense.

From this example we may estimate that the actual uncertainty of $\rho(0)$ due to the inaccuracy of $\Phi(M)$ is of the order of 10%. This uncertainty applies systematically to the entire K_z.

Comparison of K_z in Figure 3 with that derived in the preceding article illustrates the great uncertainty still attaching to the *direct* determination of K_z for distances beyond a few hundred parsecs from the galactic plane. For $z < 400$ pc there is fair agreement, as indicated in Table 6. The large differences beyond $z = 400$ pc are caused by my using a different representation of the K-star counts to 13^m and omitting the counts for 14^m as being too unreliable, while, furthermore, I have added an extension of the velocity distribution to higher velocities. But the real justification for a run of K_z as given in Figure 3 and the last column of Table 6 is that, beside giving an adequate representation of the observations, it satisfies Poisson's law, and is, therefore, consistent. By imposing this condition we greatly increase the precision with which K_z can be determined. The principal remaining uncertainty is due to the assumption, basic for the above discussion, that the average mass density of stars varies with z in the same manner as the density of K giants. However, I estimate that the uncertainty resulting from this assumption is unlikely to be greater than about 10% in the mass density and in K_z.

The value of the mass density at $z = 0$ found in the present investigation may be compared with that determined by HILL, as well as by other investigators. HILL's result is 0.13 solar masses per pc³. The agreement with my value of 0.15 solar masses is quite satisfactory, especially since HILL's result was derived from stars of 9.0 visual magnitude and brighter, while mine is mostly based on stars between 11.0 and 13.0 photographic magnitude (cf. Table 2). However, except for the addition of the halo-population II component, the same Z-velocity distribution and luminosity function were used in the two investigations.

TABLE 6

Comparison with K_z as found in the preceding article

z (pc)	K_z in units of 10^{-9} cm/sec²	
	preceding article	present article
50	1.36	1.37
100	2.75	2.50
200	5.14	4.28
300	6.08	5.40
400	6.30	6.17
600	5.70	7.10
800	4.48	7.65
1000	3.77	8.05

Other determinations of the mass density, or of $\partial K_z/\partial z$ near the galactic plane, have recently been made by KUZMIN (1955), NAHON (1957), WOOLLEY (1957) and EELSALU (1958). I wish to refer in particular to the three most recent studies.

NAHON uses the densities derived by ELVIUS (1951) from a photometric and spectrophotometric study of stars in a number of KAPTEYN's Selected Areas. He combined ELVIUS' densities with Z-velocity dispersions which he derived from the material contained in WILSON's catalogue of radial velocities. The result can be described by the value he finds for $-K_z$ at $z = 100$, viz. 4.3×10^{-9} cm/sec²; the corresponding density at $z = 0$ is 0.23 solar masses per pc³. The values derived in the present paper are 2.5×10^{-9} cm/sec² and 0.15 solar masses per pc³. NAHON's results are 1.7 and 1.5 times higher. The difference is probably mainly due to the higher velocity dispersions assumed by NAHON. For the K giants NAHON finds $\overline{Z}^2$ about 550, while HILL's velocity distribution with the addition of 1% halo population II gives $\overline{Z}^2 = 364$. The two values differ by a factor 1.51, which would just about reduce NAHON's density to mine. The higher dispersion found by NAHON may perhaps be due to his inclusion of velocities of qualities *c* and *d* in WILSON's catalogue and by his neglecting the selection effects for the fainter stars in that catalogue. HILL's determination has rather more weight, because it includes EDMONDSON's velocities of faint K giants as well as results from space velocities.

WOOLLEY's investigation is based on the density distribution and velocity dispersion of A-type stars. As these stars are considerably more concentrated to the galactic plane than the K stars, this determination of the density may be considered to be largely independent of that discussed in the present article. WOOLLEY obtains 0.18 solar masses per pc^3 at $z = 0$. He states that he has not applied a correcting factor to take account of the fact that the radial velocities used are not entirely perpendicular to the galactic plane. Strictly, this should be corrected for. If we apply the factor 0.90 given in *B.A.N.* No. 238, Table 2, the density is reduced by $(0.90)^2$, so that the corrected value becomes 0.15 solar masses per pc^3; this happens to agree exactly with the result found in the present paper.

EELSALU's result deviates widely from ours in a direction opposite to NAHON's; the same holds for KUZMIN's earlier determination. EELSALU derives the quantity $\partial K_z/\partial z$ for $z = 0$. Reduced to units of sec^{-2} he finds 4.7×10^{-30}, while my value is 8.9×10^{-30} sec^{-2}, i.e., 1.89 times higher. It would require a lengthy discussion to trace with certainty the cause of this difference, but whatever this cause is, it would appear that the weight of the result derived in the present article is considerably higher than EELSALU's. His value is based on counts of stars between 10.0 and 11.0 photographic magnitude, all spectral types being taken together. He uses a luminosity curve derived from an investigation by ÖPIK, OLMSTED, MAULBETSCH and BARNES (1933). Combined with Z-velocity dispersions derived separately for various spectral and luminosity groups these data yield the quantity $\partial K_z/\partial z$ for $z = 0$. The star counts, which are for regions between 25° and 90° latitude, must refer to regions at about 200 pc from the galactic plane. At this distance the logarithmic density difference with $z = 0$ is of the order of 0.2 (cf. Figure 4). A relatively small error in magnitude scale, or in the reduction of visual to photographic magnitudes, has a large relative effect on this density difference. Although the author has taken considerable care to eliminate errors as far as possible, it must still be feared that his result has a large percentage uncertainty. In order to illustrate how serious this uncertainty can be, I have compared EELSALU's star numbers between m_{pg} 10.0 and 11.0 for the regions above $b = 40°$ with those given by VAN RHIJN (1929). The latter are 1.38 times lower, corresponding to a decrease in $\log \Delta$ by 0.14. This is of the same order as the entire density difference on which EELSALU's determination is based; with VAN RHIJN's star counts $\partial K_z/\partial z$ would therefore have come out higher by a factor of the order of 2. Similar uncertainties in the magnitudes will exist in the material used for the present derivation of K_z. But, for K giants of $m_{pg} = 13.0$ near the galactic poles, the logarithmic mean distance from the galactic plane is around 1000 pc (cf. Table 4). At this distance $\log \Delta$ has dropped to -1.6. With such a range even an uncertainty of 0.2 in $\log \Delta$ causes an error of only about 12% in the mass density at $z = 0$.

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